ANTENNAS AND MICROWAVE ENGINEERING
(3-2 ECE, R20, JNTUA)

Prepared By

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UNIT-1(ANTENNA BASICS &WIRE ANTENNAS)

Syllabus: Antenna Basics &Wire Antennas: Definition of antenna, Radiation Mechanism – single wire, two wire, dipoles, Antenna Parameters - Radiation Patterns, Main Lobe and Side Lobes, Beam widths, Beam Area, Radiation Intensity, Beam Efficiency, Directivity, Gain and Resolution, Aperture Efficiency, Effective Height and length, Radiation from Small Electric Dipole, Quarter wave Monopole and Half wave Dipole – Current Distributions, Field Components, Radiated power, Radiation Resistance, Loop Antennas - Introduction, Small Loop, Comparison of far fields of small loop and short dipole, Radiation Resistances and Directives of small and large loops (Qualitative Treatment), Arrays with Parasitic Elements - Yagi - Uda Arrays, Folded Dipoles & their characteristics

DEFINITION OF ANTENNA

Antenna is defined as follows: Antenna is basically a wire or current carrying conductor.

(1) A device which converts electrical signal into Electromagnetic waves or vice versa.

(2) A transitional device or transducer which converts electrical energy into EM wave energy

(3) A device which converts single dimensional signal into three dimensional signal

RADIATION MECHANISM

Radiation mechanism of single wire antenna:

Conducting wires are material whose prominent characteristic is the motion of electric charges and the creation of current flow. Let us assume that an electric volume charge density, represented by q_v(coloumbs/m^3), is distributed uniformly in a circular wire of cross sectional area 'A' and volume 'V', as shown in figure below.

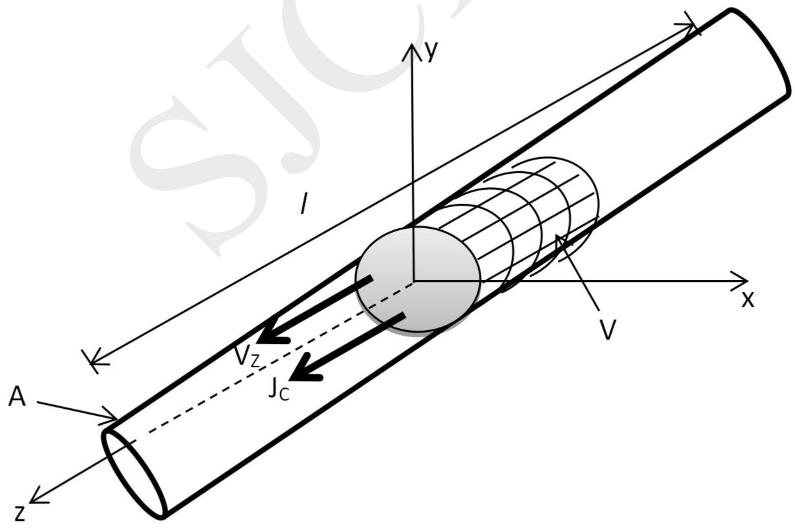


Figure 1: Charge uniformly distributed in a circular cross section

The total charge Q within volume V is moving in the z direction with a uniform velocity v_z(meters/sec). The current density J_z(ampere/m^2) over the cross section of the wire is given by

$$J_z = q_v v_z \qquad -\ (1)$$

If the wire is made of an ideal electric conductor, the current density J_s(ampere/m) resides on the surface of the wire and it is given by

$$J_s = q_s v_z \qquad -\ (2)$$

Where q_s (coulombs/m^2) is the surface charge density. If the wire is very thin(ideally zero radius),
Then the current in the wire can be represented by

$$I_z = q_l v_z \qquad - (3)$$

Where q_l (coulombs/m) is the line charge density.
Instead of examining all three current densities, we will primarily concentrate on the very thin wire. The conclusions apply to all three. If the current is time-varying then the current of equation 3 can be written as

$$\frac{dI_z}{dt} = q_l \frac{dv_z}{dt} = q_l a_z \qquad - (4)$$

Where $dv_z/dt = a_z$ (meters/sec^2) is the acceleration. If the wire is of length 'l', then equation 4 can be written as

$$l\frac{dI_z}{dt} = lq_l \frac{dv_z}{dt} = lq_l a_z \qquad - (5)$$

From equation 5, the following points can be observed:
(i) If a charge is not moving, current is not created and there is no radiation.
(ii) If charge is moving with a uniform velocity:
(a) There is no radiation if the wire is straight, and infinite in extent
(b) There is radiation if the wire is curved, bent, discontinuous, terminated, or truncated as shown in figure 2.
(iii) If charge is oscillating in a time-motion, it radiates even if the wire is straight.

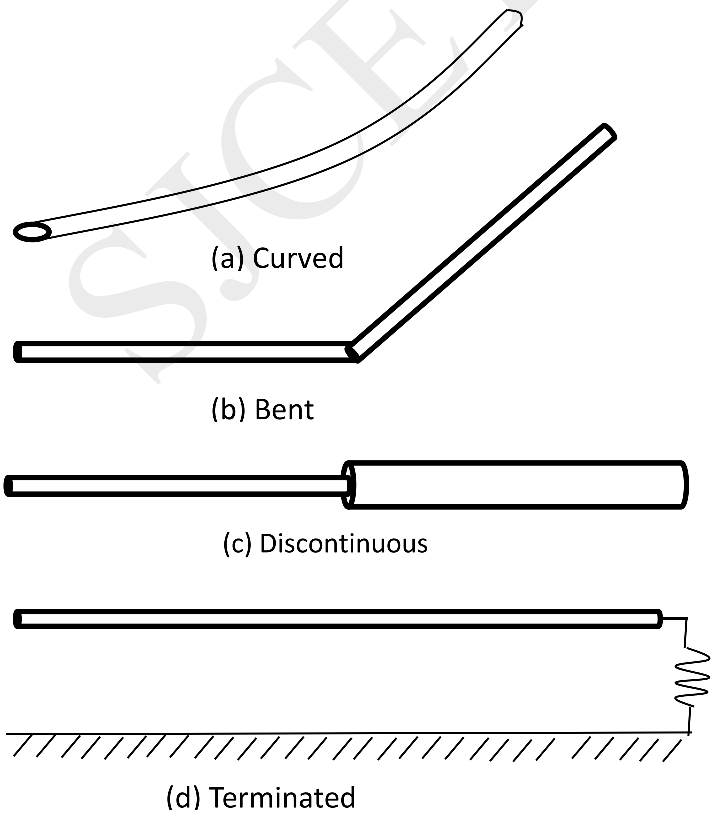


Figure 2: Wire configuration for

<u>Radiation mechanism of two-wire antenna:</u>

The two wire transmission line can also act as an antenna. To understand this let us consider the two-wire transmission line which is shown in the figure below.

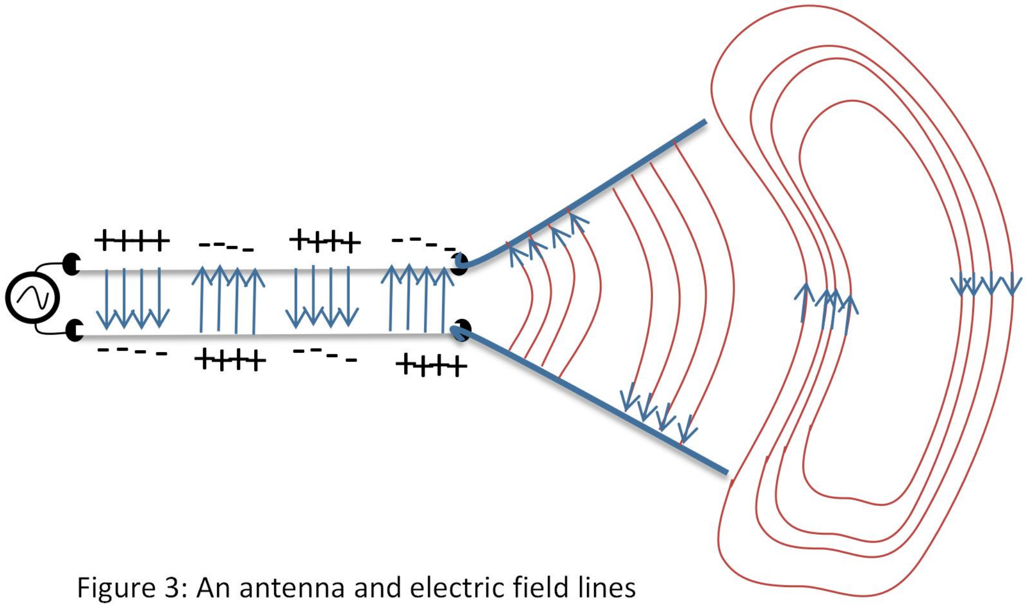


Figure 3: An antenna and electric field lines

The transmission line act as an antenna when its second port is open circuited. In case of transmission line the EM waves will present in between the two wires as shown on the figure. These EM waves will enter in to the free space through the open circuit in the form of radiation. To have the impedance matching between the transmission line and the free space, and to avoid the diffraction, the transmission line is tapered at the second port. Therefore the antenna is a transition device or transducer between a guided wave and the free space wave or vice versa.

Dipoles(Oscillating Electric dipole or short dipole):

When the charge is moving with uniform velocity along the straight conductor, it does not radiate. But a charge moving back and forth in simple harmonic motion along the straight conductor, then it radiates because there is acceleration. The dipole is nothing but a current carrying conductor contains two poles or polarities as shown in the figure 1.16 below. The short dipole is nothing but a small current carrying conductor with length less than 0.1λ.

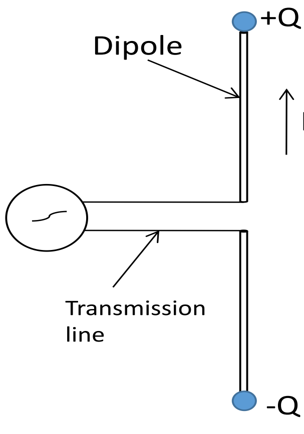


Fig1.16: Short dipole

To illustrate the radiation from dipole antenna, let us consider the dipole of figure 1.17 which has two equal charges of opposite sign oscillating up and down in harmonic motion with instantaneous separation *l (*maximu separation l_0) while focusing attention on the electric field. For clarity single electric field line is shown in the figure.

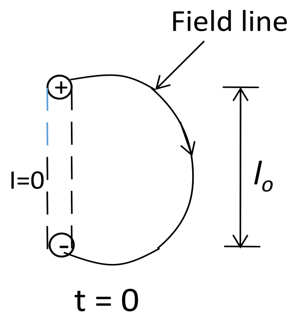


Fig 1.17(a): Electric filed line with charges at ends of dipole

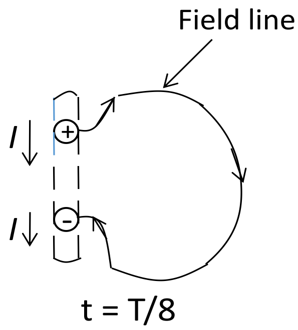


Fig 1.17(b): Wave front Moves out as charges go on

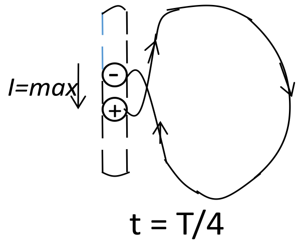


Fig 1.17(c): As the charges Pass the midpoint the field Lines cut loose

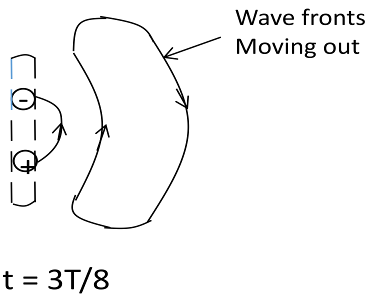


Fig 1.17(d): Wave front detached from the antenna

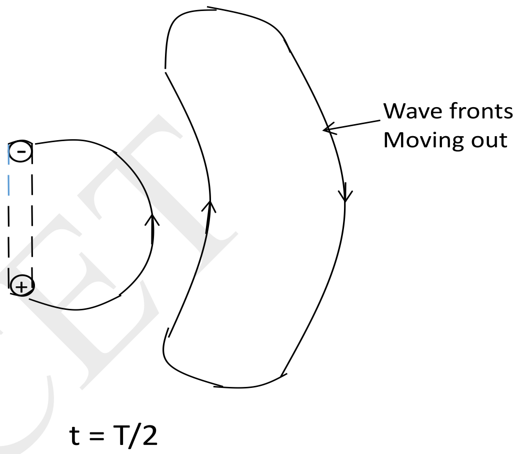


Fig 1.17(a): Wave front detached from the antenna

At time t=0, the charges are at maximum separation and undergo maximum acceleration (Fig1.17 a). At this instant the current I is zero. At 1/8 period later, the charges are moving toward each other (Fig1.17b) and at ¼ period they pass at the midpoint (Fig1.17c). As this happens, the field lines detach and new ones of opposite sign are formed. At this time the equivalent current I is maximum and the charge acceleration is zero. As time passes to a ½ period, the fields continue to move out as in fig1.17d and e.

ANTENNA PARAMETERS

Radiation Patterns:

The radiation pattern is defined as the graphical representation of radiation properties with respect to the space coordinates. The radiation properties are radiation intensity, field strength and phase or polarization. When the radiation pattern is expressed in terms of the field strength then it is called field pattern or when it is expressed in terms of power then it is called the power pattern. The radiation pattern place very important role in analyzing the performance of any practical antenna.

The examples of radiation patterns are

(i) Omnidirectional or broadcasting pattern

(ii) Pencil beam pattern

(iii) Fan beam pattern

(iv) Shaped beam pattern

In addition to these, there are other pattern shapes like Limacon, Cardioid, figure of eight or doughnut shape. The radiation pattern produced by the dipole is shown in the following figure1.1.

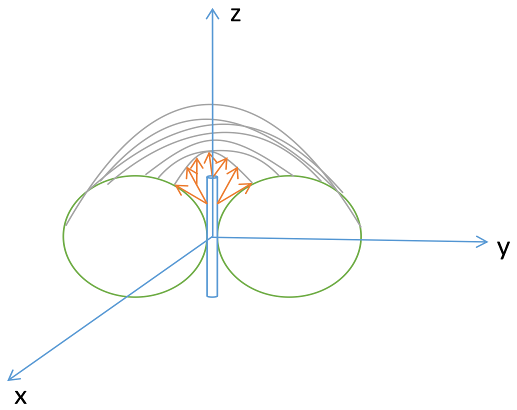


Fig1.1a: Half of the three dimensional pattern (Doughnut shape)

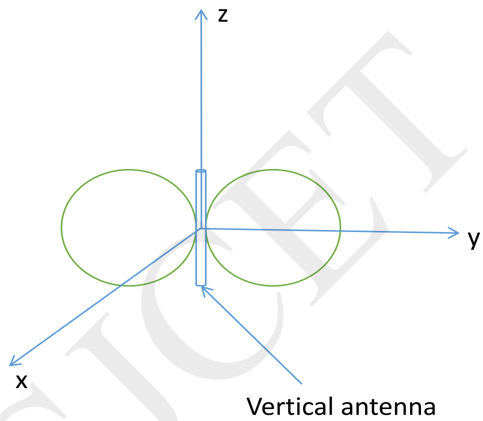


Fig1.1b: Two dimensional pattern obtained by cutting the three dimensional pattern with vertical plane along the axis of the dipole.

Main Lobe and Side Lobes(Radiation pattern lobes):

Different parts of radiation pattern are referred as the lobes. Various lobes are shown in the figure 1.4 and figure1.5 below.

Major lobe: A lobe which contains the direction of maximum radiation is called the main lobe or major lobe. The radiation pattern which contains a single major lobe is called the unidirectional radiation pattern. Whereas the radiation pattern which contains two major lobes is called the bidirectional radiation pattern.

Minor lobe: Any lobe except the main lobe is called the minor lobe. That means other than the major lobes the remaining lobes are called the minor lobes. Practically the minor lobes should be eliminated to improve the efficiency of any antenna.

Side lobe: It is a minor lobe which is existing in any direction other than the intended direction. Normally the side lobe is adjacent to the main lobe and occupies the hemisphere in the direction of the main lobe.

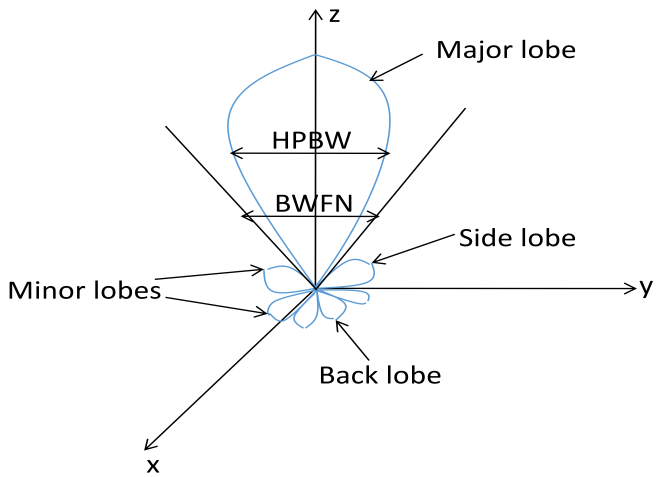


Fig1.4: Radiation pattern lobes (three dimensional)

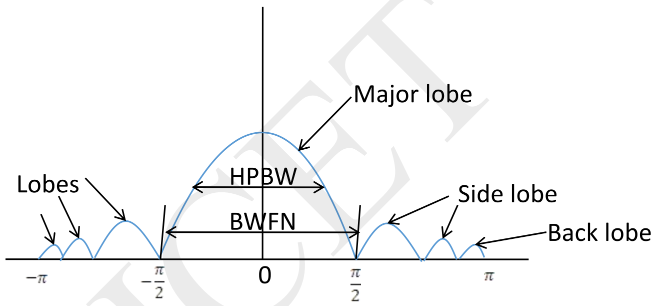


Fig1.5: Linear plot of radiation pattern (two dimensional)

Back lobe: It is a minor lobe which occupies the hemisphere in a direction exactly opposite to the main lobe.
Minor lobes are the small radiation lobes which represents the radiation in the undesired direction. In the figures 1.4 and 1.5, the terms HPBW represents the Half Power Beam Width whereas BWFN represents the Beam Width between the First Nulls. These two terms will be defined in the later sections.

Patterns in the principal planes:
The performance of any antenna can be described in terms of its patterns in the principal planes. Basically there are two types of patterns in the principal plane such as E-plane principal pattern and H-plane principal pattern. These two patterns are shown in the figure1.3 below.

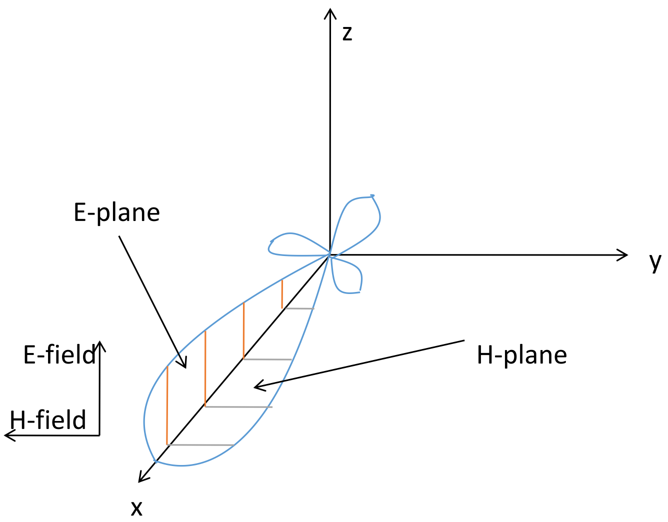


Fig1.3: E-plane and H-plane principal patterns

The radiation pattern of any antenna contains two planes such as E-plane and H-plane. The E-plane principal pattern is defined as the plane containing the electric field vector and direction of maximum radiation. Similarly the H-plane pattern is defined as the plane containing magnetic field vector and the direction of maximum radiation.

Beam widths:

In case of antennas regarding the beam width we need to define two parameters such as Half Power Beam Width (HPBW) and Beam Width between First Nulls (BWFN). These two parameters are represented in the figure 1.13.

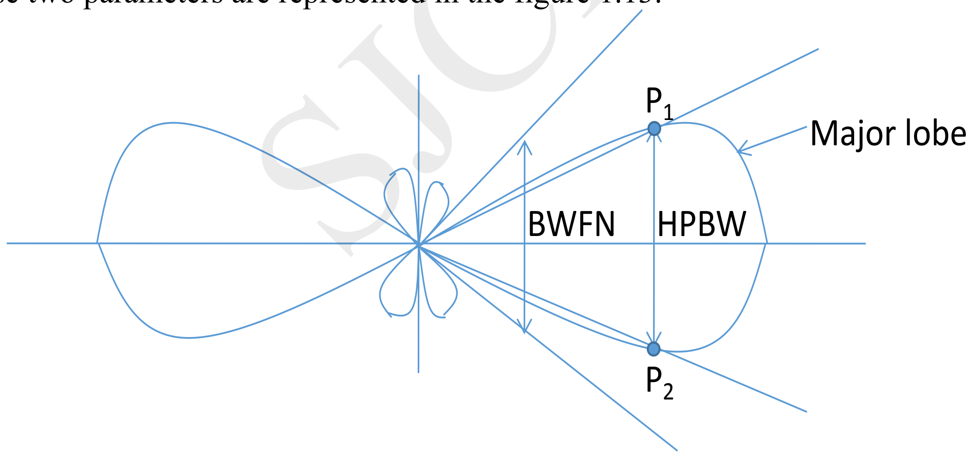


Fig 1.13: Beam Width of an antenna

The antenna beam width is the angular width in degrees measured on the main beam. The half power beam width is defined as the width of the major lobe in between the two half power points P_1 and P_2 as shown in the figure. The half power points are the points at which the power is half of the maximum value. Also the HPBW is defined as the width of the main lobe at which the electric field strength is 0.707 times or $1/\sqrt{2}$ times the maximum value. The BWFN is defined as the width of the major lobe in between the first nulls. The BWFN is approximately twice the HPBW. The directivity (D) is related with beam solid angle (dΩ) or beam area (B) as

$$D = \frac{4\pi}{d\Omega} = \frac{4\pi}{B}$$

The beam width place very important role in finding the direction of transmitting antenna. Especially the narrow beam is used in direction finding applications.

Beam Area:

Beam area is nothing but a beam solid angle. The solid angle along the radius direction is given by

$$d\Omega = \frac{ds}{r^2} = \frac{r^2 \sin\theta d\theta d\emptyset}{r^2} = \sin\theta d\theta d\emptyset$$

The beam area or beam solid angle of an antenna is given by the integral of the normalized power pattern over a sphere (4π Steradians)

$$\Omega_A = \iint P_n(\theta, \emptyset) d\Omega$$

The beam area of an antenna can be often be described approximately in terms of the angles subtended by the half power points of the main beam in the two principal planes. Thus,

$$\text{Beam Area} \cong \Omega_A \cong \theta_{HP}\emptyset_{HP}$$

Where θ_{HP} and $\emptyset_{HP}$ are the half power beam widths (HPBW) in the two principal planes, minor lobes being neglected.

Radian and steradian:

The unit of plane angle is radian. The radian is defined as the plane angle with its vertex at the centre of a circle of radius ‘r’ that is subtended by an arc whose length is ‘r’ as shown in the figure1.6 below.

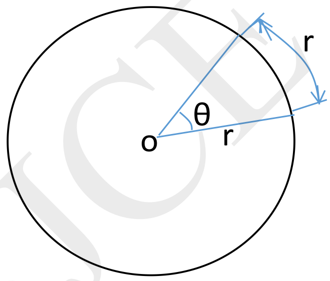


Fig1.6: Radian

The number of radians in a complete circle is given by

$$\text{Total no.of radians} = \frac{Circumference\ of\ circle}{arc\ length} = \frac{2\pi r}{r} = 2\pi$$

Therefore in the total number of radians in a complete circle is 2π.

Similarly the unit of solid angle is steradian. The steradian is defined as the solid angle with its vertex at the centre of sphere of radius ‘r’ that is subtended by the spherical surface of area which is equal to the area of square with each side of lenght ‘r’ as shown in the figure1.7 below.

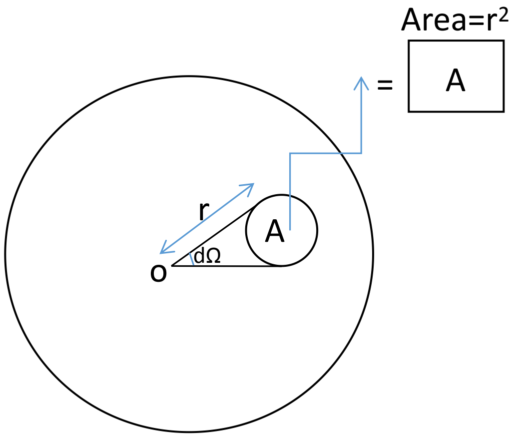


Fig1.7: Steradian

The total number of steradians in a complete sphere is given by

$$\text{Total no.of steradians} = \frac{\text{Area of the sphere}}{\text{differential surface area}} = \frac{4\pi r^2}{r^2} = 4\pi$$

$$d\Omega = 4\pi$$

Therefore in a complete sphere there are 4π steradians. The number of steradians in a sphere along the r-direction is given by

$$d\Omega = \frac{ds}{r^2} \qquad - (1)$$

But $ds = r^2 \sin\theta d\theta d\emptyset$

Therefore $$d\Omega = \frac{r^2 \sin\theta d\theta d\emptyset}{r^2} = \sin\theta d\theta d\emptyset \qquad - (2)$$

Radiation Intensity:

The radiation intensity is defined as the power per unit solid angle. The radiation intensity is represented with ϕ. Therefore

$$\Phi = \frac{\text{power}}{\text{unit solid angle}} \qquad - (1)$$

The unit of radiation intensity is watts/steradian.

We know that the poynting vector is

$$P = E \times H \text{ watts/m}^2 \qquad - (2)$$

The average poynting vector is given by

$$P_{av} = \frac{1}{2}(E \times H^*) \qquad - (3)$$

$$\text{But } \frac{E}{H} = \boldsymbol{\eta}$$

The pointing vector becomes

$$P = \frac{E^2}{\eta} \qquad - (4)$$

And the average pointing vector or radial component of pointing vector becomes

$$P_{av} = P_r = \frac{1}{2}\frac{E^2}{\eta} \qquad - (5)$$

The radiation intensity in terms of P_r is given by

$$\Phi = P_r \cdot r^2 \qquad - (6)$$

$$\text{Since } r^2 = \frac{ds}{d\Omega}$$

Substitute equation 5 in equation 6

$$\Phi = \frac{1}{2}\frac{E^2(\theta,\emptyset)}{\eta} \cdot r^2 \text{ watts/steradian}$$

The radiation intensity can also defined as

$$\Phi = \frac{\text{differential power}}{\text{differential element of solid angle}} = \frac{dW_r}{d\Omega}$$

$$\Phi = \frac{dW_r}{d\Omega} \qquad - (7)$$

$$dW_r = \phi d\Omega$$

Take integration on both sides

$$\int dW_r = \int \phi d\Omega$$

$$W_r = \int \phi d\Omega \qquad - (8)$$

The above equation represents the power radiated by the antenna. In case of isotropic radiator the solid angle is 4π (because the shape of the radiation pattern due to the isotropic radiator is sphere and hence the 4π is total steradians present in the sphere) and the radiated power becomes

$$W_r = \int 4\pi\phi$$

$$W_r = 4\pi \int \phi$$

$$W_r = 4\pi\phi_{av}$$

Where $\phi_{av} = \int \phi$

Therefore $\phi_{av} = \frac{W_r}{4\pi}$ - (11)

The above equation is called the average radiation intensity.

Beam Efficiency:

The antenna beam efficiency is defined as the ratio of main beam area (Ω_M) to the total beam area (Ω_A) i.e.

$$\text{Beam Efficiency(BE)} = \frac{\text{Main beam area}}{\text{Total beam area}} = \frac{\Omega_M}{\Omega_A}$$

The total beam area(Ω_A) = Ω_M+Ω_m

Where Ω_m is called the minor lobe area.

$$\Omega_A = \Omega_M + \Omega_m$$

Divide on both sides with Ω_A , Then

$$\frac{\Omega_A}{\Omega_A} = \frac{\Omega_M + \Omega_m}{\Omega_A}$$

$$1 = \frac{\Omega_M}{\Omega_A} + \frac{\Omega_m}{\Omega_A}$$

$$1 = \text{BE} + \text{Stray factor}$$

Therefore Beam Efficiency (BE) = 1-Stray factor.

To have the higher antenna beam efficiency always the stray factor must be as low as possible.

Directivity:

The maximum directive gain is nothing but directivity. It is denoted with D. The directivity is defined in the following ways:

$$\text{D} = \frac{\text{Maximum radiation intensity from the test antenna}}{\text{Average radiation intensuty from the test antenna}}$$

$$\text{D} = \frac{\text{Maximum radiation from the test antenna}}{\text{Radiation intensity from the reference antenna}}$$

$$\text{D} = \frac{\text{Power radiated from the test antenna}}{\text{Power radiated from the reference antenna}}$$

Gain:

In general the terms like Gain, Directive gain, Power gain, Directivity having the similar meaning. Theoretically these terms having slightly different definitions but practically all the terms are same. The gain or directivity Gain or directivity or power gain is represents the ability of transmitting antenna to radiate in to a certain direction or the ability of the receiving antenna to receive the signal from the certain direction.

Specifically the gain will be defined as follows:

$$\text{Gain(G)} = \frac{\text{Maximum radiation intensity from the test antenna}}{\text{Maximum radiation intensity from the reference antenna}}$$

And

$$\text{Gain(G)} = \frac{\text{Maximum power received by the test antenna}}{\text{Maximum power received by the reference antenna}}$$

Resolution:

The resolution of an antenna may be defined as equal to half the beamwidth between first nulls(FNBW/2). For example an antenna whose pattern FNBW = 2^0 has a resolution of 1^0 and accordingly, should be able to distinguish between transmitters on two adjacent satellites in the Clarke geostationary orbit separated by 1^0.

Antenna efficiency:

The antenna efficiency is defined as the ratio of radiated power to the power input supplied to the antenna. That is

$$\eta = \frac{\text{Power radiated}}{\text{Power input to the antenna}}$$

$$\eta = \frac{W_r}{W_T} = \frac{W_r}{W_r + W_l} \qquad - (1)$$

When the loss power(W_l) is neglected , then the antenna efficiency will be 100%.

Multiply numerator and denominator of equation 1 with $\phi(\theta,\varphi)$

$$\eta = \frac{W_r}{W_T} \times \frac{4\pi\phi(\theta,\emptyset)}{4\pi\phi(\theta,\emptyset)} = \frac{4\pi\phi(\theta,\emptyset)}{W_T} \frac{W_r}{4\pi\phi(\theta,\emptyset)}$$

$$\eta = G_P \cdot \frac{1}{G_d}$$

$$\eta = \frac{G_P}{G_d} \qquad - (2)$$

The antenna efficiency can also expressed in terms of radiation resistance(R_r) and loss resistance(R_l) as follows:

We know that

$$W_r = I^2 R_r \text{ and } W_l = I^2 R_l \qquad - (3)$$

Substitute above equation in equation 1

Then

$$\eta = \frac{W_r}{W_r + W_l} = \frac{I^2 R_r}{I^2 R_r + I^2 R_l} = \frac{I^2 R_r}{I^2 (R_r + R_l)}$$

$$\eta = \frac{R_r}{R_r + R_l} \qquad - (4)$$

Aperture Efficiency:

Aperture is nothing but area. The Physical aperture is nothing but physical area of the antenna which depends upon the physical dimensions of the antenna. The ratio in between the maximum effective area and the physical area is nothing but a absorption ratio or aperture efficiency and is given by

$$\gamma = \varepsilon_{ap} = \frac{A_{em}}{A_p}$$

Effective area or effective aperture or capture area:

The effective area or effective aperture or capture area is defined as the ratio of the power received to the poynting vector of the incident field. That is

$$A_e = \frac{\text{Power received}}{\text{Poynting vector of the incident wave}}$$

$$A_e = \frac{W}{P} \quad m^2 \qquad - (1)$$

To derive the equation for the effective area let us consider the following figure 1.8

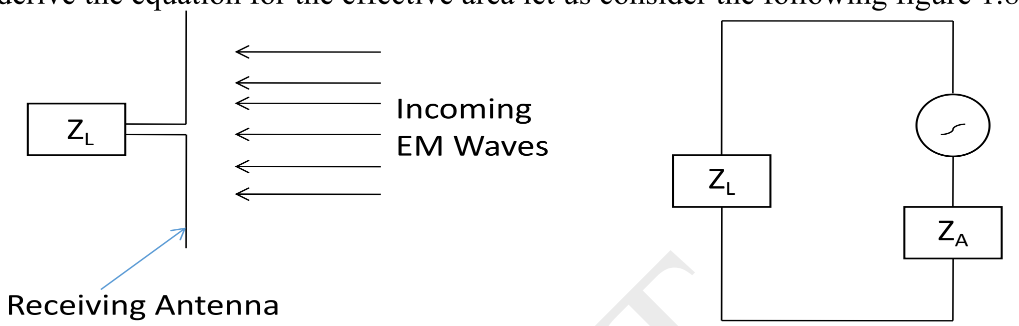


Fig1.8a: Receiving antenna   Fig1.8b: Equivalent circuit

Let I be the current flowing in the receiving antenna due to the incident EM waves, then the power received is given by

$$W = I_{rms}^2 R_L \qquad - (2)$$

To satisfy the maximum power transfer theorem only load resistance (instead of load impedance) is considered in the above equation.

Substitute equation 2 in equation 1

$$A_e = \frac{I_{rms}^2 R_L}{P} \qquad - (3)$$

From the equivalent circuit we have

$$I_{rms} = \frac{V}{Z_L + Z_A} \qquad - (4)$$

Where $Z_L = R_L + jX_L$ called the load impedance

And $Z_A = R_A + jX_A$ called the antenna impedance

$$\therefore \quad I_{rms} = \frac{V}{(R_L + jX_L) + (R_A + jX_A)}$$

$$I_{rms} = \frac{V}{R_L + jX_L + R_A + jX_A}$$

$$I_{rms} = \frac{V}{(R_L + R_A) + j(X_L + X_A)}$$

$$|I_{rms}| = \frac{V}{\sqrt{(R_L + R_A)^2 + (X_L + X_A)^2}} \qquad - (5)$$

Substitute equation 5 in the equation 3

$$A_e = \left[\frac{V}{\sqrt{(R_L + R_A)^2 + (X_L + X_A)^2}}\right]^2 \frac{R_L}{P}$$

$$A_e = \frac{V^2 R_L}{[(R_L + R_A)^2 + (X_L + X_A)^2]P} \qquad - (6)$$

To deliver the maximum power to the load we need to satisfy the maximum power transfer theorem. According to the maximum power transfer theorem the source impedance and load impedance must be complex conjugates to each other. i.e

$$\mathrm{R_L = R_A} \qquad - (7)$$

And

$$\mathrm{X_L = -\, X_A} \qquad - (8)$$

But $\mathrm{R_A = R_r + R_l = R_r}$ when $\mathrm{R_l}$ is neglected.

Therefore

$$\mathrm{R_L = R_r} \qquad - (9)$$

Substitute equations 7 and 8 in equation 6

$$A_{em} = \frac{V^2 R_L}{[(R_L + R_L)^2 + (X_L - X_L)^2]P}$$

$$A_{em} = \frac{V^2 R_L}{[(R_L + R_L)^2 + (0)^2]P}$$

$$A_{em} = \frac{V^2 R_L}{[(R_L + R_L)^2]P}$$

$$A_{em} = \frac{V^2 R_L}{[(2R_L)^2]P} = \frac{V^2 R_L}{4PR_L^2}$$

$$A_{em} = \frac{V^2}{4PR_L} \qquad - (10)$$

In addition to the above effective area we need to define some more like Scattering aperture, loss aperture, collecting aperture and physical aperture.

When the effective area is defined in terms of radiation resistance($\mathrm{R_r}$) then it is called as the Scattering aperture($\mathrm{A_s}$) and is given by

$$\mathrm{A_s} = \frac{\mathrm{V^2}}{\mathrm{4PR_r}} \qquad - (11)$$

When the effective area is defined in terms of loss resistance($\mathrm{R_{rl}}$) then it is called as the Loss aperture($\mathrm{A_l}$) and is given by

$$\mathrm{A_s} = \frac{V^2}{4PR_l} \qquad - (12)$$

The ration of loss aperture and effective aperture is called the effectiveness ratio and is given by

$$\alpha = \frac{A_l}{\mathrm{A_{em}}} \qquad - (13)$$

The collecting aperture($\mathrm{A_c}$) is nothing but a collection of all the three apertures such as effective aperture, scattering aperture and loss aperture. i.e.

$$A_c = A_e + A_s + A_l \qquad - (14)$$

The ration of scattering aperture to the effective aperture is nothing but scattering ratio and is given by

$$\beta = \frac{\mathrm{A_s}}{\mathrm{A_e}} \qquad - (15)$$

Effective Height and length:

Generally the effective height or effective length represents how far the antenna is involved either in receiving or transmitting the signal. The effective height is defined separately for receiving antenna and transmitting antenna.

For receiving antenna: In case of receiving antenna the effective height or effective length (l_e) is defined as the ratio of the induced voltage and incident electric field strength. i.e.

$$l_e = \frac{\text{Voltage induced}}{\text{Incident electric field strength}} \quad \text{m or } \lambda$$

$$l_e = \frac{V}{E} \qquad - (1)$$

We know that $$A_{em} = \frac{V^2}{4PR_L} \qquad - (2)$$

Also we know that the Poynting vector is $P = E \times H$

Or $$P = \frac{E^2}{\eta} \qquad - (3)$$

Substitute equation 3 in equation 2

$$A_{em} = \frac{V^2}{4\frac{E^2}{\eta}R_L}$$

$$\frac{V^2}{E^2} = \frac{4A_{em}R_L}{\eta}$$

$$l_e = \frac{V}{E} = \sqrt{\frac{4A_{em}R_L}{\eta}}$$

$$l_e = 2\sqrt{\frac{A_{em}R_L}{\eta}} \qquad - (4)$$

$$l_e^2 = \frac{4A_{em}R_L}{\eta}$$

Or

$$l_e^2 = \frac{4A_eR_L}{\eta}$$

$$A_e = \frac{l_e^2\eta}{4R_L} \qquad - (5)$$

For transmitting antenna: In case of transmitting antenna the effective height or effective length is simply the physical length of the antenna where the current is uniform. Theeffective length and physical length of transmitting antenna is represented in the figure 1.9 below.

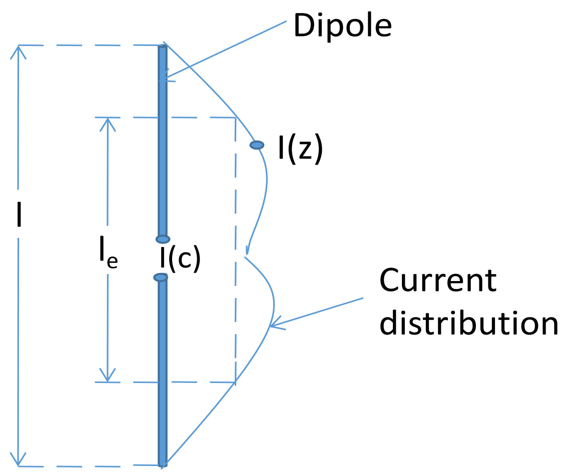


Fig 1.9: Illustration of effective length for transmitting antenna

The meaning of different letters indicated in the above figure are given by

I(c) = Current at the terminals of the actual antenna, I(z) = Current at any point 'z' of the antenna, l_e = Effective length, l = Physical length.
We know that the current element is Idl, with I is current and dl is the differential length. Therefore $$I(c)l_{et} = \int_{-l/2}^{l/2} I(z)dz$$

$$l_{et} = \frac{1}{I(c)} \int_{-l/2}^{l/2} I(z)dz$$

$$l_{et} = \frac{2}{I(c)} \int_{0}^{l/2} I(z)dz$$

The above equation represents the effective height of the transmitting antenna.

<u>Relation in between the effective area and gain or directivity:</u>

Let us consider two antennas A and B and then

D_a be the directivity of antenna A

D_b be the directivity of antenna B

A_{ema} be the maximumveffective area of the antenna A

A_{emb} be the maximum effective area of the antenna B

Practically it is found that the directivity is directly proportional to the effective area. i.e

$$D_a \propto A_{ema}$$

and $$D_b \propto A_{emb}$$

Take the ratio of the above two equations then

$$\frac{D_a}{D_b} = \frac{A_{ema}}{A_{emb}} \quad - (1)$$

Let the antenna A be the isotropic radiator, then the gain or directivity of isotropic radiator is unity. i.e. D_a=1.
Then above equation becomes

$$\frac{1}{D_b} = \frac{A_{ema}}{A_{emb}} \quad - (2)$$

$$A_{ema} = \frac{A_{emb}}{D_b} \quad - (3)$$

$$D_b = \frac{A_{emb}}{A_{ema}} \quad - (4)$$

Let the antenna B be the short dipole, then the gain or directivity of short dipole is 3/2 and maximum effective area is $\frac{3}{8\pi}\lambda^2$

i.e. $D_b = 3/2$ and $A_{emb} = \frac{3}{8\pi}\lambda^2$

Substitute above two values in the equation 3 then

$$A_{ema} = \frac{\frac{3}{8\pi}\lambda^2}{3/2}$$

$$A_{ema} = \frac{\lambda^2}{4\pi} \quad - (5)$$

Substitute equation 5 in the equation 4

$$D_b = \frac{A_{emb}}{\frac{\lambda^2}{4\pi}}$$

$$D_b = \frac{4\pi A_{emb}}{\lambda^2}$$

In general, $$D = G = \frac{4\pi A_e}{\lambda^2} \qquad - (6)$$

The above equation gives the relation in between the effective area and the gain or directivity

RADIATION FROM SMALL ELECTRIC DIPOLE

The radiation is nothing but the EM waves produced by any practical antenna. The radiation field equations of any antenna can be obtained by using the Maxwell's equations. Therefore let us discuss about the basic Maxwell's equations.

Basic Maxwell's equations:

The basic four Maxwell's equations for time varying Electromagnetic fields is given by

$$\nabla \cdot B = 0 \qquad 1.75$$

$$\nabla \cdot D = \rho_v \qquad 1.76$$

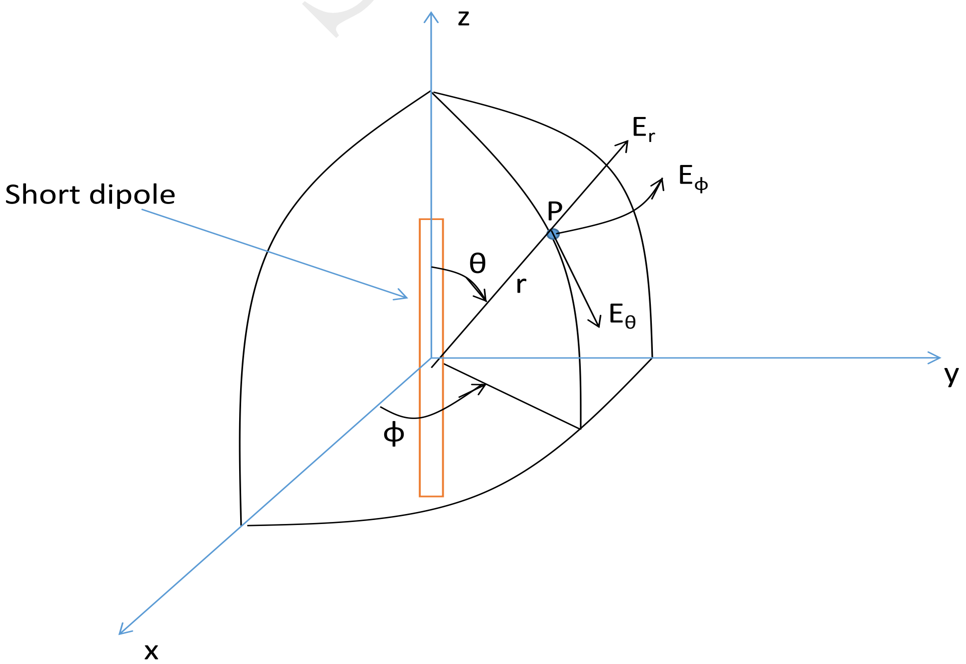
$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad 1.77$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad 1.78$$

The first Maxwell equation says that the diverging magnetic field is zero. That means the magnetic field always exists in the form of closed loops. The second Maxwell equation represents that the electric flux passing through any closed surface is equal to the total charge enclosed by that closed surface. The third Maxwell equation says that, the time varying magnetic field is able to produce the e.m.f. or voltage in a closed circuit. The fourth Maxwell equation says that, the magnetic field intensity around any closed path is equal to the conduction current density plus the displacement current density. These Maxwell equations will be used to derive the wave equation or helm holtz equation.

Field Components:

To derive the field components due to short dipole let us consider the short dipole located in the sphere as shown in the figure below.



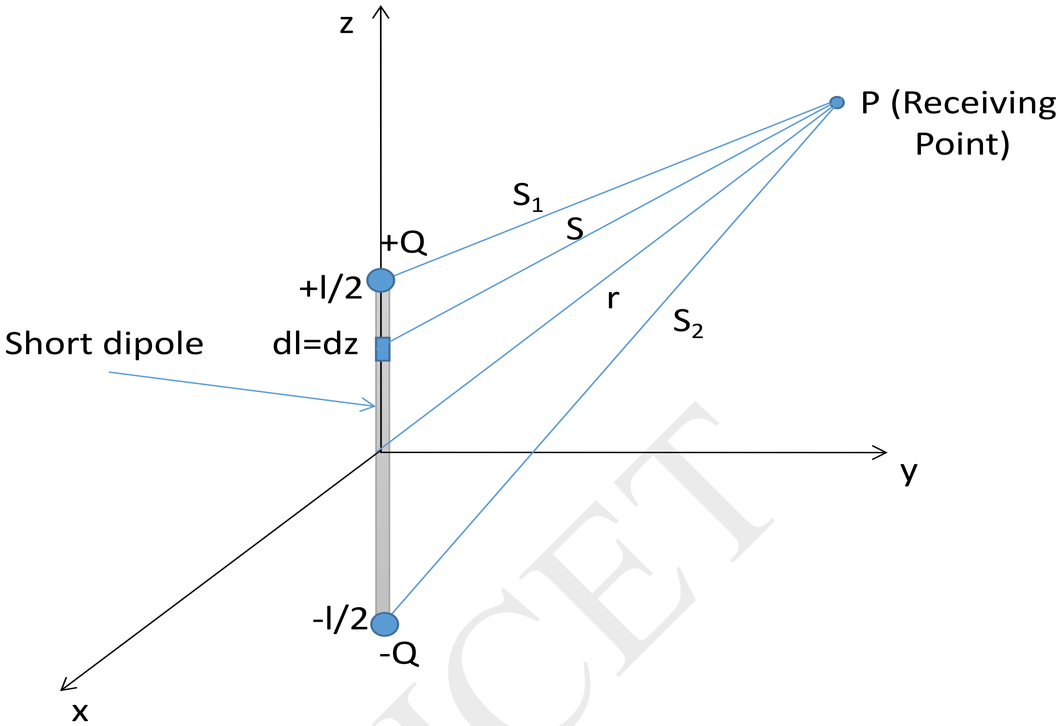
Let

$$I = I_m \sin \omega t = I_m e^{j\omega t} \qquad -1$$

be the current applied to the antenna. Then the retarded current (current at the receiving point) is given by

$$[I] = I_m\, e^{j\omega(t-r/c)} \qquad -2$$

To find out the magnetic vector potential due to short dipole consider the following figure.



The basic equation for the retarded vector potential is given by

$$[A] = \frac{\mu}{4\pi}\int \frac{[I]}{r} dl$$

But from the above figure dl = dz

$$[A] = \frac{\mu}{4\pi}\int_{-l/2}^{l/2} \frac{[I]}{r} dz = \frac{2\mu}{4\pi}\int_{0}^{l/2} \frac{[I]}{r} dz$$

$$[A] = \frac{\mu}{2\pi}\int_{0}^{l/2} \frac{[I]}{r} dz \qquad -3$$

Substitute equation 2 in equation 3.

$$[A] = \frac{\mu}{2\pi}\int_{0}^{l/2} \frac{I_m\, e^{j\omega(t-r/c)}}{r} dz$$

$$[A] = \frac{\mu}{2\pi}\frac{I_m\, e^{j\omega(t-r/c)}}{r}\int_{0}^{l/2} dz = \frac{\mu}{2\pi}\frac{I_m\, e^{j\omega(t-r/c)}}{r}\frac{l}{2}$$

$$[A] = \frac{\mu\, l\, I_m\, e^{j\omega(t-r/c)}}{4\pi r}$$

But A = A_z because the current in the short dipole flows in the z-direction. Therefore

$$[A_z] = [A] = \frac{\mu\, l\, I_m\, e^{j\omega(t-r/c)}}{4\pi r} \qquad -4$$

The above equation represents the retarded vector potential at the receiving point due to short dipole.

Now let us derive the retarded scalar potential due to short dipole. The basic equation for the retarded scalar potential is given by

$$[V] = \frac{1}{4\pi\varepsilon}\int \frac{\rho}{r} dv$$

As the short dipole contains the charges (+Q and –Q) only at the end points of the antenna, the above equation can be rewritten as

$$[V] = \frac{1}{4\pi\varepsilon}\left(\frac{Q}{S_1} - \frac{Q}{S_2}\right) \qquad -5$$

From the fundamentals, we know that

$$Q = \int [I]\, dt \qquad -6$$

Substitute equation 2 in equation 6

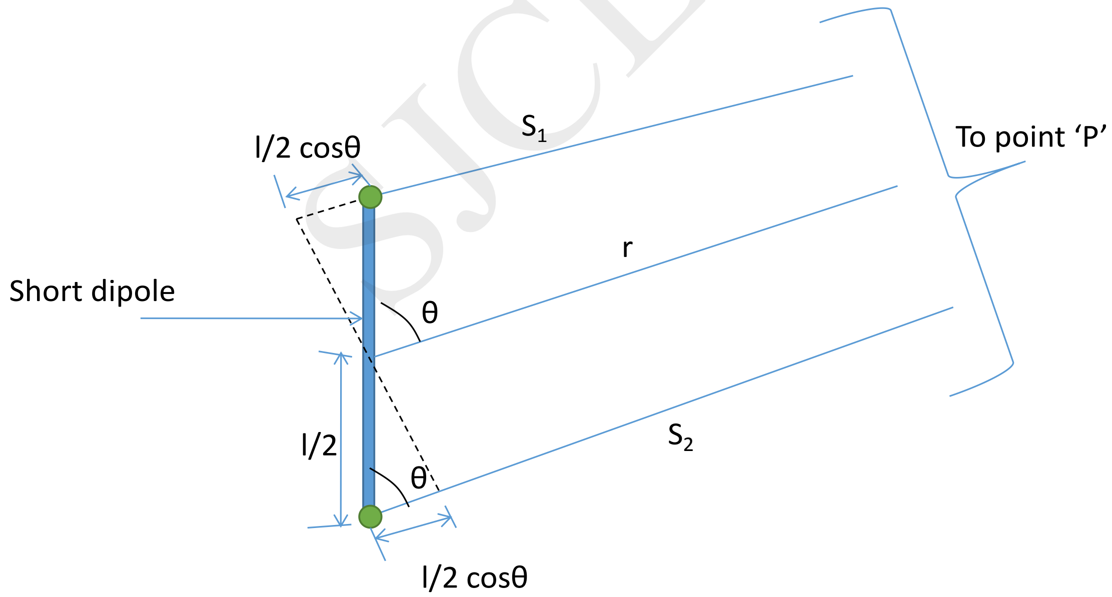
$$Q = \int I_m\, e^{j\omega(t-S/c)}\, dt$$

$$Q = \frac{I_m\, e^{j\omega(t-S/c)}}{j\omega} \qquad -7$$

Substitute equation 7 in equation 5

$$[V] = \frac{1}{4\pi\varepsilon}\left(\frac{I_m\, e^{j\omega(t-S_1/c)}}{j\omega.S_1} - \frac{I_m\, e^{j\omega(t-S_2/c)}}{j\omega.S_2}\right) \qquad -8$$

The equations for S_1 and S_2 will be obtained from the figure shown below.



From the above figure,

$$S_1 = r - \frac{l}{2}\cos\theta \qquad -9$$

$$S_2 = r + \frac{l}{2}\cos\theta \qquad -10$$

Substitute equations 9 and 10 in equation 8.

$$[V]=\frac{1}{4\pi\varepsilon}\left(\frac{I_m\, e^{j\omega\left(t-\left(r-\frac{l}{2}\cos\theta\right)/c\right)}}{j\omega.\left(r-\frac{l}{2}\cos\theta\right)}-\frac{I_m\, e^{j\omega\left(t-\left(r+\frac{l}{2}\cos\theta\right)/c\right)}}{j\omega.\left(r+\frac{l}{2}\cos\theta\right)}\right)$$

$$[V]=\frac{I_m}{4\pi\varepsilon(j\omega)}\left(\frac{e^{j\omega(t-r/c)}e^{\frac{j\omega l\cos\theta}{2c}}}{\left(r-\frac{l}{2}\cos\theta\right)}-\frac{e^{j\omega(t-r/c)}e^{\frac{-j\omega l\cos\theta}{2c}}}{\left(r+\frac{l}{2}\cos\theta\right)}\right)$$

$$[V]=\frac{I_m\, e^{j\omega(t-r/c)}}{4\pi\varepsilon(j\omega)}\left(\frac{e^{\frac{j\omega l\cos\theta}{2c}}}{\left(r-\frac{l}{2}\cos\theta\right)}-\frac{e^{\frac{-j\omega l\cos\theta}{2c}}}{\left(r+\frac{l}{2}\cos\theta\right)}\right)$$

$$[V]=\frac{I_m\, e^{j\omega(t-r/c)}}{4\pi\varepsilon(j\omega)}\left(\frac{e^{\frac{j\omega l\cos\theta}{2c}}\left(r+\frac{l}{2}\cos\theta\right)-\ e^{\frac{-j\omega l\cos\theta}{2c}}\left(r-\frac{l}{2}\cos\theta\right)}{\left(r-\frac{l}{2}\cos\theta\right)\left(r+\frac{l}{2}\cos\theta\right)}\right)$$

$$[V]=\frac{I_m\, e^{j\omega(t-r/c)}}{4\pi\varepsilon(j\omega)}\left(\frac{e^{\frac{j\omega l\cos\theta}{2c}}\left(r+\frac{l}{2}\cos\theta\right)-\ e^{\frac{-j\omega l\cos\theta}{2c}}\left(r-\frac{l}{2}\cos\theta\right)}{r^2-\left(\frac{l}{2}\cos\theta\right)^2}\right)$$

When r >> l, then $\left(\frac{l}{2}\cos\theta\right)^2$ can be neglected.

$$[V]=\frac{I_m\, e^{j\omega(t-r/c)}}{4\pi\varepsilon(j\omega)r^2}\left(e^{\frac{j\omega l\cos\theta}{2c}}\left(r+\frac{l}{2}\cos\theta\right)-\ e^{\frac{-j\omega l\cos\theta}{2c}}\left(r-\frac{l}{2}\cos\theta\right)\right)$$

$$[V]=\frac{I_m\, e^{j\omega(t-r/c)}}{4\pi\varepsilon(j\omega)r^2}\left[\left(cos\left(\frac{\omega l\cos\theta}{2c}\right)+j\sin\left(\frac{\omega l\cos\theta}{2c}\right)\right)\left(r+\frac{l}{2}\cos\theta\right)\right.$$
$$\left.-\left(cos\left(\frac{\omega l\cos\theta}{2c}\right)-j\sin\left(\frac{\omega l\cos\theta}{2c}\right)\right)\left(r-\frac{l}{2}\cos\theta\right)\right] \qquad -\ 11$$

When λ >> l, then

$$cos\left(\frac{\omega l\cos\theta}{2c}\right)=cos\left(\frac{2\pi fl\cos\theta}{2c}\right)=cos\left(\frac{\pi fl\cos\theta}{c}\right)=cos\left(\frac{\pi l\cos\theta}{\lambda}\right)\cong 1$$

Similarly

$$\sin\left(\frac{\omega l\cos\theta}{2c}\right)=sin\left(\frac{\pi l\cos\theta}{\lambda}\right)\cong\frac{\pi l\cos\theta}{\lambda}=\frac{\omega l\cos\theta}{2c}$$

Substitute above two approximations in the equation 11

$$[V]=\frac{I_m\, e^{j\omega(t-r/c)}}{4\pi\varepsilon(j\omega)r^2}\left[\left(1+\frac{j\omega l\cos\theta}{2c}\right)\left(r+\frac{l}{2}\cos\theta\right)-\left(1-\frac{j\omega l\cos\theta}{2c}\right)\left(r-\frac{l}{2}\cos\theta\right)\right]$$

$$[V]=\frac{I_m\, e^{j\omega(t-r/c)}}{4\pi\varepsilon(j\omega)r^2}\left[r+\frac{j\omega rl\cos\theta}{2c}+\frac{l}{2}\cos\theta+\frac{j\omega l^2cos^2\theta}{4c}-r+\frac{j\omega rl\cos\theta}{2c}+\frac{l}{2}\cos\theta\right.$$
$$\left.-\frac{j\omega l^2cos^2\theta}{4c}\right]$$

$$[V]=\frac{I_m\, e^{j\omega(t-r/c)}}{4\pi\varepsilon(j\omega)r^2}\left[\frac{2j\omega rl\cos\theta}{2c}+\frac{2l}{2}\cos\theta\right]$$

$$[V]=\frac{I_m\, e^{j\omega(t-r/c)}}{4\pi\varepsilon(j\omega)r^2}\left[\frac{j\omega rl\cos\theta}{c}+l\cos\theta\right]$$

$$[V] = \frac{I_m\, l\cos\theta\, e^{j\omega(t-r/c)}}{4\pi\varepsilon}\left[\frac{j\omega r}{j\omega r^2 c} + \frac{1}{j\omega r^2}\right]$$

$$[V] = \frac{I_m\, l\cos\theta\, e^{j\omega(t-r/c)}}{4\pi\varepsilon}\left[\frac{1}{cr} + \frac{1}{j\omega r^2}\right]$$

$$[V] = \frac{I_m\, l\cos\theta\, e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left[\frac{1}{r} + \frac{c}{j\omega r^2}\right] \qquad -12$$

The above equation represents the retarded scalar potential.
The equation for E in terms of A and V is given by

$$E = -j\omega A - \nabla V \qquad -13$$

Express above equation in spherical coordinate system.

$$E_r a_r + E_\theta a_\theta + E_\varphi a_\varphi = -j\omega\left(A_r a_r + A_\theta a_\theta + A_\varphi a_\varphi\right) - \left(\frac{\partial V}{\partial r}a_r + \frac{1}{r}\frac{\partial V}{\partial \theta}a_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}a_\varphi\right)$$

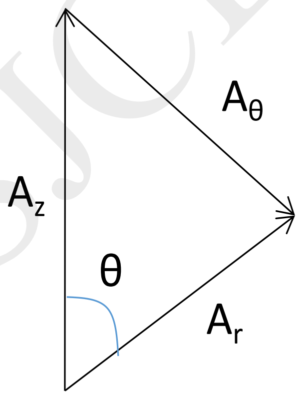
Equate individual components on both sides

$$E_r = -j\omega A_r - \frac{\partial V}{\partial r} \qquad -14$$

$$E_\theta = -j\omega A_\theta - \frac{1}{r}\frac{\partial V}{\partial \theta} \qquad -15$$

$$E_\varphi = -j\omega A_\varphi - \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi} \qquad -16$$

The z-component of retarded vector potential (A_z) given in equation 4 can be resolved in to A_r, A_θ, and $A\varphi$ from the following figure.



From the above figure

$$A_r = A_z\cos\theta \qquad -17$$

$$A_\theta = -A_z\sin\theta \qquad -18$$

$$A_\varphi = 0 \qquad -19$$

Substitute equation 4 in equations 17 and 18

$$A_r = \frac{\mu\, l\, I_m\, e^{j\omega(t-r/c)}}{4\pi r}\cos\theta = \frac{\mu\, l\, \cos\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi r} \qquad -20$$

$$A_\theta = -\frac{\mu\, l\, I_m\, e^{j\omega(t-r/c)}}{4\pi r}\sin\theta = -\frac{\mu\, l\, \sin\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi r} \qquad -21$$

From equation 12

$$\frac{\partial V}{\partial r} = \frac{\partial}{\partial r}\left[\frac{I_m\, l\cos\theta\, e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left(\frac{1}{r} + \frac{c}{j\omega r^2}\right)\right]$$

$$\frac{\partial V}{\partial r} = \frac{I_m l cos\theta}{4\pi\varepsilon c}\left[e^{j\omega(t-r/c)}\left(-\frac{j\omega}{c}\right)\left(\frac{1}{r} + \frac{c}{j\omega r^2}\right) + e^{j\omega(t-r/c)}\left(\frac{-1}{r^2} - \frac{2c}{j\omega r^3}\right)\right]$$

$$\frac{\partial V}{\partial r}=\frac{I_m l cos\theta e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left[\frac{-j\omega}{cr}-\frac{1}{r^2}-\frac{1}{r^2}-\frac{2c}{j\omega r^3}\right]$$

$$\frac{\partial V}{\partial r}=\frac{I_m l cos\theta e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left[\frac{-j\omega}{cr}-\frac{2}{r^2}-\frac{2c}{j\omega r^3}\right]$$

$$\frac{\partial V}{\partial r}=-\frac{I_m l cos\theta e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left[\frac{j\omega}{cr}+\frac{2}{r^2}+\frac{2c}{j\omega r^3}\right] \qquad -22$$

Substitute equations 20 and 22 in equation 14.

$$E_r=-j\omega\left(\frac{\mu\, l\, \cos\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi r}\right)-\left(-\frac{I_m l cos\theta e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left[\frac{j\omega}{cr}+\frac{2}{r^2}+\frac{2c}{j\omega r^3}\right]\right)$$

$$E_r=-\frac{j\omega\mu\, l\, \cos\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi r}+\frac{I_m l cos\theta e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left[\frac{j\omega}{cr}+\frac{2}{r^2}+\frac{2c}{j\omega r^3}\right]$$

$$E_r=\frac{l\, \cos\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi}\left[\frac{-j\omega\mu}{r}+\frac{1}{\varepsilon c}\left(\frac{j\omega}{cr}+\frac{2}{r^2}+\frac{2c}{j\omega r^3}\right)\right]$$

But $\mu=\frac{1}{c^2\varepsilon}$ because $c=\frac{1}{\sqrt{\mu\varepsilon}}$

Then

$$E_r=\frac{l\, \cos\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi}\left[\frac{-j\omega}{r}\frac{1}{c^2\varepsilon}+\frac{1}{\varepsilon c}\left(\frac{j\omega}{cr}+\frac{2}{r^2}+\frac{2c}{j\omega r^3}\right)\right]$$

$$E_r=\frac{l\, \cos\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left[\frac{-j\omega}{cr}+\frac{j\omega}{cr}+\frac{2}{r^2}+\frac{2c}{j\omega r^3}\right]$$

$$E_r=\frac{l\, \cos\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left[\frac{2}{r^2}+\frac{2c}{j\omega r^3}\right]$$

$$E_r=\frac{2l\, \cos\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left[\frac{1}{r^2}+\frac{c}{j\omega r^3}\right]$$

$$E_r=\frac{l\, \cos\theta\, I_m\, e^{j\omega(t-r/c)}}{2\pi\varepsilon c}\left[\frac{1}{r^2}+\frac{c}{j\omega r^3}\right]$$

$$E_r=\frac{l\, \cos\theta\, I_m\, e^{j\omega(t-r/c)}}{2\pi\varepsilon}\left[\frac{1}{cr^2}+\frac{c}{cj\omega r^3}\right]$$

$$\boxed{E_r=\frac{l\, \cos\theta\, I_m\, e^{j\omega(t-r/c)}}{2\pi\varepsilon}\left[\frac{1}{cr^2}+\frac{1}{j\omega r^3}\right] \qquad -23}$$

From equation 12

$$\frac{\partial V}{\partial \theta}=\frac{\partial}{\partial \theta}\left(\frac{I_m\, l\cos\theta\, e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left(\frac{1}{r}+\frac{c}{j\omega r^2}\right)\right)$$

$$\frac{\partial V}{\partial \theta}=\frac{I_m\, l(-\sin\theta) e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left(\frac{1}{r}+\frac{c}{j\omega r^2}\right)$$

$$\frac{\partial V}{\partial \theta}=-\frac{I_m\, l\sin\theta\, e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left(\frac{1}{r}+\frac{c}{j\omega r^2}\right) \qquad -24$$

Substitute equations 21 and 24 in equation 15

$$E_\theta = -j\omega\left(-\frac{\mu\, l\, \sin\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi r}\right) - \frac{1}{r}\left(-\frac{I_m\, l \sin\theta\, e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left(\frac{1}{r}+\frac{c}{j\omega r^2}\right)\right)$$

$$E_\theta = \frac{j\omega\mu\, l\, \sin\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi r} + \frac{I_m\, l \sin\theta\, e^{j\omega(t-r/c)}}{4\pi\varepsilon c}\left(\frac{1}{r^2}+\frac{c}{j\omega r^3}\right)$$

$$E_\theta = \frac{I_m\, l \sin\theta\, e^{j\omega(t-r/c)}}{4\pi}\left(\frac{j\omega\mu}{r} + \frac{1}{\varepsilon c}\left(\frac{1}{r^2}+\frac{c}{j\omega r^3}\right)\right)$$

But $\mu = \frac{1}{c^2\varepsilon}$ because $c = \frac{1}{\sqrt{\mu\varepsilon}}$

Then

$$E_\theta = \frac{I_m\, l \sin\theta\, e^{j\omega(t-r/c)}}{4\pi}\left(\frac{j\omega}{r}\frac{1}{c^2\varepsilon} + \frac{1}{\varepsilon c}\left(\frac{1}{r^2}+\frac{c}{j\omega r^3}\right)\right)$$

$$\boxed{E_\theta = \frac{I_m\, l \sin\theta\, e^{j\omega(t-r/c)}}{4\pi\varepsilon}\left(\frac{j\omega}{c^2 r} + \frac{1}{c r^2} + \frac{1}{j\omega r^3}\right) \qquad -25}$$

Since $A_\varphi = 0$, The component E_φ is zero.

That is

$$\boxed{E_\varphi = 0}$$

Similarly let us derive the field components of H

We know the relation

$$B = \nabla \times A$$
$$\mu H = \nabla \times A$$
$$H = \frac{1}{\mu}(\nabla \times A)$$

Express above equation in spherical coordinate system

$$H_r a_r + H_\theta a_\theta + H_\varphi a_\varphi = \frac{1}{\mu}\left(\frac{1}{r^2 \sin\theta}\begin{vmatrix} a_r & r a_\theta & r\sin\theta a_\varphi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi} \\ A_r & rA_\theta & r\sin\theta A_\varphi \end{vmatrix}\right)$$

$$H_r a_r + H_\theta a_\theta + H_\varphi a_\varphi = \frac{1}{\mu}\left(\frac{1}{r^2 \sin\theta}\begin{vmatrix} a_r & r a_\theta & r\sin\theta a_\varphi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & 0 \\ A_r & rA_\theta & 0 \end{vmatrix}\right)$$

Because $A_\varphi = 0$

Equate individual components on both sides

$$H_r = \frac{1}{\mu r^2 \sin\theta}\left(\frac{\partial}{\partial\theta}(0) - rA_\theta(0)\right) = 0$$

$$\boxed{H_r = 0}$$

$$H_\theta = -\frac{1}{\mu r^2 \sin\theta}\left(r\left[\frac{\partial}{\partial r}(0) - A_r(0)\right]\right) = 0$$

$$\boxed{H_\theta = 0}$$

$$H_\varphi = \frac{1}{\mu r^2 \sin\theta}\left[r\sin\theta\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial\theta}(A_r)\right)\right]$$

$$H_\varphi = \frac{1}{\mu r}\left[\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial\theta}(A_r)\right)\right] \qquad -26$$

From equation 21

$$rA_\theta = -\frac{\mu\, l\, \sin\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi}$$

$$\frac{\partial}{\partial r}(rA_\theta) = \frac{\partial}{\partial r}\left(-\frac{\mu\, l\, \sin\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi}\right)$$

$$\frac{\partial}{\partial r}(rA_\theta) = -\frac{\mu\, l\, \sin\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi}\left(-\frac{j\omega}{c}\right)$$

$$\frac{\partial}{\partial r}(rA_\theta) = \frac{j\omega\mu\, l\, \sin\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi c} \qquad -27$$

From equation 20

$$\frac{\partial}{\partial\theta}(A_r) = \frac{\partial}{\partial\theta}\left(\frac{\mu\, l\, \cos\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi r}\right)$$

$$\frac{\partial}{\partial\theta}(A_r) = -\frac{\mu\, l\, \sin\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi r} \qquad -28$$

Substitute equations 27 and 28 in equation 26

$$H_\varphi = \frac{1}{\mu r}\left[\left(\frac{j\omega\mu\, l\, \sin\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi c} - \left(-\frac{\mu\, l\, \sin\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi r}\right)\right)\right]$$

$$H_\varphi = \frac{1}{\mu r}\left[\frac{j\omega\mu\, l\, \sin\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi c} + \frac{\mu\, l\, \sin\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi r}\right]$$

$$H_\varphi = \frac{\mu\, l\, \sin\theta\, I_m\, e^{j\omega(t-r/c)}}{\mu r(4\pi)}\left[\frac{j\omega}{c} + \frac{1}{r}\right]$$

$$\boxed{H_\varphi = \frac{l\, \sin\theta\, I_m\, e^{j\omega(t-r/c)}}{4\pi}\left[\frac{j\omega}{cr} + \frac{1}{r^2}\right] \qquad -29}$$

Finally the field components due to small electric dipole are given by

$$E_r = \frac{l\, \cos\theta\, I_m\, e^{j\omega\left(t-\frac{r}{c}\right)}}{2\pi\varepsilon}\left[\frac{1}{cr^2} + \frac{1}{j\omega r^3}\right]$$

$$E_\theta = \frac{I_m\, l \sin\theta\, e^{j\omega(t-r/c)}}{4\pi\varepsilon}\left(\frac{j\omega}{c^2 r} + \frac{1}{cr^2} + \frac{1}{j\omega r^3}\right)$$

$$E_\varphi = 0$$
$$H_r = 0$$
$$H_\theta = 0$$

$$H_\varphi = \frac{l\ \sin\theta\, I_m\ e^{j\omega(t-r/c)}}{4\pi}\left[\frac{j\omega}{cr} + \frac{1}{r^2}\right]$$

Filed components for radiation zone or far field components:

The general field components due to small electric dipole are given by

$$E_r = \frac{l\ \cos\theta\, I_m\ e^{j\omega\left(t-\frac{r}{c}\right)}}{2\pi\varepsilon}\left[\frac{1}{cr^2} + \frac{1}{j\omega r^3}\right]$$

$$E_\theta = \frac{I_m\, l \sin\theta\ e^{j\omega(t-r/c)}}{4\pi\varepsilon}\left(\frac{j\omega}{c^2 r} + \frac{1}{cr^2} + \frac{1}{j\omega r^3}\right)$$

$$E_\varphi = 0$$

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\varphi = \frac{l\ \sin\theta\, I_m\ e^{j\omega(t-r/c)}}{4\pi}\left[\frac{j\omega}{cr} + \frac{1}{r^2}\right]$$

We know that, the far field related to long distance and near field related to small distance. If we observe the general filed components, they are proportional to the terms $\frac{1}{r}, \frac{1}{r^2}$ *and* $\frac{1}{r^3}$. When the distance (r) is large, then the terms of the filed components which depend upon $\frac{1}{r^2}$ *and* $\frac{1}{r^3}$ can be neglected. Therefore the field components for the radiation zone are given by

$$E_r = 0$$

$$E_\theta = \frac{I_m\, l \sin\theta\ e^{j\omega(t-r/c)}}{4\pi\varepsilon}\left(\frac{j\omega}{c^2 r}\right) = \frac{j\omega I_m\, l \sin\theta\ e^{j\omega(t-r/c)}}{4\pi\varepsilon c^2 r}$$

$$= \frac{j\beta I_m\, l \sin\theta\ e^{j\omega(t-r/c)}}{4\pi\varepsilon c r}$$

$$E_\varphi = 0$$

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\varphi = \frac{l\ \sin\theta\, I_m\ e^{j\omega(t-r/c)}}{4\pi}\left[\frac{j\omega}{cr}\right] = \frac{j\omega l\ \sin\theta\, I_m\ e^{j\omega(t-r/c)}}{4\pi c r}$$

$$= \frac{j\beta l\ \sin\theta\, I_m\ e^{j\omega(t-r/c)}}{4\pi r}$$

Power radiated and radiation resistance due to small electric dipole:

The general equation for the power radiated due to any antenna is given by

$$W = \int P_{av} \cdot ds \qquad -1$$

Where P_{av} is the average poynting vector.

From the poynting theorem we have

$$P_{av} = \frac{1}{2}(E \times H) = \frac{1}{2}\eta |H|^2 \qquad -2$$

The magnetic field intensity due to small electric dipole for radiation zone is given by

$$H = H_{\varphi} = \frac{j\omega l \ \sin\theta \, I_m \, e^{j\omega\left(t-\frac{r}{c}\right)}}{4\pi cr}$$

$$|H| = \frac{\omega l \ \sin\theta \, I_m}{4\pi cr} \qquad -3$$

Substitute equation 3 in equation 2

$$P_{av} = \frac{1}{2}\eta\left(\frac{\omega l \ \sin\theta \, I_m}{4\pi cr}\right)^2 \qquad -4$$

Substitute equation 4 in equation 1

$$W = \int \frac{1}{2}\eta\left(\frac{\omega l \ \sin\theta \, I_m}{4\pi cr}\right)^2 \cdot ds$$

But $ds = r^2 \sin\theta d\theta d\varphi$

$$W = \int \frac{1}{2}\eta\left(\frac{\omega l \ \sin\theta \, I_m}{4\pi cr}\right)^2 r^2 \sin\theta d\theta d\varphi$$

$$W = \frac{1}{2}\eta\frac{\omega^2 I_m^2 l^2}{16\pi^2 c^2}\int_0^{2\pi} d\varphi \int_0^{\pi} sin^3\theta d\theta$$

$$W = \frac{1}{2}\eta\frac{\omega^2 I_m^2 l^2}{16\pi^2 c^2}(2\pi)\left(\frac{4}{3}\right)$$

$$W = \eta\frac{\omega^2 I_m^2 l^2}{12\pi c^2} = \eta\frac{\beta^2 I_m^2 l^2}{12\pi}$$

$$W = \frac{\eta(\beta I_m l)^2}{12\pi} \qquad - 5$$

The above equation represents the power radiated by the small electric dipole.
In general the average power applied to the antenna is given by

$$W = \frac{1}{2}I_m^2 R_r \qquad -6$$

Where I_m is the peak value of current applied to the antenna and R_r is the radiation resistance.
Equate equations 5 and 6

$$\frac{1}{2}I_m^2 R_r = \frac{\eta\left((\beta I_m l)\right)^2}{12\pi}$$

$$\frac{1}{2}I_m^2 R_r = \frac{\eta\beta^2 I_m^2 l^2}{12\pi}$$

$$R_r = \frac{\eta\beta^2 l^2}{6\pi} = \frac{(120\pi)}{6\pi}\left(\frac{2\pi}{\lambda}\right)^2 l^2$$

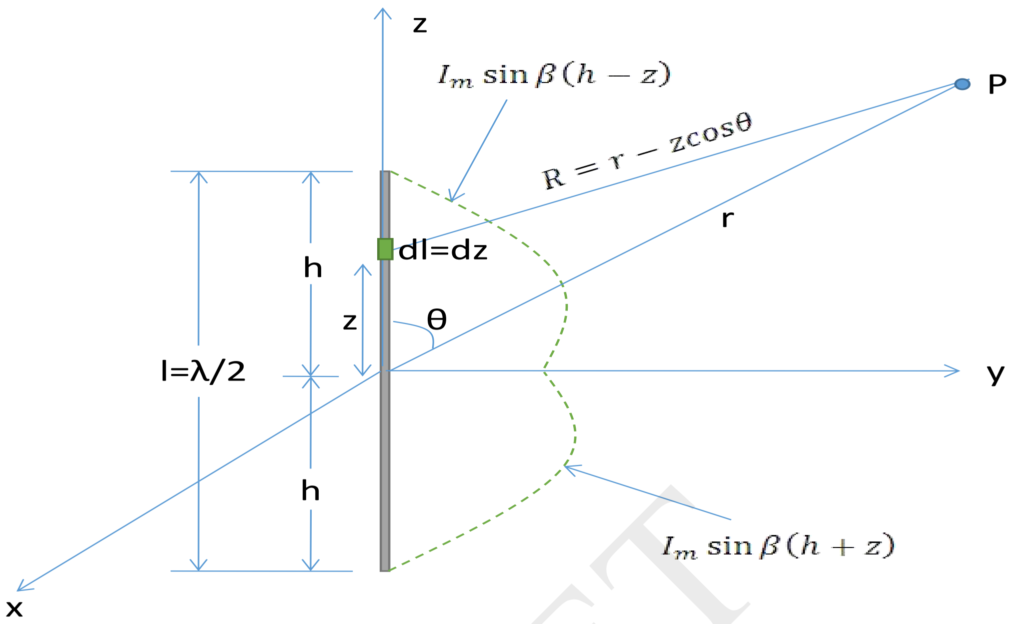
$$R_r = 80\pi^2\left(\frac{l}{\lambda}\right)^2 \qquad -7$$

The above equation represents the radiation resistance of small electric dipole.

RADIATION FROM HALF WAVE DIPOLE OR MONOPOLE

Field components due to half wave dipole or quarter wave monopole:

The field components due to half wave dipole can be derived from the following figure.



The current distribution along the half wave dipole is given by

$$I = I_m \sin\beta(h-z) \quad for\ z > 0 \qquad -1$$

$$I = I_m \sin\beta(h+z) \quad for\ z < 0 \qquad -2$$

The general equation for the retarded vector potential is given by

$$[A] = \frac{\mu}{4\pi}\int \frac{[I]}{r} dl$$

The above equation can be written for the half wave dipole as

$$[A] = \frac{\mu}{4\pi}\left[\int_{-h}^{0} \frac{I_m \sin\beta(h+z)\, e^{-j\beta R}}{R} dz + \int_{0}^{h} \frac{I_m \sin\beta(h-z)\, e^{-j\beta R}}{R} dz\right]$$

$$[A] = \frac{\mu I_m}{4\pi}\left[\int_{-h}^{0} \frac{\sin\beta(h+z)\, e^{-j\beta R}}{R} dz + \int_{0}^{h} \frac{\sin\beta(h-z)\, e^{-j\beta R}}{R} dz\right] \qquad -3$$

From figure,

$$R = r - z\cos\theta$$

$$R \cong r \quad when\ \ z\cos\theta\ is\ small$$

Substitute above relations in equation 3

$$[A] = \frac{\mu I_m}{4\pi}\left[\int_{-h}^{0} \frac{\sin\beta(h+z)\, e^{-j\beta(r - z\cos\theta)}}{r} dz + \int_{0}^{h} \frac{\sin\beta(h-z)\, e^{-j\beta(r-z\cos\theta)}}{r} dz\right] - 5$$

From figure,

$$h = \frac{\lambda}{4}$$

Then

$$\sin\beta(h+z) = \sin(\beta h + \beta z) = \sin\left(\frac{2\pi}{\lambda}\frac{\lambda}{4} + \beta z\right)$$

$$= \sin\left(\frac{\pi}{2} + \beta z\right) = \cos(\beta z)$$

$$\sin\beta(h+z) = \cos\beta z \qquad -6$$

Similarly

$$\sin\beta(h-z) = \cos\beta z \qquad -7$$

Substitute equations 6 and 7 in equation 5

$$[A] = \frac{\mu I_m}{4\pi}\left[\int_{-h}^{0} \frac{\cos\beta z\, e^{-j\beta(r - z\cos\theta)}}{r} dz + \int_{0}^{h} \frac{\cos\beta z\, e^{-j\beta(r - z\cos\theta)}}{r} dz\right]$$

$$[A] = \frac{\mu I_m}{4\pi r}\left[\int_{-h}^{0} \cos\beta z\, e^{-j\beta(r - z\cos\theta)}\, dz + \int_{0}^{h} \cos\beta z\, e^{-j\beta(r - z\cos\theta)} dz\right]$$

$$[A] = \frac{\mu I_m}{4\pi r}\left[\int_{-h}^{0} \cos\beta z\, e^{-j\beta r} e^{j\beta z\cos\theta}\, dz + \int_{0}^{h} \cos\beta z\, e^{-j\beta r} e^{j\beta z\cos\theta} dz\right]$$

$$[A] = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{-h}^{0} \cos\beta z\, e^{j\beta z\cos\theta}\, dz + \int_{0}^{h} \cos\beta z\, e^{j\beta z\cos\theta} dz\right]$$

$$[A] = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{h} \cos\beta z\, e^{-j\beta z\cos\theta}\, dz + \int_{0}^{h} \cos\beta z\, e^{j\beta z\cos\theta} dz\right]$$

When the limits are interchanged, the –ve sign to be added to the exponential term.

Let $z =- z$ then

$$dz =- dz$$

$$\int_{-h}^{0} \cos\beta z\, e^{j\beta z\cos\theta}\, dz = \int_{h}^{0} \cos\beta z\, e^{j\beta(-z)\cos\theta}\, (-dz)$$

$$=- \int_{h}^{0} \cos\beta z\, e^{-j\beta z\cos\theta}\, dz$$

$$=+ \int_{0}^{h} \cos\beta z\, e^{-j\beta z\cos\theta}\, dz$$

$$[A] = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{h} \cos\beta z\left(e^{-j\beta z\cos\theta} + e^{j\beta z\cos\theta}\right) dz\right]$$

$$[A] = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{h} \cos\beta z(\cos(\beta z\cos\theta) - j\sin(\beta z\cos\theta) + \cos(\beta z\cos\theta) + j\sin(\beta z\cos\theta))\, dz\right]$$

$$[A] = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{h} \cos\beta z(2\cos(\beta z\cos\theta))\, dz\right]$$

$$[A] = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{h} 2\cos\beta z\cos(\beta z\cos\theta)\, dz\right] \qquad -8$$

But

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

The equation 8 becomes

$$[A] = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{h} (\cos(\beta z + \beta z\cos\theta) + \cos(\beta z - \beta z\cos\theta)) dz\right]$$

$$[A] = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{h} (\cos\beta z(1+\cos\theta) + \cos\beta z(1-\cos\theta)) dz\right]$$

$$[A]=\frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_0^{h=\lambda/4}\cos\beta z(1+\cos\theta)\,dz+\int_0^{h=\lambda/4}\cos\beta z(1-\cos\theta)\,dz\right]$$

$$[A]=\frac{\mu I_m e^{-j\beta r}}{4\pi r}\left(\left[\frac{\sin\beta z(1+\cos\theta)}{\beta(1+\cos\theta)}\right]_0^{\lambda/4}+\left[\frac{\sin\beta z(1-\cos\theta)}{\beta(1-\cos\theta)}\right]_0^{\lambda/4}\right)$$

$$[A]=\frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{1+\cos\theta}+\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{1-\cos\theta}\right]$$

$$[A]=\frac{\mu I_m e^{-j\beta r}\cos\left(\frac{\pi}{2}\cos\theta\right)}{4\pi\beta r}\left[\frac{1}{1+\cos\theta}+\frac{1}{1-\cos\theta}\right]$$

$$[A]=\frac{\mu I_m e^{-j\beta r}\cos\left(\frac{\pi}{2}\cos\theta\right)}{4\pi\beta r}\left[\frac{(1-\cos\theta)+(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}\right]$$

$$[A]=\frac{\mu I_m e^{-j\beta r}\cos\left(\frac{\pi}{2}\cos\theta\right)}{4\pi\beta r}\left[\frac{1-\cos\theta+1+\cos\theta}{1-cos^2\theta}\right]$$

$$[A]=\frac{\mu I_m e^{-j\beta r}\cos\left(\frac{\pi}{2}\cos\theta\right)}{4\pi\beta r}\left[\frac{2}{sin^2\theta}\right]$$

$$[A]=[A_z]=\frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{sin^2\theta} \qquad -9$$

The above equation represents the retarded vector potential due to half wave dipole
Now let us concentrate only on far field components. We know that the radiation zone contains only two components such as E_θ *and* H_φ. First let us find H_φ and then from H_φ we can find E_θ
by using relation $E_\theta / H_\varphi = \eta$
We know the relation

$$B=\nabla\times A$$
$$\mu H=\nabla\times A$$
$$H=\frac{1}{\mu}(\nabla\times A)$$

Express above equation in spherical coordinate system

$$H_r a_r+H_\theta a_\theta+H_\varphi a_\varphi=\frac{1}{\mu}\left(\frac{1}{r^2\sin\theta}\begin{vmatrix}a_r & ra_\theta & r\sin\theta a_\varphi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi}\\ A_r & rA_\theta & r\sin\theta A_\varphi\end{vmatrix}\right)$$

$$H_r a_r+H_\theta a_\theta+H_\varphi a_\varphi=\frac{1}{\mu}\left(\frac{1}{r^2\sin\theta}\begin{vmatrix}a_r & ra_\theta & r\sin\theta a_\varphi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & 0\\ A_r & rA_\theta & 0\end{vmatrix}\right)$$

Because $A_\varphi=0$

Equate individual components on both sides

$$H_r=\frac{1}{\mu r^2\sin\theta}\left(\frac{\partial}{\partial\theta}(0)-rA_\theta(0)\right)=0$$

$$\boxed{H_r = 0}$$

$$H_\theta = -\frac{1}{\mu r^2 \sin\theta}\left(r\left[\frac{\partial}{\partial r}(0) - A_r(0)\right]\right) = 0$$

$$\boxed{H_\theta = 0}$$

$$H_\varphi = \frac{1}{\mu r^2 \sin\theta}\left[r\sin\theta\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}(A_r)\right)\right]$$

$$H_\varphi = \frac{1}{\mu r}\left[\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}(A_r)\right)\right]$$

For radiation zone the term $\frac{\partial}{\partial\theta}(A_r)$ can be neglected because it will be proportional to $1/r^2$

Therefore

$$H_\varphi = \frac{1}{\mu r}\left[\frac{\partial}{\partial r}(rA_\theta)\right] \qquad -10$$

But

$$A_\theta = -A_z \sin\theta \qquad -11$$

Substitute equation 9 in equation 11

$$A_\theta = -\left(\frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\ \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{sin^2\theta}\right)\sin\theta$$

$$A_\theta = -\frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\ \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{sin\theta} \qquad -12$$

Substitute equation 12 in equation 10

$$H_\varphi = -\frac{1}{\mu r}\left[\frac{\partial}{\partial r}\left(r\frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\ \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{sin\theta}\right)\right]$$

$$H_\varphi = -\frac{1}{r}\left[\frac{\partial}{\partial r}\left(\frac{I_m e^{-j\beta r}}{2\pi\beta}\ \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{sin\theta}\right)\right]$$

$$H_\varphi = -\frac{I_m \cos\left(\frac{\pi}{2}\cos\theta\right) e^{-j\beta r}(-j\beta)}{2\pi\beta r\sin\theta}$$

$$\boxed{H_\varphi = \frac{jI_m}{2\pi r}\ \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} e^{-j\beta r} \qquad -13}$$

We know that,

$$\frac{E_\theta}{H_\varphi} = \eta$$

$$E_\theta = \eta H_\varphi \qquad -14$$

Substitute equation 13 in equation 14

$$E_\theta = \eta \frac{jI_m}{2\pi r} \; \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} e^{-j\beta r}$$

But

$$\eta = 120\pi$$

$$E_\theta = \frac{(120\pi)jI_m}{2\pi r} \; \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} e^{-j\beta r}$$

$$\boxed{E_\theta = \frac{j60I_m}{r} \; \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} e^{-j\beta r} \qquad - 15}$$

<u>Power radiated and radiation resistance due to half wave dipole:</u>

The general equation for the power radiated due to any antenna is given by

$$W = \int P_{av} \cdot ds \qquad - 1$$

Where P_{av} is the average poynting vector.
From the poynting theorem we have

$$P_{av} = \frac{1}{2}(E \times H) = \frac{1}{2}\eta|H|^2 \qquad - 2$$

The magnetic field intensity due to hale wave dipole for radiation zone is given by

$$H = H_\varphi = \frac{jI_m}{2\pi r} \; \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} e^{-j\beta r}$$

$$|H| = \frac{I_m}{2\pi r} \; \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \qquad - 3$$

Substitute equation 3 in equation 2

$$P_{av} = \frac{1}{2}\eta\left(\frac{I_m}{2\pi r} \; \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right)^2$$

$$P_{av} = \frac{1}{2}(120\pi)\frac{I_m^2 cos^2\left(\frac{\pi}{2}\cos\theta\right)}{4\pi^2 r^2 sin^2\theta}$$

$$P_{av} = \frac{15I_m^2 cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\pi r^2 sin^2\theta}$$

$$P_{av} = \frac{15\left(\sqrt{2}I_{rms}^2\right)^2 cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\pi r^2 sin^2\theta}$$

$$P_{av} = \frac{30I_{rms}^2 cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\pi r^2 sin^2\theta} \qquad - 4$$

Substitute equation 4 in equation1

$$W = \int \frac{30I_{rms}^2 cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\pi r^2 sin^2\theta} \cdot ds$$

But $ds = r^2 \sin\theta d\theta d\varphi$

$$W = \int \frac{30 I_{rms}^2 cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\pi r^2 sin^2\theta} \cdot r^2 \sin\theta d\theta d\varphi$$

$$W = \int \frac{30 I_{rms}^2 cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\pi \sin\theta} d\theta d\varphi$$

$$W = \frac{30 I_{rms}^2}{\pi} \int_0^{2\pi} d\varphi \int_0^{\pi} \frac{cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

$$W = \frac{30 I_{rms}^2}{\pi} (2\pi) \int_0^{\pi} \frac{cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

$$W = 60\, I_{rms}^2 \int_0^{\pi} \frac{cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

In above equation the integral part can be evaluated by using either analytical methods or numerical methods such as Simpson's or Trapezoidal method. The value of the integral part is equal to 1.219

$$W = 60\, I_{rms}^2 (1.219)$$

$$W = 73\, I_{rms}^2 \qquad -5$$

The above equation represents power radiated due to half wave dipole.
We know that

$$W = I_{rms}^2 R_r \qquad -6$$

By comparing equations 5 and 6 we can say that the radiation resistance is

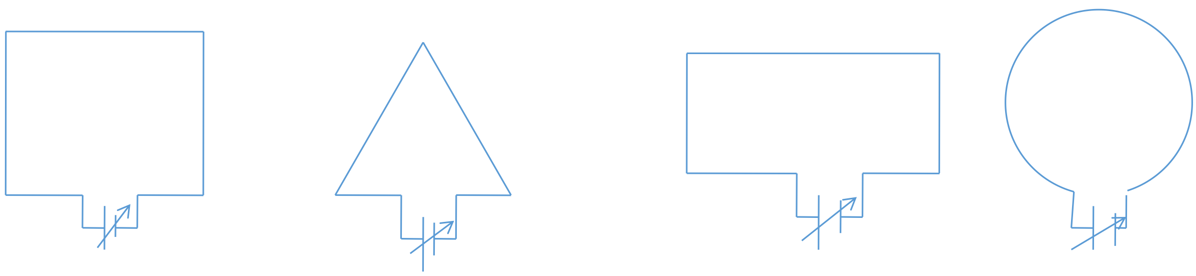
$$R_r = 73\, \Omega$$

The radiation resistance due to quarter monopole is 36.5 Ω

LOOP ANTENNAS

Introduction:

Loop antenna is defined as a radiating coil of any convenient cross section of one or more turns carrying RF (Radio Frequency) current. It may assume any shape such as rectangular, square, triangular, hexagonal and circular. The following figure shows the loop antennas of different shapes.



Square loop Triangular loop Rectangular loop Circular loop

Small Loop:

A loop antenna is said to be small if its cross sectional is small. Let us consider the two cases of small loop antenna such as receiving case and transmitting case.

(i) Receiving loop antenna:

To derive the equation for the e.m.f or voltage induced in the receiving loop antenna, let us consider the rectangular loop antenna shown in the figure below.

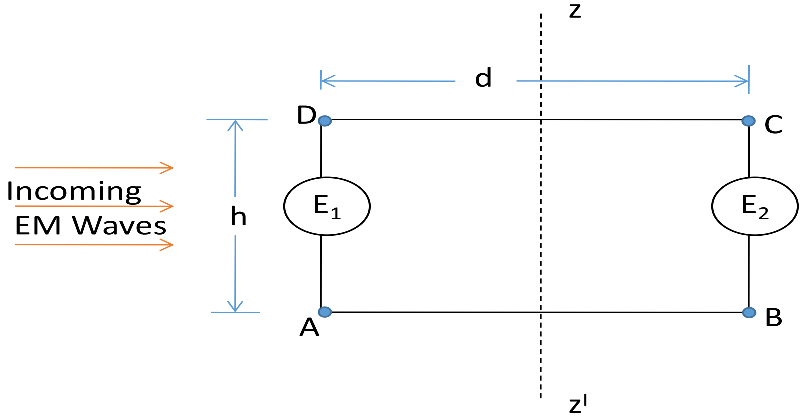
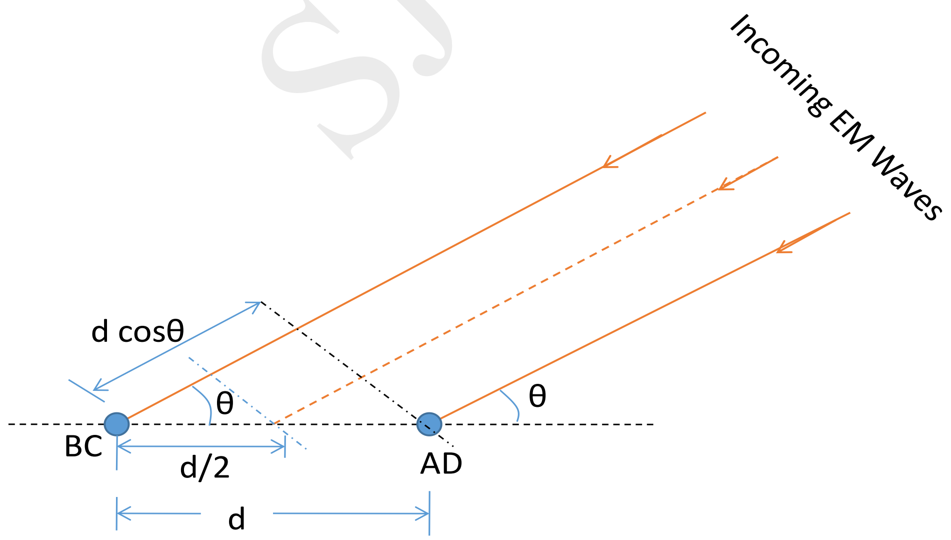


Fig: Loop antenna with rotational axis

For the sake of explanation of let us assume that, the incoming EM waves are vertically polarized. The rectangular loop antenna can be imagined as a combination of two vertical antennas (AD & BC) and two horizontal antennas (AB & CD). When the axis of the plane of the loop is perpendicular with respect to the incoming EM waves, then the two vertical antennas will receive the same amount of signal but the resultant voltage induced is zero because the vertical antennas will be at equidistance with respect to incoming waves. When the axis of the plane of the loop is in parallel with incoming EM waves, then the there will be maximum induced voltage.
From the figure shown below we can find the voltage induced in the loop antenna.



The path difference between the waves due to antenna AD w.r.t center or waves due to antenna BC w.r.t center is given by

$$Path\ difference(P.D) = \frac{d}{2}cos\theta \qquad -1$$

The phase angle difference due to path difference is given by

$$\alpha = \frac{2\pi}{\lambda}(P.D) = \frac{2\pi}{\lambda}\left(\frac{d}{2}cos\theta\right) = \frac{\pi dcos\theta}{\lambda} \qquad -2$$

Le the incident electric field at the center of the two antennas is $E_m sin\omega t$, then the voltage induced in the antenna AD is given by

$$V_1 = E_m \sin(\omega t + \alpha).h \qquad -3$$

Similarly the voltage induced in antenna BC is given by

$$V_2 = E_m \sin(\omega t - \alpha).h \qquad -4$$

The resultant voltage induced is given by

$$e = V_1 - V_2 \qquad -5$$

Substitute equations 3 and 4 in 5

$$e = E_m \sin(\omega t + \alpha).h - E_m \sin(\omega t - \alpha).h$$

$$e = E_m h[(\sin(\omega t + \alpha) - \sin(\omega t - \alpha)]$$

$$e = E_m h[sin\omega t.cos\alpha + cos\omega t.sin\alpha - (sin\omega t.cos\alpha + cos\omega t.sin\alpha)]$$

$$e = 2E_m h\, cos\omega t.sin\alpha \qquad -6$$

Now substitute equation 2 in equation 6

$$e = 2E_m h\, cos\omega t.sin\left(\frac{\pi dcos\theta}{\lambda}\right)$$

But when d << λ, then $sin\left(\frac{\pi dcos\theta}{\lambda}\right) = \left(\frac{\pi dcos\theta}{\lambda}\right)$

$$e = 2E_m h\, cos\omega t.\left(\frac{\pi dcos\theta}{\lambda}\right)$$

$$e = \frac{2\pi hd\,cos\theta}{\lambda}E_m cos\omega t$$

$$e = V_m cos\omega t$$

Where V_m called as magnitude of the induced voltage and is given by

$$V_m = \frac{2\pi hd\,cos\theta}{\lambda}E_m = \frac{2\pi AN\,cos\theta}{\lambda}E_m$$

Where A = hd is known as area of the loop and N is the no.of turns of the loop antenna.

(ii) Transmitting loop Antenna:

To derive the far field components of the loop antenna under the transmitting mode, let us consider the square loop located at the center of the spherical coordinate system as shown in the figure below.

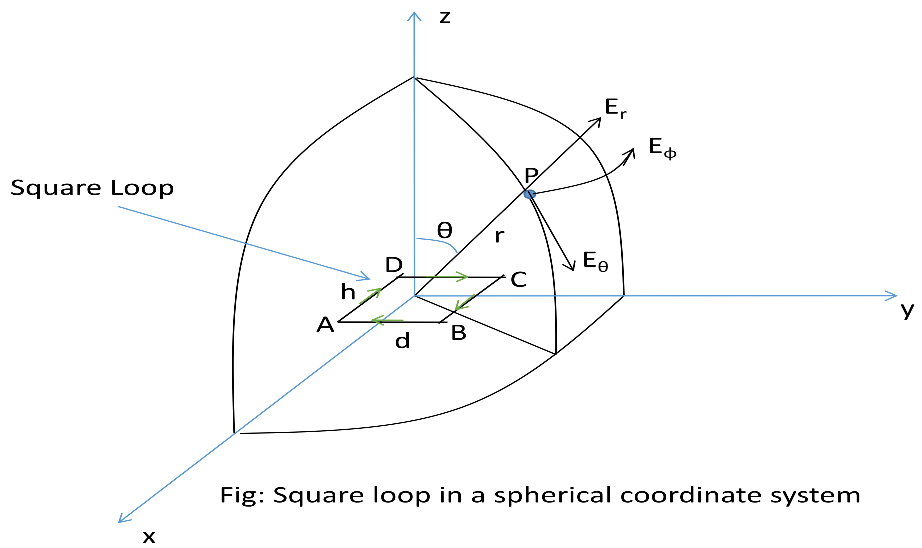
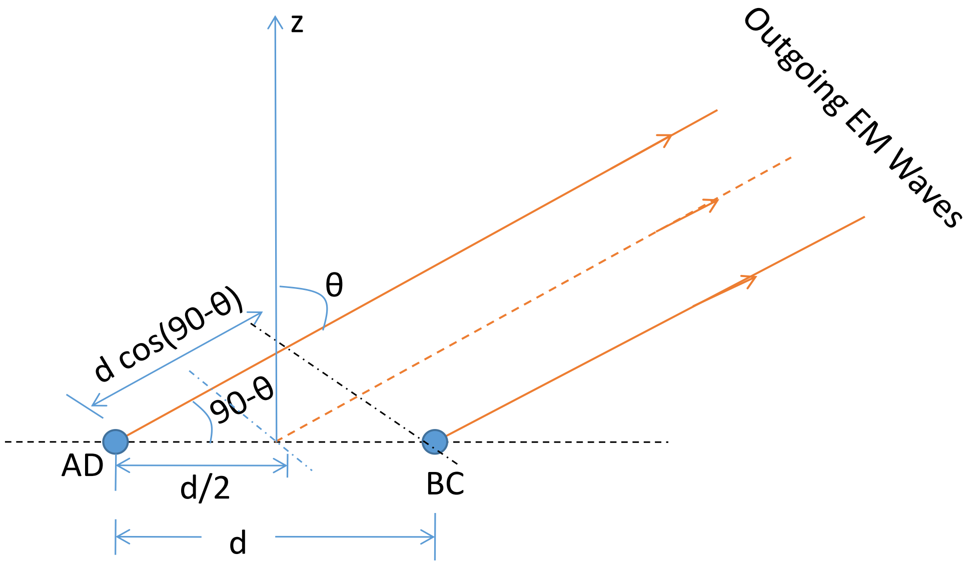


Fig: Square loop in a spherical coordinate system



From the second figure shown in above, we can derive the field components of the loop antenna.

The path difference between the waves due to two antennas AD and BC is given by

$$Path\ difference(P.D) = d\cos(90^0 - \theta) = d\ \sin\theta \qquad -1$$

$$\Psi = \frac{2\pi}{\lambda}(P.D) = \frac{2\pi}{\lambda}(d\ \sin\theta) = \beta\ d\sin\theta \qquad -2$$

The total electric field at the receiving point due two antennas AD and BC is given by

$$E_\emptyset = E_0 e^{-j\Psi/2} - E_0 e^{j\Psi/2} =- E_0\left(e^{j\Psi/2} - e^{-j\Psi/2}\right)$$

$$E_\emptyset =- \frac{2j}{2j}E_0\left(e^{j\Psi/2} - e^{-j\Psi/2}\right) =- 2jE_0\left(\frac{e^{j\Psi/2} - e^{-j\Psi/2}}{2j}\right) =- 2jE_0 sin(\Psi/2) \qquad -3$$

Substitute equation 2 in equation 3

$$E_\emptyset =- 2jE_0 sin\left(\frac{\beta\ d\sin\theta}{2}\right) =- 2jE_0\left(\frac{\beta\ d\sin\theta}{2}\right) \quad when\ d \ll \lambda$$

$$E_\emptyset =- jE_0\beta d\ \sin\theta \qquad -4$$

In above equation E_0 is known as individual field component which can be obtained from the short dipole.

We know that, the far field component of short dipole is

$$E_\theta = E_0 = \frac{j60\pi[I]h}{r\lambda} \qquad -5$$

Substitute equation 5 in equation 4

$$E_\emptyset =- j\left(\frac{j60\pi[I]h}{r\lambda}\right)\beta d\ \sin\theta = \frac{120\pi^2[I]A\ \sin\theta}{r\lambda^2} \qquad -6$$

We know that

$$\frac{E}{H} = \eta$$

$$\frac{E_\emptyset}{H_\theta} = \eta$$

Then,

$$H_\theta = \frac{E_\emptyset}{\eta} = \frac{E_\emptyset}{120\pi} \qquad -7$$

Substitute equation 6 in equation 7

$$H_\theta = \frac{E_\emptyset}{120\pi} = \frac{\frac{120\pi^2[I]A\,sin\theta}{r\lambda^2}}{120\pi} = \frac{\pi[I]A\,sin\theta}{r\lambda^2}$$

Comparison of far fields of small loop and short dipole:

The following table gives the comparison of far fields of small loop and short dipole. From the table it can be observed the following points.

(i) The field components of short dipole includes the parameter j indicates that, the field components due to short dipole are in time phase quadrature as compared with the filed components due to loop antenna.

(ii) The field components due to loop antenna are inversely proportional to the square of the wave length λ where as the field components due to short dipole are inversely proportional to wave length λ

Field	Short dipole	Loop antenna
Electric filed	$E_\theta = \frac{j60\pi[I]l\,sin\theta}{r\lambda}$	$E_\emptyset = \frac{120\pi^2[I]A\,sin\theta}{r\lambda^2}$
Magnetic filed	$\boldsymbol{H}_\emptyset = \frac{j[I]l\,sin\theta}{2r\lambda}$	$H_\theta = \frac{\pi[I]A\,sin\theta}{r\lambda^2}$

Radiation Resistances and Directives of small and large loops (Qualitative Treatment):

The radiation resistance of loop antenna is given by

$$R_r = 31{,}200\left(\frac{NA}{\lambda^2}\right)^2 \quad when\ the\ loop\ is\ small$$

$$R_r = 592\ C_\lambda \quad when\ the\ loop\ is\ large$$

Where C_λ is known as circumference in wavelength. For circular loop $C_\lambda = 2\pi a/\lambda$ where 'a' is the radius of the loop.

The directivity of the loop antenna is given by

$$Directiviyt(D) = \frac{3}{2} \qquad when\ the\ loop\ is\ small\ i.e\ when\ C_\lambda < 1/3$$

$$Directiviyt(D) = 0.68\ C_\lambda \qquad when\ the\ loop\ is\ small\ i.e\ when\ C_\lambda > 2$$

Applications of Loop Antennas:

The following are the list of applications of loop antennas

(i) In direction finding applications
(ii) Radio receivers
(iii) UHF transmitters
(iv) Aircraft receivers

ARRAYS WITH PARASITIC ELEMENTS

Yagi - Uda Arrays:

Array is defined as the method of combining the radiations from the group or array of antennas by involving the wave interference. Parasitic array or ‘array with parasitic elements’ is an array which contains one driven element and number of parasitic or passive elements. Example of parasitic array is the Yagi-Uda array. The Yagi-Uda array is invented by S.Uda and H.Yagi. The structure of 3-element yagi uda array is shown in the figure below.

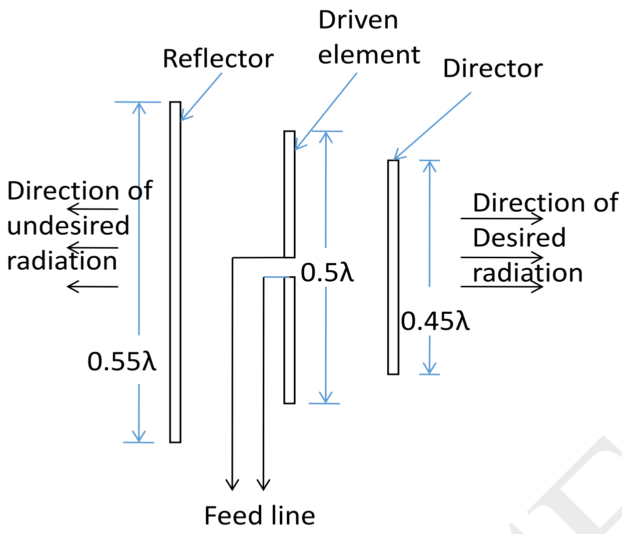


Fig : 3-element yagi-uda array

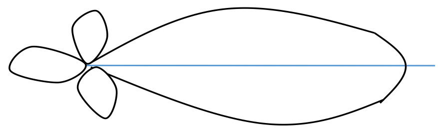


Fig : Radiation pattern

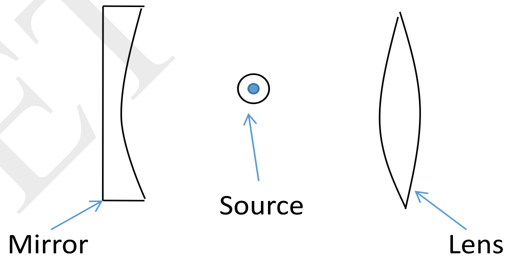


Fig : Optical equivalent

The radiation pattern and optical equivalent also shown in the figure. The principle of operation of the yagi-uda array can be explained as follows:

(i) 3-element Yagi-Uda antenna consists of one driven element, one reflector and one director.

(ii) The input signal will be supplied to the driven element and the two passive elements (reflector and director) are parasitically or electromagnetically coupled to the driven element.

(iii)The function of the reflector is to reflect back the signal and the function of the director is to further forward the signal in the forward direction.

(iv)The reflector having the nature of inductive where as the director having the nature of capacitive. The reason for this is the length of the reflector is greater than the driven element and the length of the director is smaller than the driven element.

(v) By selecting the proper length of the elements and proper spacing between the elements we can produce the highly directional beam.

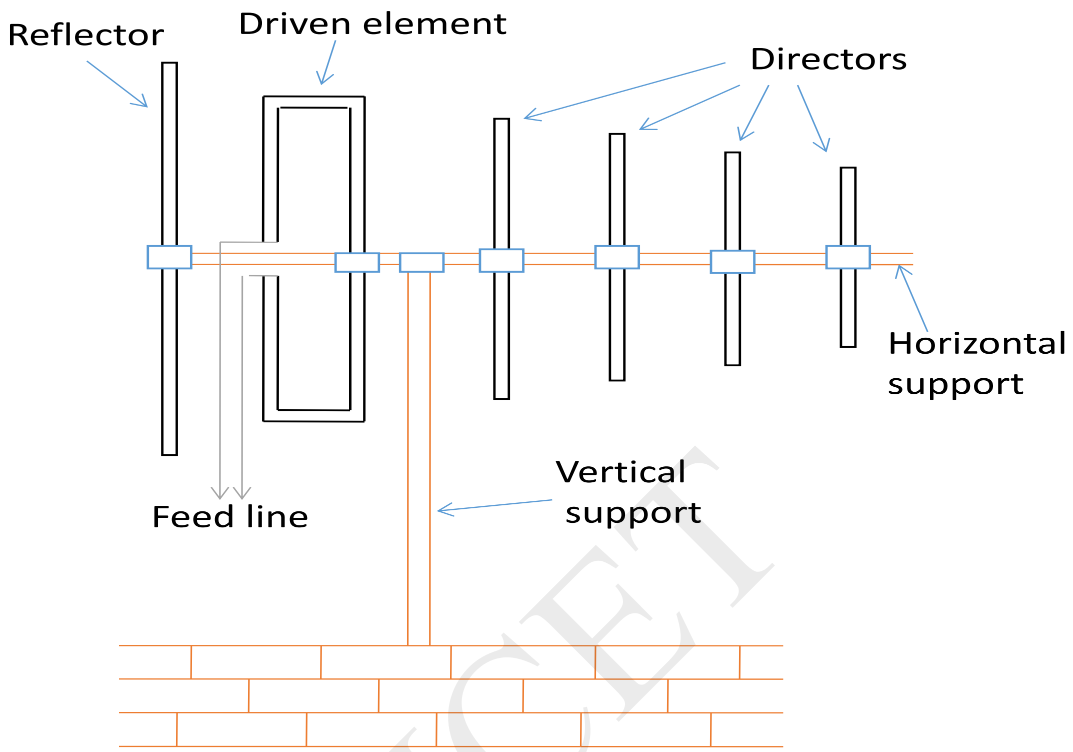
(vi)The Yagi-Uda antenna is a hi9gh gain antenna. It provides the gain in the order of 8 dB and Front to Back Ratio (FBR) of about 20 dB.

(vii) To achieve the greater directivity, the number directors can be increased.

(viii) The approximate formulae for the length of the driven element, reflector and director is given by

Length of the reflector = 500/f(MHz) feet
Length of the driven element = 475/f(MHz) feet
Length of the director = 455/f(MHz) feet

The practical structure of 6-element Yagi-Uda antenna is shown in the figure below.



Folded Dipoles &their characteristics:

A very important variation of conventional half wave dipole is the folded dipole which is shown in the figure below.

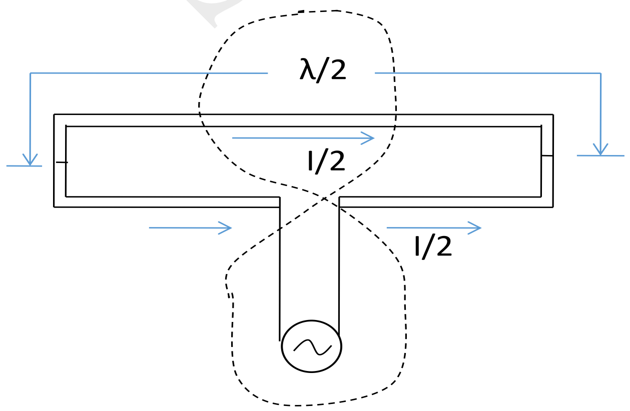


Fig : Folded dipole with current distribution and radiation pattern

The following are the important points to be noted about the folded dipole:

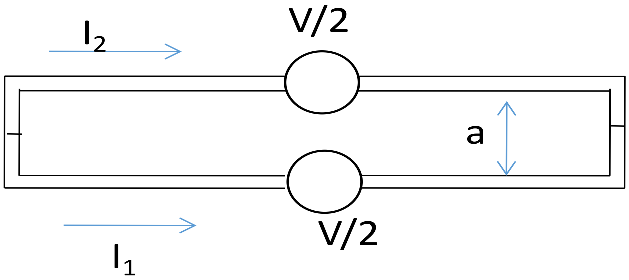
(i) Folded dipole is a combination of two half wave dipoles, one is continuous and other is splitted at the center.

(ii) The advantages of folded dipole as compared with conventional half wave dipole are high input impedance, wide band in frequency and act as a built in reactance compensations network.

(iii) The shape of radiation pattern due to the folded dipole is figure of eight shape or doughnut shape.

(iv) The folded dipole with conductors of equal radius can provide impedance up to 292 ohms and folded tripole (combination of three half wave dipoles) can provide impedance up to 657 ohms.

The equation for the input impedance of the folded dipole is obtained from the following figure:



$$\frac{V}{2} = I_1 Z_{11} + I_2 Z_{12}$$

But $I_1 = I_2$ because the two conductors are in series.

$$\frac{V}{2} = I_1 Z_{11} + I_1 Z_{12} = I_1(Z_{11} + Z_{12})$$

When the spacing between the two conductors is small, then $Z_{11} = Z_{12}$

$$\frac{V}{2} = I_1(Z_{11} + Z_{11}) = 2I_1 Z_{11}$$

$$Z = \frac{V}{I_1} = 4Z_{11} = 4(73) = 292\ Ohms.$$

In general

$$Z = n^2 Z_{11}$$

Where n represents the number of half wave dipoles used.

When the folded dipole is made with conductors of unequal radii, then the input impedance can be obtained by the following formula.

$$Z = Z_{11}\left(1 + \frac{r_2}{r_1}\right)^2$$

Where, r_2 and r_1 represent the radii of conductors.

SOLVED PROBLEMS

1.Find out the directivity or gain of short dipole or oscillating electric dipole

Ans: The directivity or directive gain is defined as

$$\mathrm{D} = \mathrm{G} = \frac{\text{Power density from the test anteena(Short dipole)}}{\text{Power density from the reference antenna(Isotropic radiator)}}$$

$$D = \frac{P_r}{\frac{W_r}{4\pi r^2}} = 4\pi r^2 \frac{P_r}{W_r} \qquad \text{------} \qquad 1$$

Where

P_r=Average pointing vector or power density due to short dipole

$\frac{W_r}{4\pi r^2}$ = power density radiated by the isotropic radiator.

r = distance from the transmitting antenna to the receiving point.

We know that,

$$P_r = \frac{1}{2}(E \times H^*)$$

$$P_r = \frac{1}{2}(|E||H|) \qquad \text{----- } 2$$

But

$$\frac{E}{H} = \eta \quad or\ E = \eta H \qquad \text{---- } 3$$

Substitute equation 3 in equation 2

$$P_r = \frac{1}{2}\eta|H|^2 \qquad \text{----- } 4$$

The magnetic field intensity due to short dipole is given by

$$H = H_\emptyset = \frac{j\beta I_m l \sin\theta e^{j\omega\left(t-\frac{r}{c}\right)}}{4\pi r}$$

$$|H| = \frac{\beta I_m l \sin\theta}{4\pi r}$$

For maximum value of H, θ=90^0

$$|H| = \frac{\beta I_m l}{4\pi r} \qquad \text{--------- } 5$$

Substitute equation 5 in equation 4

$$P_r = \frac{1}{2}\eta\left(\frac{\beta I_m l}{4\pi r}\right)^2 = \frac{1}{2}\eta\frac{\beta^2 I_m^2 l^2}{16\pi^2 r^2}$$

But β=2π/λ

$$P_r = \frac{1}{2}\eta\frac{\left(\frac{2\pi}{\lambda}\right)^2 I_m^2 l^2}{16\pi^2 r^2}$$

$$P_r = \frac{\eta I_m^2 l^2}{8\lambda^2 r^2} \qquad \text{------- } 6$$

The power radiated is given by

$$W_r = I_{rms}^2 R_r \qquad \text{--------- } 7$$

Where R_r is the radiation resistance

But the radiation resistance due to short dipole is given by

$$R_r = 80\pi^2\left(\frac{l}{\lambda}\right)^2 \qquad \text{------------ } 8$$

Where *l* is the length of the short dipole.

Substitute equation 8 in equation 7

$$W_r = I_{rms}^2 80\pi^2\left(\frac{l}{\lambda}\right)^2$$

But $I_{rms} = \frac{I_m}{\sqrt{2}}$

$$W_r = \left(\frac{I_m}{\sqrt{2}}\right)^2 80\pi^2\left(\frac{l}{\lambda}\right)^2 = \frac{80 I_m^2 \pi^2 l^2}{2\lambda^2} \qquad \text{--- } 9$$

Substitute equations 6 and 9 in equation 1, then

$$D = 4\pi r^2 \frac{\eta I_m^2 l^2}{8\lambda^2 r^2} \times \frac{2\lambda^2}{80 I_m^2 \pi^2 l^2}$$

But $\eta = 120\pi$

$$D = \frac{\eta}{80\pi} = \frac{3}{2} = 1.5$$

The directivity in dB is given by

$$D = 10\log(1.5) = 1.76\,dB$$

Therefore the gain or directivity of short dipole is 1.5 or 1.76 dB.

2. Calculate the maximum effective area of short dipole

Ans: Consider the short dipole shown in the figure 1.20 below. The short dipole is used as the receiving antenna.

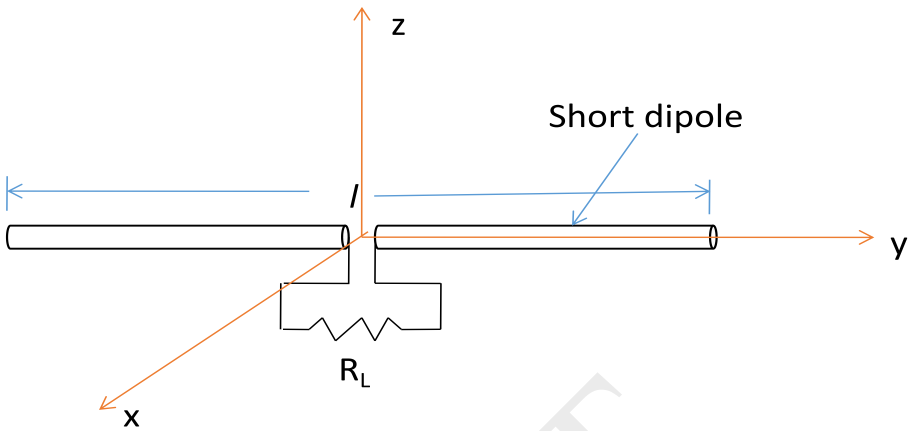


Fig1.20:The short dipole with uniform current along its entire Length and terminated by load resistance R_L

We know that the maximum effective area is

$$A_{em} = \frac{V^2}{4PR_r} \qquad \text{-----1}$$

The induced voltage (V) in a short dipole due to incident electric field (E) is given by

$$V = E \cdot l \qquad \text{----- 2}$$

In above equation it is assumed that, the incident electric field is uniform about the entire length of short dipole.

The pointing vector (P) is given by

$$P = E \times H = \frac{E^2}{\eta} \qquad \text{---- 3}$$

Also the radiation resistance due to short dipole is given b y

$$R_r = 80\pi^2\left(\frac{l}{\lambda}\right)^2 \qquad \text{---- 4}$$

Substitute equations 2,3&4 in equation 1

$$A_{em} = \frac{(E \cdot l)^2}{4\frac{E^2}{\eta}80\pi^2\left(\frac{l}{\lambda}\right)^2}$$

$$A_{em} = \frac{E^2 l^2 \eta \lambda^2}{4E^2 80\pi^2 l^2}$$

$$A_{em} = \frac{\eta\lambda^2}{4 \times 80\pi^2} = \frac{120\pi\lambda^2}{4 \times 80\pi^2} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2$$

$$\therefore A_{em} = 0.119\lambda^2$$

3. Calculate the directivity or gain of the half wave dipole

Ans: The half wave dipole is the antenna having the physical length of λ/2.

The directivity or directive gain is defined as

$$\mathrm{D} = \mathrm{G} = \frac{\text{Power density from the test anteena(half wave dipole)}}{\text{Power density from the reference antenna(Isotropic radiator)}}$$

$$D = \frac{P_r}{\frac{W_r}{4\pi r^2}} = 4\pi r^2 \frac{P_r}{W_r} \qquad \text{------} \qquad 1$$

Where

P_r=Average pointing vector or power density due to short dipole

$\frac{W_r}{4\pi r^2}$ = power density radiated by the isotropic radiator.

r = distance from the transmitting antenna to the receiving point.

We know that,

$$P_r = \frac{1}{2}(E \times H^*)$$

$$P_r = \frac{1}{2}(|E||H|) \qquad \text{-----} \qquad 2$$

But

$$\frac{E}{H} = \eta \quad or\ E = \eta H \qquad \text{----} \qquad 3$$

Substitute equation 3 in equation 2

$$P_r = \frac{1}{2}\eta |H|^2 \qquad \text{-----} \qquad 4$$

The magnitude of magnetic field intensity due to half wave dipole is given by

$$|H| = \frac{I_m}{2\pi r}\left\{\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right\}$$

To have the maximum value for H, take $\theta=90^0$, then above equation becomes

$$|H| = \frac{I_m}{2\pi r} \qquad \text{--}\ 5$$

Substitute equation 5 in equation4

$$P_r = \frac{1}{2}\eta\left(\frac{I_m}{2\pi r}\right)^2 = \frac{\eta I_m^2}{2(4\pi^2 r^2)}$$

$$P_r = \frac{\eta I_m^2}{8\pi^2 r^2} \qquad \text{--} \qquad 6$$

The power radiated is given by

$$W_r = I_{rms}^2 R_r \qquad \text{---------} \qquad 7$$

Where R_r is the radiation resistance

But the radiation resistance due to half wave dipole is given by

$$R_r = 73\Omega \qquad \text{---}\ 8$$

Substitute equation 8 in equation 7, then

$$W_r = I_{rms}^2(73)$$

But $I_{rms} = \frac{I_m}{\sqrt{2}}$

$$W_r = \left(\frac{I_m}{\sqrt{2}}\right)^2 (73)$$

$$W_r = \frac{73 I_m^2}{2} \qquad \text{--} \qquad 9$$

Substitute equations 6 and 9 in equation1

$$D = 4\pi r^2 \frac{\frac{\eta I_m^2}{8\pi^2 r^2}}{\frac{73 I_m^2}{2}} = 4\pi r^2 \frac{\eta I_m^2}{8\pi^2 r^2} \times \frac{2}{73 I_m^2}$$

$$D = \frac{\eta}{73\pi}$$

But $\eta = 120\pi$

$$D = \frac{120\pi}{73\pi} = \frac{120}{73} = 1.64$$

The directivity in dB is given by

$$D = 10 \log (1.64) = 2.148\ dB$$

Therefore the gain or directivity of half wave dipole is 1.64 or 2.148 dB.

4. Calculate the maximum effective area of half wave dipole

We know that the maximum effective area is

$$A_{em} = \frac{V^2}{4PR_r} \qquad \text{-----1}$$

To find out the voltage induced in the half wave dipole due to incident electric field we need to consider the current distribution on the half wave dipole which is shown in the figure1.21 below

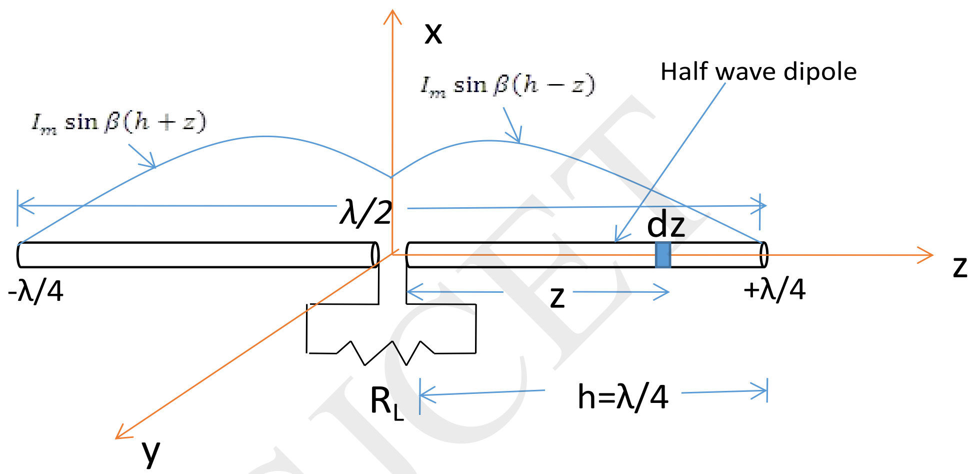


Fig1.21: Half wave dipole and its current distribution

$$I = I_m \sin \beta(h - z) \qquad for \quad z > 0$$
$$I = I_m \sin \beta(h + z) \qquad for \quad z < 0$$

But h = λ/4

$$I = I_m \sin \beta(h - z) = I_m \sin \beta \left(\frac{\lambda}{4} - z\right) = I_m \sin \left(\beta\left(\frac{\lambda}{4}\right) - \beta z\right)$$

$$I = I_m \sin \left(\frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) - \beta z\right) = I_m \sin \left(\frac{\pi}{2} - \beta z\right)$$

$$I = I_m \cos \beta z \qquad -2$$

The above equation can also written as

$$dI = dI_m \cos \beta z$$

Or

$$dV = dV_m \cos \beta z \qquad -3$$

We know that,

$$V = E \cdot l$$

Or

$$dV_m = E \cdot dl = E \cdot dz \qquad -4$$

Substitute equation 4 in equation 3

$$dV = E \cdot dz \cos \beta z$$

$$dV = E\, cos\beta z\, dz$$

Take integration on both sides

$$\int dV = \int E\, cos\beta z\, dz$$

$$V = \int_{-\lambda/4}^{\lambda/4} E\, cos\beta z\, dz$$

$$V = 2\int_{0}^{\lambda/4} E\, cos\beta z\, dz$$

$$V = 2E\left[\frac{sin\beta z}{\beta}\right]_0^{\lambda/4}$$

$$V = \frac{2E}{\beta}(\sin\beta(\lambda/4) - \sin\beta(0))$$

$$V = \frac{2E}{\frac{2\pi}{\lambda}}\left(\sin\frac{2\pi}{\lambda}\frac{\lambda}{4} - 0\right)$$

$$V = \frac{E\lambda}{\pi} \qquad -5$$

$$P = E \times H = \frac{E^2}{\eta} \qquad -6$$

Substitute equation 5 and 6 in equation 1

$$A_{em} = \frac{\left(\frac{E\lambda}{\pi}\right)^2}{4\frac{E^2}{\eta}R_r} \qquad -7$$

The radiation resistance due to half wave dipole is given by

$$R_r = 73\ \Omega \qquad -8$$

Substitute equation 8 in equation 7

$$A_{em} = \frac{\left(\frac{E\lambda}{\pi}\right)^2}{4\frac{E^2}{\eta}(73)} = \frac{E^2\lambda^2\eta}{4E^2\pi^2(73)} = \frac{\lambda^2\eta}{4\pi^2(73)}$$

But $\eta = 120\ \pi$

$$A_{em} = \frac{\lambda^2 120\pi}{4\pi^2(73)} = 0.13\ \lambda^2$$

5. Calculate the effective length of the half wave dipole

Sol:

The effective length is given by

$$l_e = 2\sqrt{\frac{A_{em}R_r}{\eta}} \qquad -1$$

For half wave dipole

$$R_r = 73\ \Omega \qquad -2$$

$$A_{em} = 0.13\ \lambda^2 \qquad -3$$

Substitute equations 2 and 3 in equation 1

$$l_e = 2\sqrt{\frac{0.13\ \lambda^2(73)}{120\pi}} = 0.3174\ \lambda$$

6. Find out the directivity of isotropic radiator

Sol:

The formula for directivity is given by

$$D = \frac{4\pi}{Beam\ area} = \frac{4\pi}{4\pi} = 1$$

7. Calculate the maximum effective aperture of a microwave antenna which has a directivity of 900.

Sol:

The relation between the directivity and effective area is given by

$$D = G = \frac{4\pi A_e}{\lambda^2}$$

$$A_e = \frac{D\,\lambda^2}{4\pi} = \frac{900\,\lambda^2}{4\pi} = 71.619\,\lambda^2$$

8. An antenna has a radiation resistance of 72 ohms, a loss resistance of 8 ohms and power gain of 12 dB. Determine the antenna efficiency and its directivity.

Sol:

Given data:

$$R_r = 72\,\Omega,\ R_l = 8\Omega,\ G_p = 12\,dB = 15.85$$

The antenna efficiency factor is given by

$$\eta = \frac{R_r}{R_r + R_l} = \frac{72}{72+8} = 0.9$$

Antenna efficiency = 0.9 X 100 = 90%

The relation between the power gain and directive gain is given by

$$\eta = \frac{G_p}{G_d} = \frac{G_p}{D}$$

$$G_d = D = \frac{G_p}{\eta} = \frac{15.85}{0.9} = 17.611$$

Or

$$D = 10\log(17.611) = 12.458\,dB$$

9. A Low frequency transmitting antenna has a radiation resistance of 0.5 ohms and a total loss of 2.5 ohms. Calculate the radiated power, power input and antenna efficiency if the current applied to the antenna is 100 A(rms).

Sol:

Given data:

$$R_r = 0.5\,\Omega,\ R_l = 2.5\Omega,\ I_{rms} = 100\,A$$

We know that, the power radiated is given by

$$W_r = I_{rms}^2 R_r = 100^2(0.5) = 5\,kW$$

Power input is given by

$$W_T = I_{rms}^2(R_A) = I_{rms}^2(R_r + R_l) = 100^2(0.5+2.5) = 30\,kW$$

The antenna efficiency is given by

$$\eta = \frac{R_r}{R_r + R_l} \times 100 = \frac{0.5}{0.5+2.5} \times 100 = 16.6\%$$

10. An antenna has a field pattern given by

$$E(\theta) = \cos^2\theta, \quad \textbf{for} \quad 0^0 \le \theta \le 90^0$$

Find Half Power Beamwidth(HPBW)

Sol:

$$E(\theta) = \cos^2\theta, \quad \text{for} \quad 0^0 \le \theta \le 90^0$$

At half power points, the electric field will be

$$E(\theta) = 0.707$$
$$0.707 = \cos^2\theta$$
$$\cos\theta = \sqrt{0.707}$$
$$\theta = \cos^{-1}\left(\sqrt{0.707}\right) = 33^0$$
$$HPBW = 2\times\theta = 2\times 33 = 66^0$$

11. An antenna has a field pattern given by

$$E(\theta) = \cos\theta\cos 2\theta, \quad \textbf{for} \;\; 0^0 \le \theta \le 90^0$$

Find (a) Half Power Beamwidth(HPBW) and (b) Beamwidth between First Nulls(FNBW)

Sol:

$$E(\theta) = \cos\theta\cos 2\theta, \quad \textbf{for} \;\; 0^0 \le \theta \le 90^0$$

At half power points, the electric field will be

$$E(\theta) = 0.707 = \frac{1}{\sqrt{2}}$$
$$\frac{1}{\sqrt{2}} = \cos\theta\cos 2\theta$$
$$\cos 2\theta = \frac{1}{\sqrt{2}\cos\theta}$$
$$2\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta}\right)$$
$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta'}\right)$$

By iterating with $\theta' = 0$ as a first guess,

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos(0)}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 22.5^0$$

Let $\theta' = 22.5^0$, then

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos(22.5)}\right) = 20.03^0$$

Let $\theta' = 20.03^0$, then

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos(20.03)}\right) = 20.59^0$$

Let $\theta' = 20.59^0$, then

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos(20.59)}\right) = 20.47^0$$

Let $\theta' = 20.47^0$, then

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos(20.47)}\right) = 20.47^0 \cong 20.5^0$$

Therefore,

$$\theta = \theta' = 20.5^0$$

(a) The Half Power Beamwidth is given by

$$HPBW = 2\times\theta = 2\times 20.5 = 41^0$$

(b) The First Null Beam Width (FNBW) is obtained as follows:
At FNBW, the electric filed will be zero.

$$E(\theta)=0$$

$$0=\cos\theta\cos 2\theta$$

$$\cos 2\theta=0$$

$$2\theta=\cos^{-1}(0)=90^0$$

$$\theta=\frac{90}{2}=45^0$$

$$FNBW=2\times\theta=2\times 45=90^0$$

12. Find the number of square degrees in the solid angle Ω on a spherical surface that is between θ = 20⁰ and θ = 40⁰ and between ϕ = 30⁰ and ϕ = 70⁰.

Sol:

We know that,

$$\mathrm{ds}=\mathrm{r}^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\emptyset$$

Solid angle is given by

$$\mathrm{d}\Omega=\frac{\mathrm{ds}}{\mathrm{r}^2}$$

$$\mathrm{d}\Omega=\frac{\mathrm{r}^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\emptyset}{\mathrm{r}^2}=\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\emptyset$$

$$\mathrm{d}\Omega=\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\emptyset$$

Take integration on both sides

$$\Omega=\int\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\emptyset$$

$$\Omega=\int_{20}^{40}\sin\theta\,\mathrm{d}\theta\int_{30}^{70}\mathrm{d}\emptyset$$

$$\Omega=[-\cos\theta]_{20}^{40}\times[\emptyset]_{30}^{70}$$

$$\Omega=(-\cos 40+\cos 20)\times(70-30)\times\frac{\pi}{180}=0.121\text{ steradians}$$

$$\Omega=0.121\times\left(\frac{180}{\pi}\right)^2=397\text{ square degrees}$$

13. An antenna has a field pattern given by

$$E(\theta)=\cos^2\theta,\quad \textbf{for}\ \ 0^0\le\theta\le 90^0$$

Find the beam area of this pattern

Sol:

$$E(\theta)=\cos^2\theta,\quad \textbf{for}\ \ 0^0\le\theta\le 90^0$$

At half power points, the electric field will be

$$E(\theta)=0.707$$

$$0.707=\cos^2\theta$$

$$\cos\theta=\sqrt{0.707}$$

$$\theta=\cos^{-1}\left(\sqrt{0.707}\right)=33^0$$

$$HPBW=2\times\theta=2\times 33=66^0$$

$$\theta_{HP}=\emptyset_{HP}=66^0$$

The approximate formula for beam area is given by

$$\Omega_{\mathrm{A}}=\theta_{HP}\emptyset_{HP}$$

$$\Omega_A = 66 \times 66 = 4356 \text{ Square degrees}$$

But one square radian is equal to 3283 square degrees.

$$\Omega_A = \frac{4356}{3283} = 1.33 \text{ steradian}$$

14. A radio link has a 15-W transmitter connected to an antenna of 2.5 m^2 effective aperture at 5 GHz. The receiving antenna has an effective aperture of 0.5 m^2 and is located at a 15-km line-of-sight distance from the transmitting antenna. Assume lossless, matched antennas, find the power delivered to the receiver.

Sol:

Transmitted power (P_T) = 15 W

Effective aperture of transmitting antenna (A_{eT}) = 2.5 m^2

Frequency of operation (f) = 5 GHz

Wavelength (λ) = c/f = ($3X10^8$)/($5X10^9$) = 0.06

Effective aperture of receiving antenna (A_{eR}) = 0.5 m^2

Distance between transmitter and receiver (R) = 15 km

$$G_T = \frac{4\pi A_{eT}}{\lambda^2}$$

$$G_R = \frac{4\pi A_{eR}}{\lambda^2}$$

The power received by receiver is given by

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi R}\right)^2$$

$$P_R = P_T \left(\frac{4\pi A_{eT}}{\lambda^2}\right)\left(\frac{4\pi A_{eR}}{\lambda^2}\right)\left(\frac{\lambda}{4\pi R}\right)^2$$

$$P_R = P_T \frac{A_{eT} A_{eR}}{R^2 \lambda^2}$$

$$P_R = 15 \times \frac{2.5 \times 0.5}{15^2 \times 0.06^2}$$

$$P_R = 23\mu W$$

15. An elliptically polarized wave traveling in the positive z direction in air has x and y components.

$$\boldsymbol{E_x = 3\sin(\omega t - \beta x) \quad V/m}$$

$$\boldsymbol{E_y = 6\sin(\omega t - \beta x + 75^0) \quad V/m}$$

Find the average power per unit area conveyed by the wave.

Sol:

$$E_x = 3\sin(\omega t - \beta x) \quad V/m$$

$$E_y = 6\sin(\omega t - \beta x + 75^0) \quad V/m$$

From poynting theorem, the average poynting vector(Average power per unit area) is given by

$$P_{av} = \frac{1}{2}\frac{E^2}{\eta} = \frac{1}{2}\frac{E_x^2 + E_y^2}{\eta}$$

$$P_{av} = \frac{1}{2}\frac{E^2}{\eta} = \frac{1}{2} \times \frac{3^2 + 6^2}{377} = 60\ mW/m^2$$

16. Calculate the physical height of a half wave dipole (λ/2) having antenna Q of 30 and bandwidth of 10 Mhz.

Sol:

Quality factor (Q) = 30

Bandwidth (BW) = 10 MHz

$$Q = \frac{f}{BW}$$

$$f = Q \times BW = 30 \times 10 \times 10^6 = 3 \times 10^8\,Hz$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^8} = 1m$$

The physical height of half wave dipole is

$$l = \frac{\lambda}{2} = \frac{1}{2} = 0.5m$$

17. A resonant half wave length dipole is made out of copper (σ = 10X10^7 siemen/m). Calculate the conduction dielectric radiation efficiency of the dipole antenna at f = 100 MHz if the radius of the wire is r_0 = 3 X 10^{-4} λ and radiation resistance of the λ/2 dipole is 73 ohms.

Sol:

Conductivity (σ) = 10X10^7 siemen/m

Frequency (f) = 100 MHz

Wavelength is given by

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{100 \times 10^6} = 3m$$

Radius of the wire is(r_0) = 3 X 10^{-4} λ

Radiation resistance (R_r) = 73 ohms

The loss resistance at high frequency is given by

$$R_{hf} = R_l = \frac{l}{2\pi r_0} R_s = \frac{l}{2\pi r_0}\sqrt{\frac{\omega\mu}{2\sigma}}$$

$$R_l = \frac{\lambda/2}{2\pi r_0}\sqrt{\frac{2\pi f\mu}{2\sigma}}$$

$$R_l = \frac{3/2}{2\pi \times 3 \times 10^{-4} \times 3}\sqrt{\frac{2\pi \times 100 \times 10^6 \times 4\pi \times 10^{-7}}{2 \times 10 \times 10^7}} = 0.698\Omega$$

Antenna efficiency is given by

$$\eta = \frac{R_r}{R_r + R_l} \times 100 = \frac{73}{73 + 0.698} \times 100 = 99.052\%$$

18. A dipole antenna with length equal to 25 cm and carrying a current of 2 A at a frequency of 8.5 MHz radiates in to free space. Calculate the total power radiated by that antenna.

Sol:

Given data:

$$lenght\ of\ the\ dipole\ (l) = 25\ cm = 0.25\ m$$

$$Current\ (I_m) = 2\ A$$

$$Frequency\ (f) = 8.5\ MHz, \lambda = \frac{c}{f} = \frac{3 \times 10^8}{8.5 \times 10^6} = 35.294\ m$$

The power radiated due to short dipole is given by

$$W = \frac{\eta\left((\beta I_m l)\right)^2}{12\pi} = \frac{\eta \beta^2 I_m^2 l^2}{12\pi} = \frac{\eta\left(\frac{2\pi}{\lambda}\right)^2 I_m^2 l^2}{12\pi}$$

$$W = \frac{120\pi \times 4\pi^2 \times 2^2 \times 0.25^2}{12\pi \times 35.294} = 0.0792\ watts$$

19. At what frequency the 65 cm length antenna produces a radiation resistance of 0.75 Ω

Sol:

Given data:

$$length\ (l) = 65\ cm = 0.65\ m$$
$$Radiation\ resistance (R_r) = 075\ \Omega$$

We know that,

$$R_r = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2 \frac{l^2}{\lambda^2}$$

$$\lambda^2 = \frac{80\ \pi^2 l^2}{R_r}$$

$$\lambda = \sqrt{\frac{80\ \pi^2 l^2}{R_r}}$$

But λ = c/f

$$\frac{c}{f} = \sqrt{\frac{80\ \pi^2 l^2}{R_r}}$$

$$f = c\sqrt{\frac{R_r}{80\ \pi^2 l^2}} = 3 \times 10^8 \sqrt{\frac{0.75}{80\ \pi^2 \times 0.65^2}} = 14.3\ MHz$$

20. Calculate the radiation resistance of a dipole antenna having length λ/8, if the equivalent loss resistance accounting for the heat loss in the antenna due to finite conductivity of the dipole is 1.5 Ω. Also find the efficiency of the antenna.

Sol:

Given data:

$$Length\ of\ the\ dipole(l) = \frac{\lambda}{8}$$
$$Loss\ resistance\ (R_l) = 1.5\ \Omega$$

$$R_r = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2 \frac{l^2}{\lambda^2}$$

$$R_r = 80\pi^2 \frac{\left(\frac{\lambda}{8}\right)^2}{\lambda^2} = 98.75\ \Omega$$

Antenna efficiency is given by

$$\%\ \eta = \frac{R_r}{R_r + R_l} \times 100 = \frac{98.75}{98.75 + 1.5} \times 100 = 98.5\%$$

21. A radiating element of 1 cm carries an effective current of 0.5 Amp at 3 GHz. Calculate the radiated power

Sol:

Length of the element (l) = 1 cm = 0.01 m
Current (I_m) = 0.5 Amp
Frequency (f) = 3 GHz
Wavelength is given by

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^9} = 0.1m$$

The radiated power is given by

$$W = \frac{\eta(\beta I_m l)^2}{12\pi} = \frac{\eta\beta^2 I_m^2 l^2}{12\pi} = \frac{\eta\left(\frac{2\pi}{\lambda}\right)^2 I_m^2 l^2}{12\pi}$$

$$W = \frac{120\pi\times\left(\frac{2\pi}{0.1}\right)^2\times(0.5)^2\times(0.01)^2}{12\pi} = 0.986w$$

22. Design 3 element Yagi-Uda array with frequency of operation 64 MHz.

Sol:

$$Length\ of\ the\ reflector = \frac{500}{f(MHz)}\quad feet = \frac{500}{64} = 7.812\ feet$$

$$Length\ of\ the\ driven\ element = \frac{475}{f(MHz)}\ feet = \frac{475}{64} = 7.421\ feet$$

$$Length\ of\ the\ director = \frac{455}{f(MHz)}\ feet = \frac{455}{64} = 7.11\ feet$$

23. Design Yagi-Uda antenna of six elements to provide a gain of 12 dBi if the operating frequency is 200 MHz.

Sol:

Given

Frequency (f) = 200 MHz

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{200\times10^6} = 1.5\ m$$

$$Length\ of\ reflector = 0.475\ \lambda = 0.475\times1.5 = 0.7125\ m$$

$$Length\ of\ driven\ element = 0.46\ \lambda = 0.46\times1.5 = 0.69\ m$$

$$Length\ of\ first\ director = 0.44\ \lambda = 0.44\times1.5 = 0.66\ m$$

$$Length\ of\ second\ director = 0.44\ \lambda = 0.44\times1.5 = 0.66\ m$$

$$Length\ of\ third\ director = 0.43\ \lambda = 0.43\times1.5 = 0.645\ m$$

$$Length\ of\ fourth\ director = 0.40\ \lambda = 0.40\times1.5 = 0.6\ m$$

$$Spacing\ between\ reflector\ and\ driven\ element = 0.25\ \lambda = 0.25\times1.5 = 0.375\ m$$

$$Spacing\ between\ driven\ element\ and\ first\ director = 0.31\ \lambda = 0.31\times1.5 = 0.465\ m$$

$$Spacing\ between\ two\ directors = 0.31\ \lambda = 0.31\times1.5 = 0.465\ m$$

$$Length\ of\ the\ array = 1.5\ \lambda = 1.5\times1.5 = 2.25\ m$$

UNIT-2 (VHF, UHF AND MICROWAVE ANTENNAS)

Syllabus: Helical Antennas-Helical Geometry, Helix modes, Horn Antennas- Types, Fermat's Principle, Optimum Horns, Design considerations of Pyramidal Horns, Micro strip Antennas- Introduction, features, advantages and limitations, Rectangular patch antennas Geometry and parameters, characteristics of Micro strip antennas, reflector antennas - Introduction, corner reflectors, parabola reflectors- geometry, pattern characteristics, Feed Methods, Reflector Types - Related Features, Lens Antennas - Geometry of Non-metallic Dielectric Lenses, Zoning , Tolerances, Applications

HELICAL ANTENNAS

Helical Geometry:

The geometry of constructional features of helical antenna is shown in the figure below.

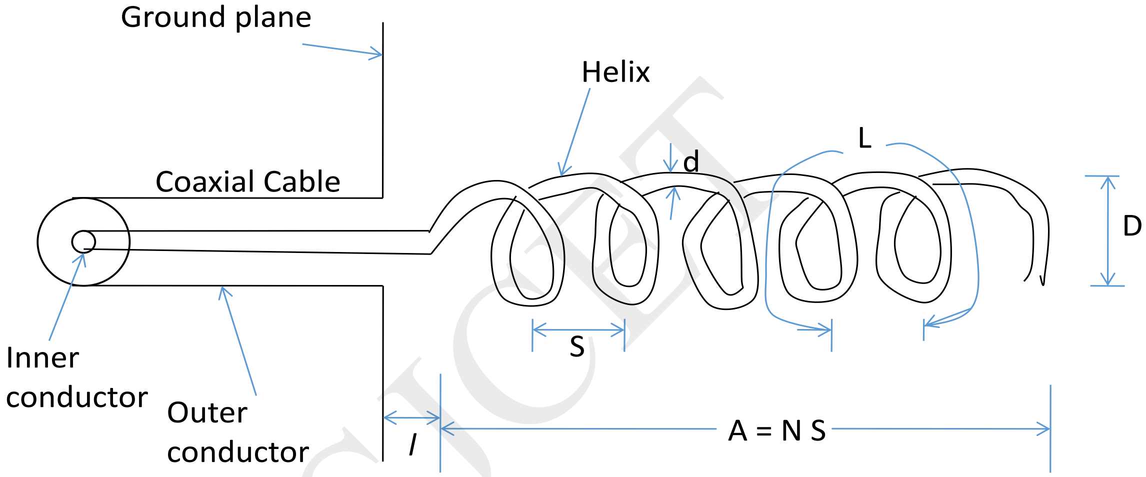


Fig : Helical antenna

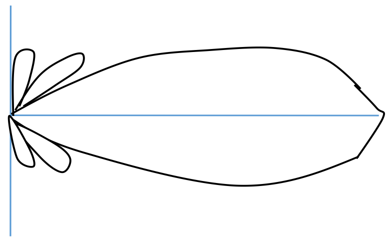
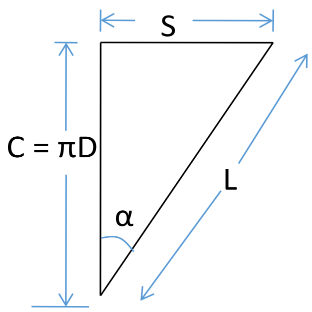


Fig : Radiation pattern



Helical antenna consists of a helix of thick copper wire or tubing wound in the shape of a screw thread and used as an antenna in conjunction with a flat metal plate called a ground plate. It is the simplest antenna to produce the circularly polarized waves. It is a broadband antenna. The physical parameters of the helical antenna are listed below:

C = Circumference of the helix
d = Diameter of the helix conductor
D = Diameter of the helix
A = NS = Axial length or length of the helix

N = Number of turns
S = Spacing between the turns
L = Length of each turn
l = Spacing between the helix and the ground plate.
$\alpha = \tan^{-1}(S/C)$ = Pitch angel of the helix.

The pitch angle (α) of the helix is defined as the angle between the line Tangent to the helix wire and the plane normal to the helix axis.

Helix modes:

Helical antenna can be operated in two modes such as

(i) Normal mode or perpendicular mode of operation
(ii) Axial mode or beam mode of radiation

Normal Mode: The helical antenna can be operated in the normal mode when the dimensions of the helix are smaller than the wavelength (λ). The helical antenna is a combination of loop antenna and short dipole as shown in the figure below.

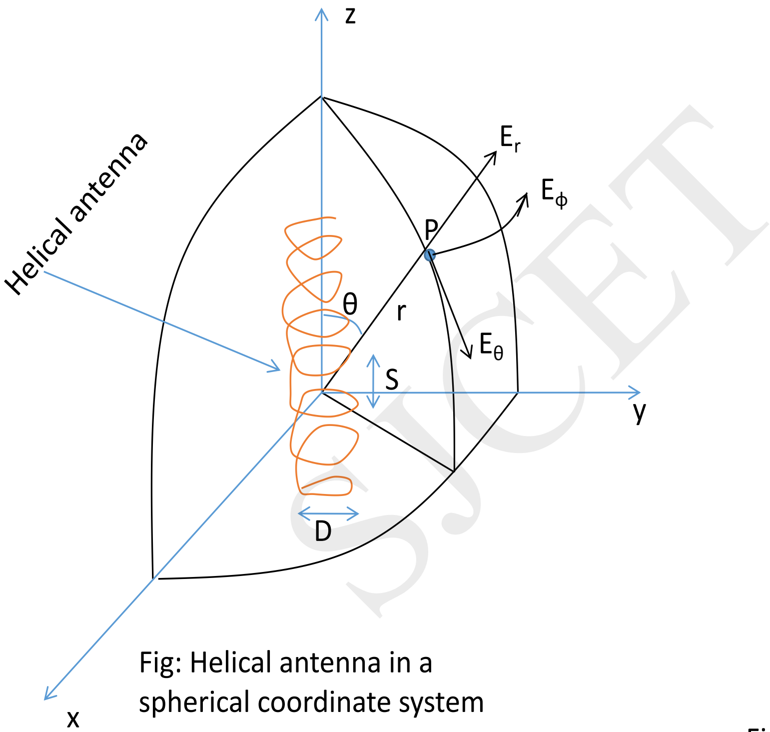


Fig: Helical antenna in a spherical coordinate system

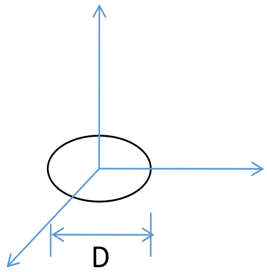


Fig : Loop when $\alpha = 0^0$

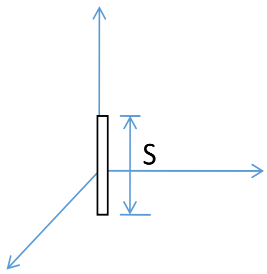


Fig : Short dipole when $\alpha = 90^0$

In this mode of operation, the direction of maximum radiation is in perpendicular or in normal direction with respect to the helix axis. The shape of the radiation pattern in this mode is bidirectional. When the spacing 'S' of the helical antenna tends to zero or when the pitch angle is equal to 0^0, then the helix reduces to loop antenna as shown in the figure above. Similarly when the diameter 'D' of the helix tends to zero or pitch angle is equal to 90^0, then the helical antenna reduced to short dipole with length 'S'. Therefore the far field equations of the helical antenna van be obtained from the far field equations of short dipole and loop antenna.

The electric filed component due to the short dipole of length 'S' is given by

$$E_\theta = \frac{j60\pi[I]sin\theta}{r}.\frac{S}{\lambda}$$

Similarly the electric field component due to the loop antenna is given by

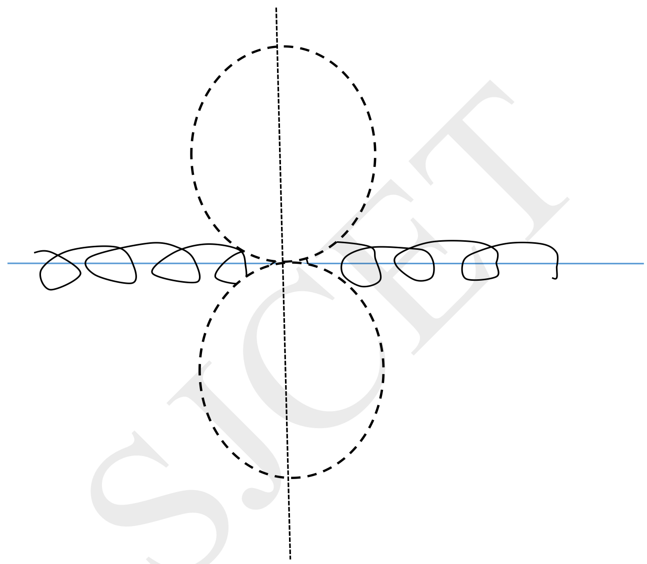
$$E_{\emptyset} = \frac{120\pi^2 [I] sine\theta}{r} . \frac{A}{\lambda^2}$$

Where A = area of the loop antenna.
For circular loop with diameter 'D' the area is given by $A = \pi D^2/4$
The axial ratio (AR) of helical antenna is given by

$$AR = \frac{|E_\theta|}{E_\emptyset} = \frac{S\lambda}{2\pi A} = \frac{2S\lambda}{\pi^2 D^2} = \frac{2S\lambda}{C^2}$$

AR = 1, for circular polarization
AR > 1, for elliptical polarization
AR = 0, for linear horizontal polarization
AR = ∞, for linear vertical polarization

The radiation pattern of the helical antenna in the normal mode is shown in the figure below.



Axial Mode: The helical antenna can be operated in the axial mode when the dimensions of the helix are larger than the wavelength (λ). Axial mode is also known as beam mode of radiation. In this mode the maximum radiation will be in the direction of the helix axis. The radiation pattern produced in this mode is unidirectional. This mode of operation is preferable used to produce the circular polarization. The radiation pattern of the helical antenna under the axial mode is shown in the figure below.

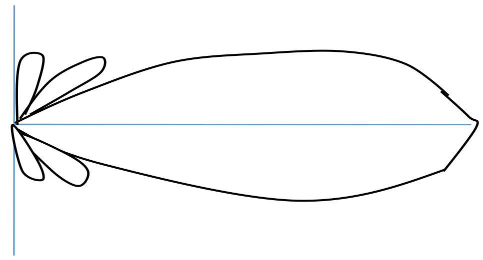


Fig : Radiation pattern

Practical Design considerations for Monofilar Helical Antenna in Axial and Normal Modes:

When the helical antenna is made from the single conductor, then it is known as monofilar helical antenna. Practical design considerations of monofilar helical antenna are given below.

Axial length (A) = N S

Spacing between the helix and ground plane (l) = S/2

Length of each turn (L) = $\sqrt{S^2 + C^2}$

Circumference of the helix (C) = π D

Pitch angel (α) = $\tan^{-1}$(S/C)

Axial Ratio (AR) = $2S\lambda/C^2$

Half Power Beam Width (HPBW) = $\frac{52}{C}\sqrt{\frac{\lambda^3}{NS}}$

Beam Width between First Nulls (BWFN) = $\frac{115}{C}\sqrt{\frac{\lambda^3}{NS}}$

Directivity (D) = $\frac{15\, NSC^2}{\lambda^3}$

HORN ANTENNAS

Types:

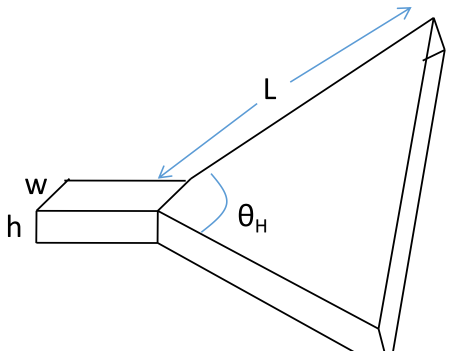
Horn antenna is a opened out or flared out waveguide. There are three advantages of flaring, such as

(i) Impedance matching between waveguide and free space is obtained
(ii) Diffraction problem will be eliminated.
(iii) The EM waves can easily convert from guiding media (waveguide) into unguiding media (free space).

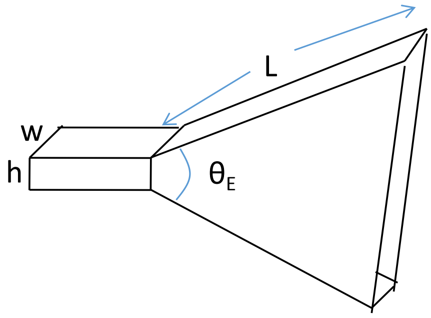
The types of horn antennas are given by

(i) Sectorial H-Plane horn antenna
(ii) Sectorial E-Plane horn antenna
(iii) Pyramidal horn antenna
(iv) Exponentially tapered pyramidal horn antenna
(v) Conical horn antenna
(vi) Exponentially tapered conical horn antenna

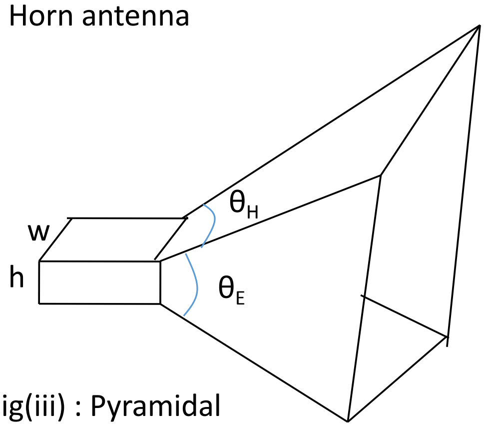
The structures of all the above antennas are shown in the figure below.\



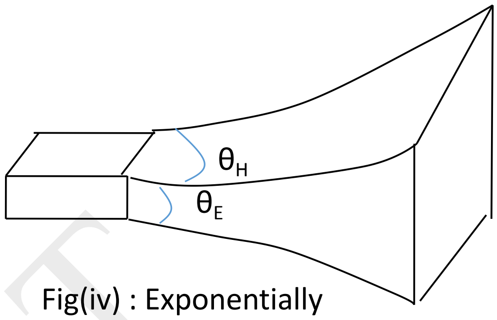
Fig(i) : Sectorial H-Plane Horn antenna



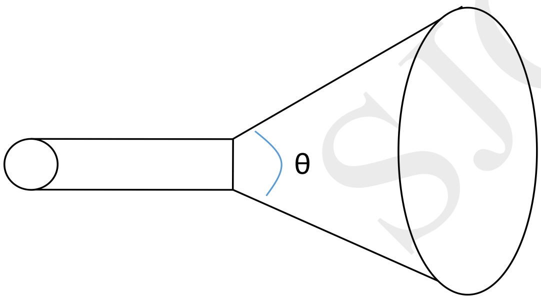
Fig(ii) : Sectorial E-Plane Horn antenna



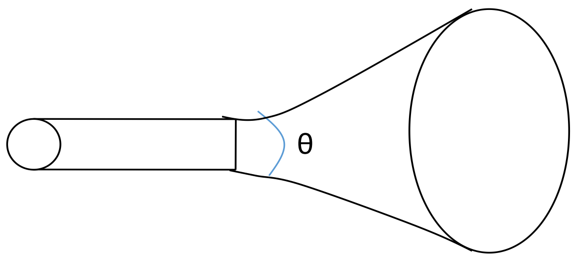
Fig(iii) : Pyramidal Horn antenna



Fig(iv) : Exponentially tapered Horn antenna



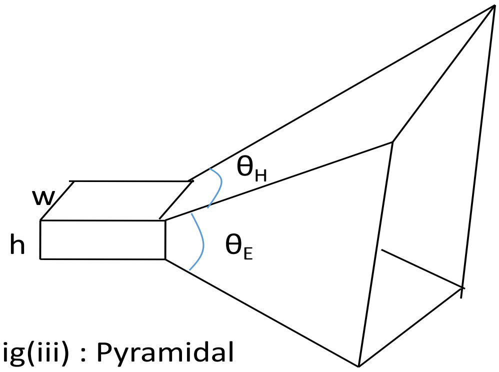
Fig(v) : Conical horn antenna



Fig(vi) : Exponentially Conical horn antenna

<u>Fermat's Principle and Optimum horns:</u>

The statement of Fermat's principle is "equality of physical path lengths or equality of electrical path lengths". To get the maximum filed strength at the receiving point, the Fermat's principle must be satisfied. In case of horn antenna, the Fermat's principle is not satisfied exactly because there is deviation (δ) between the waves at the center and waves at the edges but satisfied with some relaxation. The pyramidal horn antenna and its cross section is shown in figure below.



Fig(iii) : Pyramidal Horn antenna

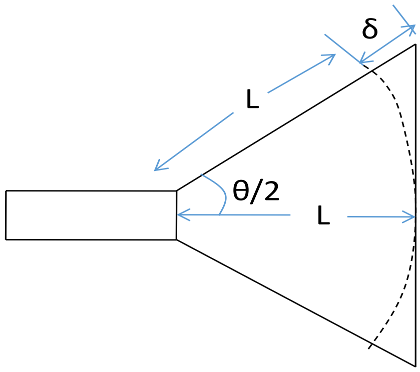
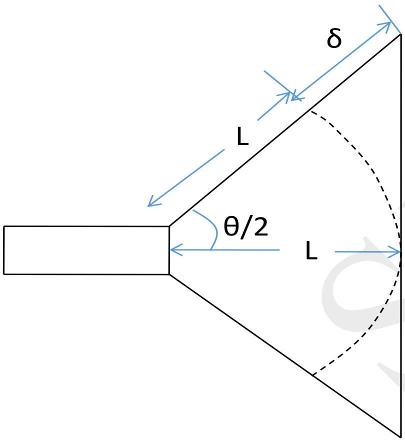
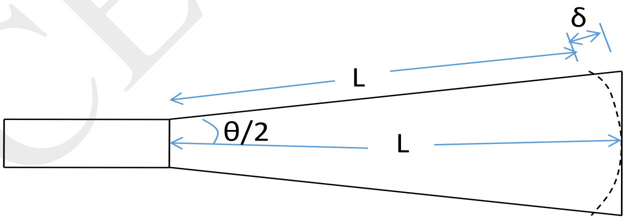


Fig : Cross section

The relaxation is such that, the deviation (δ_E) along the E-Plane should not be greater than 0.25λ and the deviation (δ_H) along H-Plane should not be greater than 0.4λ.
In case of horn antenna, the parameters flare angel (θ) and length (L) should be properly selected otherwise certain problems will arise. When the flare angle (θ) is large and length (L) is small, then there will be more deviation (δ), but the advantage is the construction is easy. When the length (L) is large and flare angle (θ) is small, then it is difficult to construct such large length but the advantage is less deviation (δ). These two situations are shown in the figures below



Fig(i) : Cross section of Pyramidal Horn antenna when θ is large and L is small



Fig(ii) : Cross section of Pyramidal Horn antenna when θ is small and L is large

Therefore we need to compromise between these two satiations. That is we need to select optimum values for flare angle and length. The equations for the optimum values of deviation (δ_o) and length (L) is given by

$$\delta_o = \frac{L}{cos\left(\frac{\theta}{2}\right)} - L$$

$$L = \frac{\delta_o cos\left(\frac{\theta}{2}\right)}{1 - cos\left(\frac{\theta}{2}\right)}$$

Design considerations of Pyramidal Horns:

Pyramidal horn antenna is a combination of sectorial E-plane horn and sectorial H-plane horn. Therefore designing of pyramidal horn is nothing but designing of sectorial E-plane and sectorial H-plane horan antennas. Let us consider the designing of these two antennas.

Sectorial E-Plane Horn antenna: The cross section of sectorial E-polane horn antenna is shown in the figure below.

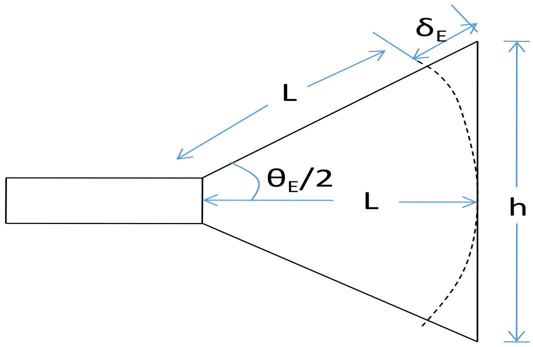


Fig : Cross section of Sectorial E-Plane Horn antenna

From the above figure,

$$\cos\left(\frac{\theta_E}{2}\right) = \frac{L}{L+\delta_E}$$

$$\theta_E = 2\cos^{-1}\left(\frac{L}{L+\delta_E}\right)$$

Or

$$\sin\left(\frac{\theta_E}{2}\right) = \frac{h/2}{L+\delta_E}$$

$$\theta_E = 2\,sin^{-1}\left(\frac{h}{2(L+\delta_E)}\right)$$

Or

$$\tan\left(\frac{\theta_E}{2}\right) = \frac{h/2}{L}$$

$$\theta_E = 2\tan^{-1}\left(\frac{h}{2L}\right)$$

From figure,

$$(L+\delta_E)^2 = L^2 + \left(\frac{h}{2}\right)^2$$

$$L^2 + {\delta_E}^2 + 2L\delta_E = L^2 + \frac{h^2}{4}$$

$$L = \frac{h^2}{8\delta_E}$$

Sectorial H-Plane Horn antenna: The cross section of sectorial H-polane horn antenna is shown in the figure below.

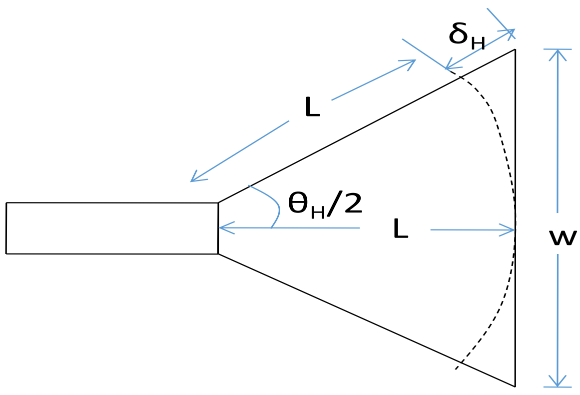


Fig : Cross section of Sectorial H-Plane Horn antenna

From the above figure,

$$\cos\left(\frac{\theta_H}{2}\right) = \frac{L}{L+\delta_H}$$

$$\theta_H = 2\cos^{-1}\left(\frac{L}{L+\delta_H}\right)$$

Or

$$\sin\left(\frac{\theta_H}{2}\right) = \frac{w/2}{L+\delta_H}$$

$$\theta_H = 2\,sin^{-1}\left(\frac{w}{2(L+\delta_H)}\right)$$

Or

$$\tan\left(\frac{\theta_H}{2}\right) = \frac{w/2}{L}$$

$$\theta_H = 2\tan^{-1}\left(\frac{w}{2L}\right)$$

From figure,

$$(L+\delta_H)^2 = L^2 + \left(\frac{w}{2}\right)^2$$

$$L^2 + {\delta_H}^2 + 2L\delta_H = L^2 + \frac{w^2}{4}$$

$$L = \frac{w^2}{8\delta_H}$$

The Half Power Beam Width (HPBW) along E-Plane and H-plane directions are given by

$$\theta_{E(HPBW)} = \frac{56\lambda}{h}\ degree$$

$$\theta_{H(HPBW)} = \frac{67\lambda}{w}\ degree$$

The directivity of pyramidal horn antenna is given by

$$D = \frac{7.5\ h.w}{\lambda^2} = \frac{7.5\ A}{\lambda^2}$$

The power gain (G_P) of pyramidal horn antenna is given by

$$G_P = \frac{4.5\, h.w}{\lambda^2} = \frac{4.5\, A}{\lambda^2}$$

MICROSTRIP ANTENNAS

Features:

(i) The microstrip antenna was first proposed by G.A.Deschamps in 1953.
(ii) Microstrip antennas are similar to patch antennas.
(iii) The microstrip antenna is contained conducting strip or patch suspended over a ground plane
(iv) Microstrip antennas are simple to fabricate
(v) These antennas are constructed using lithographic patterning on a printed circuit boards.
(vi) The simplest patch antenna uses a half wavelength long patch with a larger ground plane.
(vii) The simple microstrip antenna generates a linearly polarized wave.
(viii) It is a narrowband, wide-beam antenna.
(ix) To achieve higher bandwidth, a relatively thick substrate is used.
(x) The microstrip antennas are often used where thickness and conformability to the surface of mount or platform are the key requirements.
(xi) The microstrip antennas are available with different shapes such as square, rectangular, circular, triangular or elliptical.
(xii) The size of the microstrip antenna is inversely proportional to its frequency.
(xiii) The following figure1 illustrate different shapes of microstrip antennas.

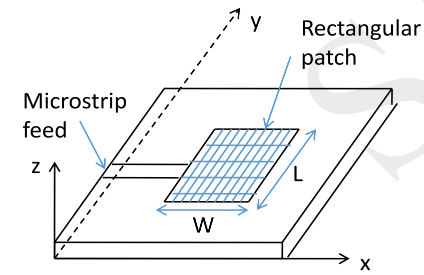


Fig.1: Rectangular patch antenna

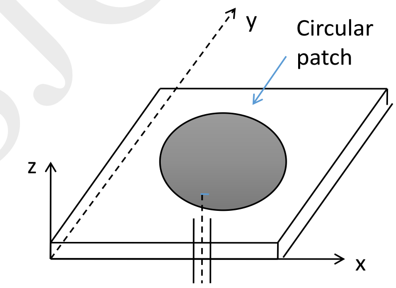


Fig.2: Circular patch antenna

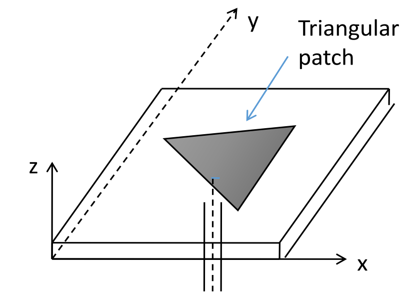


Fig.1: Triangular patch antenna

Advantages and limitations:

Advantages :

(i) Light weight
(ii) Smaller size
(iii) Lesser volume
(iv) Low profile planar configuration
(v) They can be easily molded to any desired shape
(vi) Simple to fabricate
(vii) Their fabrication process is compatible with Microwave Monolithic Integrated Circuit (MMIC) and Optoelectronic Integrated Circuit (OEIC) technologies.
(viii) These can support both linear and circular polarizations.
(ix) They are mechanically robust when mounted on rigid surfaces.

(x) With microstrip antennas it is easy to form large arrays with half wavelength or lesser spacing.

Limitations:

(i) Low bandwidth
(ii) Low efficiency
(iii) Low gain
(iv) Low power handling capacity
(v) Complicated design due to smaller size
(vi) These are resonant devices by its inherent nature
(vii) They suffer from the radiation effects due to feeds and junctions
(viii) These are poor end-fire radiators

<u>Rectangular patch antennas- Geometry and parameters:</u>

The basic structure of rectangular microstrip antennas is shown in the figure1 below. The distribution of electric field in the patch antenna is shown in the figure 2 below. The dimensions are shown in the figure above. The length of the strip is preferably half wavelength. The strip also having thickness and width which will be smaller than the length. The radiating edges are at the ends of L dimension which sets up the single polarization.

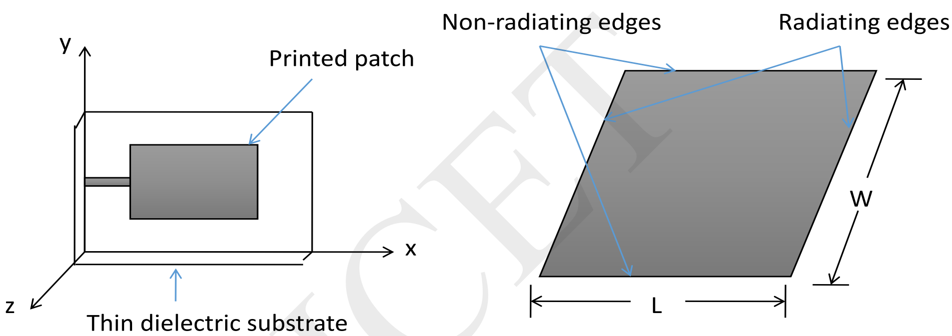


Fig 1(a): Patch layout on a substrate

Fig 1(b): Details of patch

Fig 1: Basic structure of a rectangular patch antenna

Fig 2(a): Sinusoidal variation of E

Fig 2(a): Uniform variation of E

Fig 2: Patch antenna with E field distribution

The geometry of microstrip antenna is shown in figure 3 below.

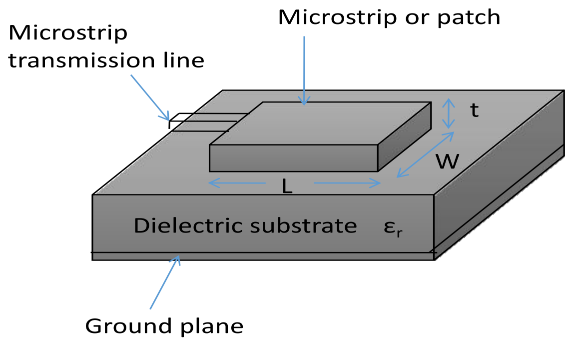


Fig 3: Geometry of rectangular patch antenna

The radiation that occurs at the ends of the W-dimension is very less and is referred to as the cross polarization. In the radiation zone perpendicular to the substrate, the radiation from two sides added up because the fields are in phase. In other directions (off-bore-sight) the fields will cancelled. Therefore the radiation intensity of microstrip antenna is depends upon the direction it is viewed as it has gain and directivity. The rectangular shape is simplest and most widely used configuration for fabrication of microstrip antennas.It is fed by microstrip transmission line. The conducting strip, transmission line and the ground plane is made from good conductors. The patch having the length L, width W and sitting on top of dielectric substrate of thickness h with permittivity ε_r. The critical or center frequency of patch antenna is given by

$$f_c = \frac{c}{2L\sqrt{\varepsilon_r}} = \frac{1}{2L\sqrt{\varepsilon_0 \varepsilon_r \mu_0}}$$

Where c is the velocity of light, ε_0 and μ_0 are the permittivity and permeability of free space respectively and ε_r is the permittivity of the dielectric substrate. The expression for dominant mode is given by

$$f_{r,nm} = \frac{c}{2(L + 2\Delta L)\sqrt{\varepsilon_{r,eff}}}$$

Where ΔL and ΔW are the incremental length and width whixh account for the fringing of field at the respective edges. The radiation patterns of microstrip antennas are shown in figure 4 below.

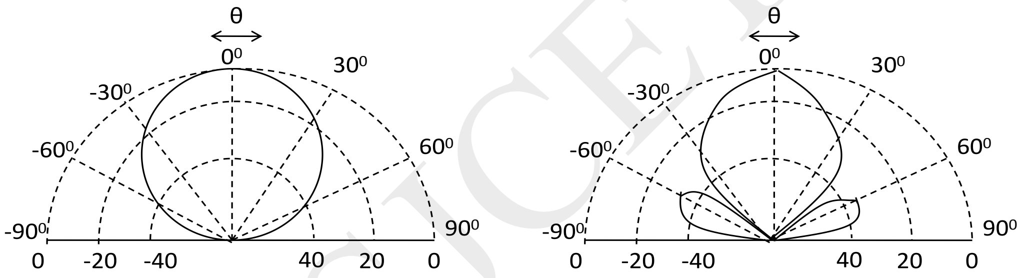


Fig 4(a): ϕ = 0⁰

Fig 4(b): ϕ = 90⁰

Fig 4: Radiation pattern for microstrip antenna

Characteristics of Micro strip antennas:

1. *Radiation pattern:* The following figure shows radiation pattern of microstrip antenna in $\varphi=0$ direction and $\varphi=90^0$. The power radiated at 180^0 is about 15dB less than the power in the center of the beam i.e.at 90^0. The beam width is about 65^0 and the gain is about 9 dBi.

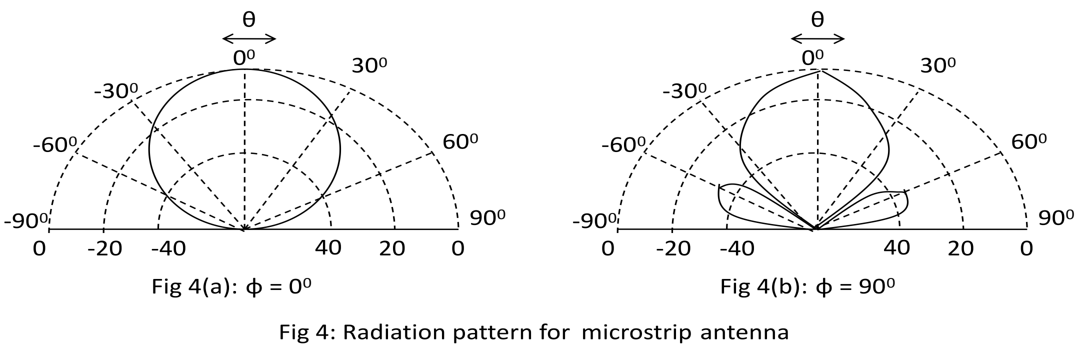


Fig 4(a): ϕ = 0⁰

Fig 4(b): ϕ = 90⁰

Fig 4: Radiation pattern for microstrip antenna

2. *Beam width:* The microstrip antennas have very wide bandwidth, both in azimuth and elevation.
3. *Directivity:* The directivity of microstrip antennas is given by

$$D = \frac{2h^2 E_0^2 W'^2 K_0^2}{P_r \pi \eta_0}$$

Where h thickness of the substrate, P_r is the radiated power, $W' = W + h$, $\eta_0 = 120\pi$, K_0 is the wave number and E_0 is the magnitude of the z-directed electric field.

4. *Gain:* The gain of the rectangular microstrip patch antenna will be in between 7 to 9 dB.
5. *Bandwidth:* The bandwidth decreases with increase of Quality factor. The impedance bandwidth of a patch antenna is influenced by the spacing between the patch and the ground palne. The bandwidth of the microstrip antenna is given by

$$Bandwidth = \frac{S-1}{Q_0\sqrt{S}}$$

Where S is the voltage standing wave ratio and Q_0 is the unloaded radiation quality factor.

Quality factor: The microstrip antennas have a very high quality factor. The quality factor 'Q' represents the losses associated with the antenna. When the quality factor is
large then the bandwidth is low.

6. *Efficiency:* The loss factor of a microstrip antenna is given by

$$L_T = L_c + L_d + L_r$$

Where L_c is the conductor losses, L_d is the dielectric losses and L_r is the radiation losses.

The efficiency of the microstrip antenna is given by

$$\eta = \frac{P_r}{P_c + P_d + P_r}$$

Where P_r is the radiated power, P_c is the power dissipated due to conductor losses, and P_d is the power dissipated due to dielectric.

7. *Polarization:* The main advantage of microstrip antenna is polarization diversity. The microstrip antennas can be designed to generate EM waves with different types of polarizations such as vertical, horizontal, circular polarizations. Circular polarization can be obtained from the patch antenna by exciting the square patch with two feeds with their inputs having 90^0 phase shift.
8. *Return loss:* The return loss is defined as the ratio of the Fourier transforms of the incident pulse and the reflected signal.
9. *Radar Cross-section:* The GPS guidance system requires low radar cross section (RCS) platforms. A standard technique used to reduce the RCS of a conventional patch antenna is to cover the patch with a magnetic absorbing material.

Impact of different parameters on characteristics:

The different parameters of microstrip antenna like length (L), width (W), height of the substrate (h), dielectric constant (ε_r), etc will affect the antenna properties. Therefore the nature and quantum of impact of these parameters is to be properly accounted for an efficient design. The dimensions of the patch or conducting strip will control the resonant frequency of the antenna. The wider the patch becomes, the lower will be the input impedance. The best choice for the dimension W is given by

$$W = \frac{c}{2f_0\sqrt{\{(\varepsilon_r + 1)/2\}}}$$

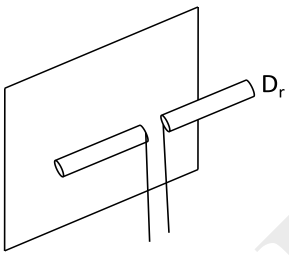
The permittivity (ε_r) of substrate will controls the fringing field. Lower the ε_r, wider more will be the fringing and better will be the radiation. The antenna bandwidth and efficiency will be high when the ε_r is low. Therefore the value of ε_r should be properly selected. The equation for the length (L) is given by

$$L = \frac{1}{2f_c\sqrt{\varepsilon_0\varepsilon_r\mu_0}}$$

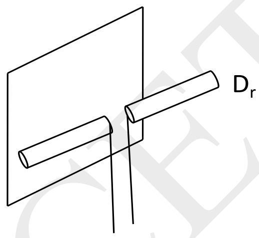
REFLECTOR ANTENNAS:

Introduction:

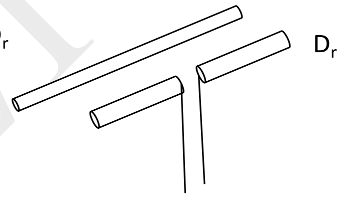
(i) Reflectors are made from good conductor.
(ii) Reflectors are used to modify the radiation pattern of the antenna.
(iii) The reflectors are also used to eliminate the back lobe radiation
(iv) The reflectors are used to convert the bidirectional radiation pattern in to unidirectional radiation pattern.
(v) There are various types of reflectors such a flat sheet reflector, thin linear reflector, Corner reflector, parabolic reflector, elliptical reflector, circular reflector, etc.
(vi) The structure of various types of reflectors are shown in the figure below



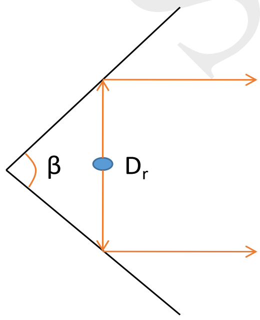
(a) Large flat sheet reflector



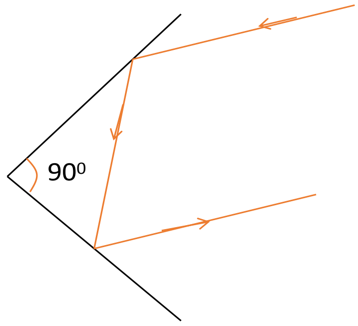
(a) Small flat sheet reflector



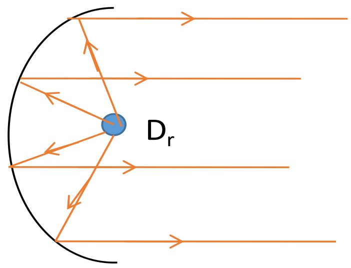
(a) Thin linear reflector



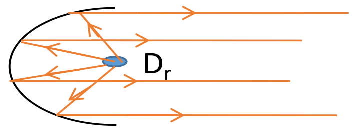
(d) Active corner reflector



(e) Passive corner or retro reflector



(f) Parabolic reflector

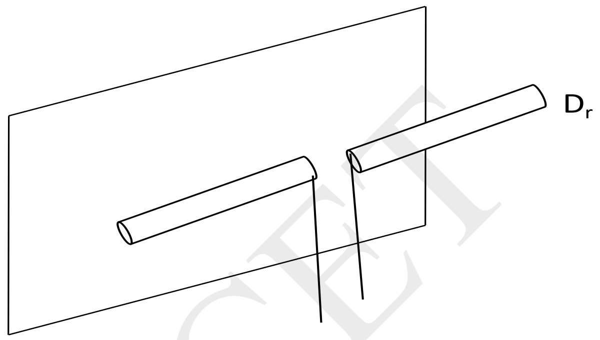


(g) Elliptical reflector

FLAT SHEET REFLECTORS:

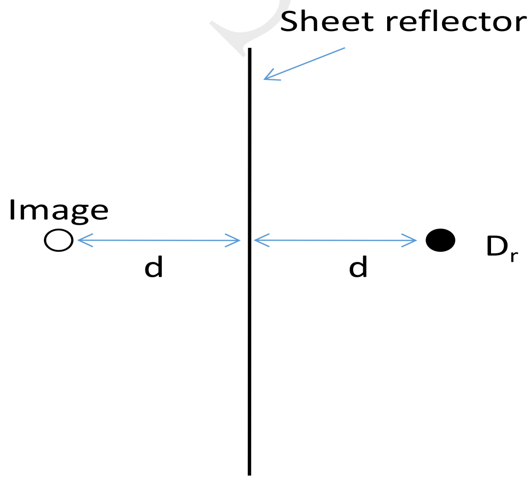
(i) When a flat metal sheet is used at backside of the antenna (driven element) to reflect back the signal then it is known as flat sheet reflector.

(ii) The structure of flat sheet reflector antenna is shown in figure below.



flat sheet reflector

(iii)The reflector antennas will be analyzed by using the method of images.



(iv)Flat sheet reflector antenna can be analyzed by imagining as a combination of two antennas separated by certain distance (d).

(v) A large flat sheet reflector can convert bidirectional pattern in to unidirectional pattern.

(vi)The distance(d) between the reflector and the driven element will decide the directional properties of antenna

CORNER REFLECTORS:

(i) When two flat metal sheets are meeting at an angle or corner, then it is known as corner reflector antenna.

(ii) The basic structure of corner reflector antenna is shown in the figure below.

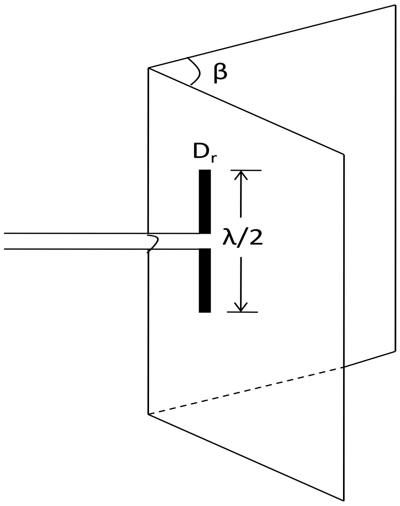


Fig: Corner reflector antenna

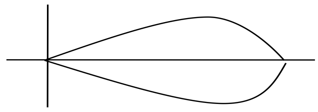


Fig: Radiation pattern

(iii) There are two types of corner reflector antennas such as active corner reflector antenna and passive corner reflector antenna.

(iv) The active corner reflector contains the driven element, where as passive corner reflector do not contain the driven element.

(v) The corner angle β is given by

$$\beta = \frac{180^0}{n}$$

Where n = an integer = 1,2,3,……

(vi) When corner angle is equal to 180^0(when n = 1), then it is called as flat sheet reflector.

(vii) When the corner angle is equal to 90^0 (when n = 2), then it is known as square corner reflector.

Square corner reflector:

- The structure of square corner reflector is shown in the figure below

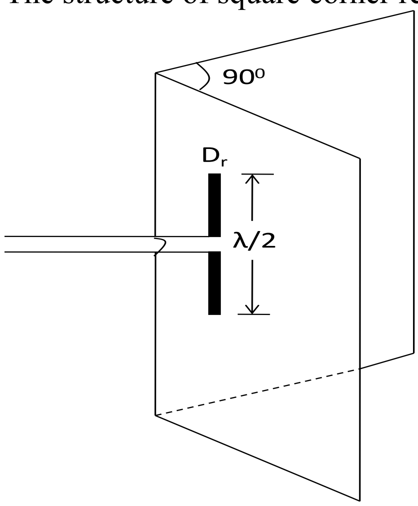


Fig: Square Corner reflector antenna

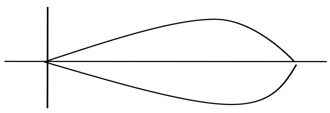
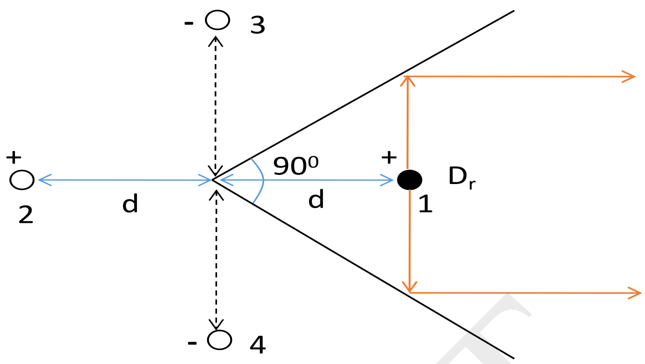


Fig: Radiation pattern

- The square corner reflector antenna contains corner reflector and driven element.
- The driven element will be preferably half wave dipole.
- A square corner reflector without driven element is called passive reflector or retro reflector.
- As per the method of images, square corner reflector can be imagined as combination of four antennas (three images and one driven element) as shown in figure below.



- It is a combination of two two-element arrays.
- The gain of square corner reflector is derived as follows:

The filed pattern $E_\varphi(\theta)$ in the horizontal plane at a large distance r from the antenna is given by

$$E_\varphi(\theta) = k' I_1[cos(\beta d cos\theta) - cos(\beta d sin\theta)] \qquad -(1)$$

Where k' is the constant involving distance r

I_1 is the current in each element, $\beta = 2\pi/\lambda$ called phase constant, d is the distance between the driven element and the corner.

The terminal voltage at the centre of the driven element (half wave dipole) is given by

$$V_1 = I_1 Z_{11} + I_2 Z_{12} - I_1 Z_{13} - 1Z_{14}$$

But $Z_{13} = Z_{14}$,

Then $$V_1 = I_1(Z_{11} + Z_{12} - 2Z_{14})$$

Where Z_{11} is the self impedance of driven element, Z_{12} is the mutual impedance between the element 1 and 2, Z_{14} is the mutual impedance between element 1 and 4.

The above equation can also be written as

$$V_1 = I_1(R_{11} + R_{12} - 2R_{14})$$

Now if P be the power supplied to the driven antenna, then

$$P = I_1^2 R$$

$$I_1 = \sqrt{\frac{P}{R}} = \sqrt{\frac{P}{R_{11} + R_{12} - 2R_{14}}} \qquad -(2)$$

By substituting equation (2) in equation (1),

$$E_\varphi(\theta) = k' \sqrt{\frac{P}{R_{11} + R_{12} - 2R_{14}}} \times [cos(\beta d cos\theta) - cos(\beta d sin\theta)] \qquad -(3)$$

When the reflector is removed then equation (3) becomes

$$E_\varphi(\theta)_{\lambda/2} = k' \sqrt{\frac{P}{R_{11}}} \qquad -(4)$$

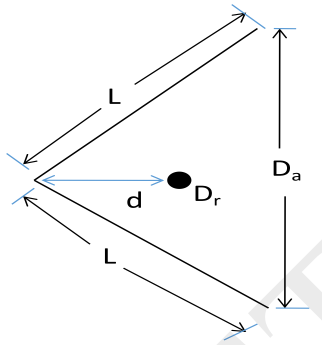
Above equation represents the electric field strength of half wave dipole which will be used as the reference for obtaining the gain of the corner reflector antenna. The ratio between the equations (3) and (4) gives the gain of the square corner reflector antenna and is given by

$$G = \frac{E_{\varphi}(\theta)}{E_{\varphi}(\theta)_{\lambda/2}} = \sqrt{\frac{R_{11}}{R_{11} + R_{12} - 2R_{14}}} \times [cos(\beta dcos\theta) - cos(\beta dsin\theta)]$$

In above equation the term $cos(\beta dcos\theta) - cos(\beta dsin\theta)$ is called as pattern factor and the term $\sqrt{\frac{R_{11}}{R_{11}+R_{12}-2R_{14}}}$ is called as coupling factor.

Design consideration of square corner reflector:

The dimensions of the square corner reflector is shown in the figure below

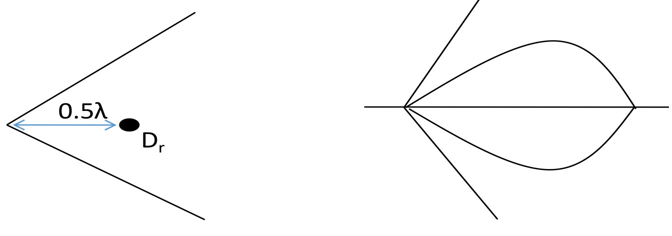


From the above figure the following formulae can be deduced

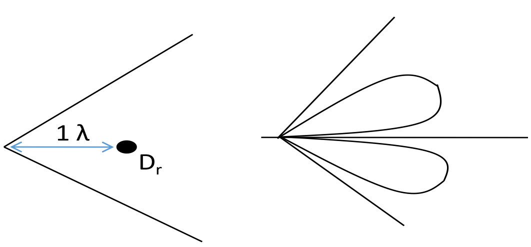
$$d = \frac{L}{2}$$
$$L = 2d$$
$$D_a = \sqrt{L^2 + L^2} = L\sqrt{2} = 1.414\, L$$
$$D_a = 1.414\, L = 1.414\,(2d) = 2.828\, d$$

Effect of spacing between the driven element and corner of the reflector (d) on the pattern:

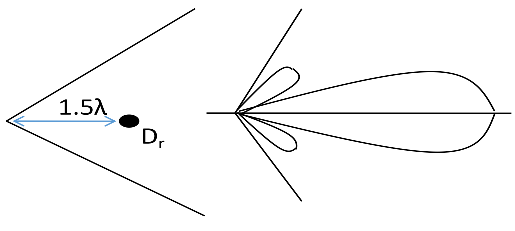
When the spacing (d) between the driven element and corner of the reflector is varied, the pattern characteristics such as beam width, gain, minor lobes, etc will be affected. This effect can be observed from the following figure;



Fig(a): When d = 0.5 λ



Fig(b): When d = 1 λ



Fig(c): When d = 1.5 λ

Retro or passive Square corner reflector:

- The square corner reflector without any driven element is known as retro reflector of passive square corner reflector.
- The structure of passive square corner reflector is shown in the following figure

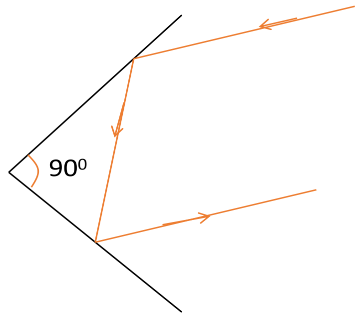


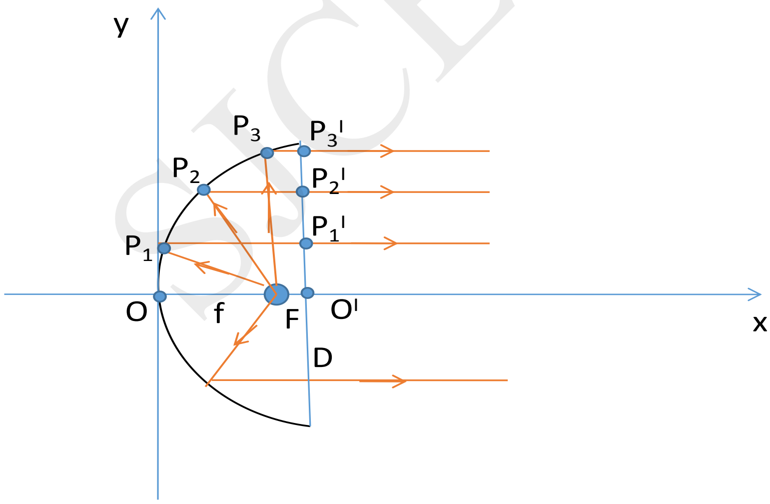
Fig:Passive corner or retro reflector

- The corner angel of retro reflector is 90^0.
- The retro reflector will be used as the target for the radar system.

PARABOLA REFLECTORS:

Geometry:

- The geometry or constructional features of parabola reflector is shown in the figure below.



- In above figure, OF represents Focal length (f), F is called Focus, O is called Vertex, D is called Directrix or aperture size and OO^1 is called axis of parabola.
- Parabola is defined as the locus of a point which moves in such a way that, its distance from a fixed distance called focus plus its distance from a straight line called directrix is constant i.e.

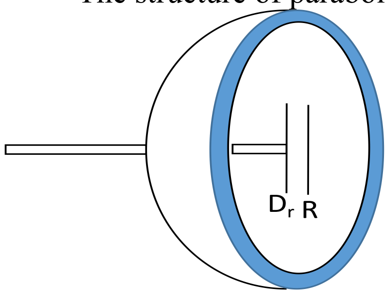
$$FP_1 + P_1P_1^{/} = FP_2 + P_2P_2^{/} = FP_3 + P_3P_3^{/} = constant$$

- The equation of parabola in terms of its coordinates is given by

$$y^2 = 4fx$$

- The ratio of focal length (f) to Aperture size (D) is known as f over D ratio or simple f/D.

- When a parabola is rotated about its axis OO[1], then it is known as paraboloidal reflector.
- The parabola is a two dimensional and paraboloid is three dimensional.
- The structure of paraboloidal reflector is shown in the figure below.



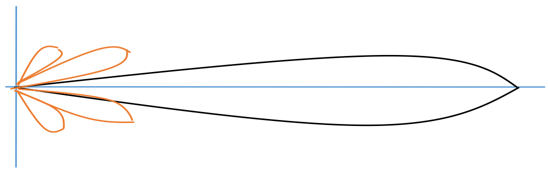


Fig: Radiation Pattern

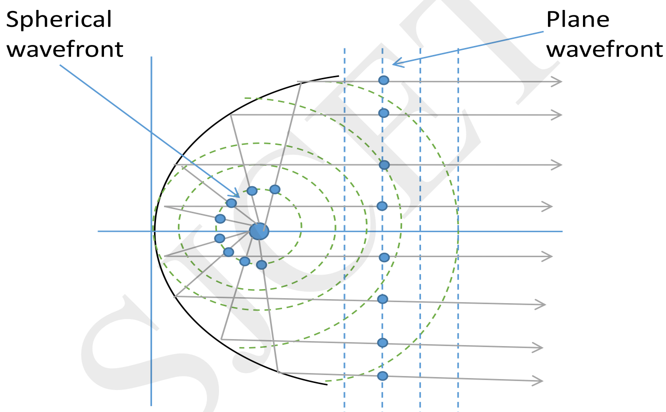
Fig: Paraboloidal reflector

- The equation of the paraboloidal reflector is given by

$$y^2 + z^2 = 4fx$$

<u>Pattern characteristics:</u>

- The parabolic reflector converts spherical wavefront into plane wavefront as shown in the figuer below.



- It will convert bidirectional radiation pattern in to unidirectional pattern.
- The radiation pattern characteristics or directional characteristics of reflector antenna depend upon the f over D ratio.
- When f/D ratio is small (as shown in figure below), then all the waves radiated by the driven element will be reflected by the reflector, but uniform illumination of reflector by the source (driven element) is not possible.

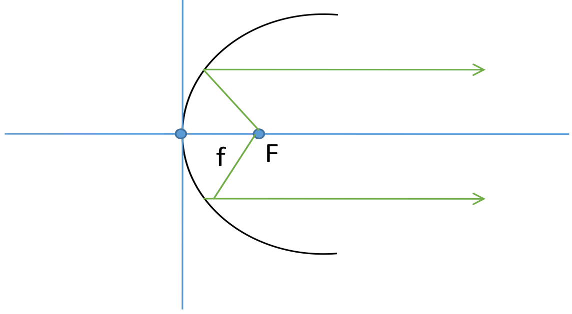


Fig: f/D < 1/4

- When f/D ratio is large as shown in the figure below, some of the waves will escape without reflection by the reflector and will constitute a spill over.

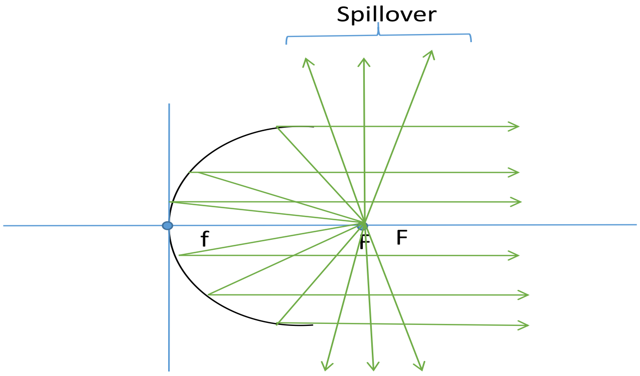


Fig: f/D > 1/4

- Practically f/D ratio will be selected in between small and large, typically f/D =1/4 as shown in the figure below.

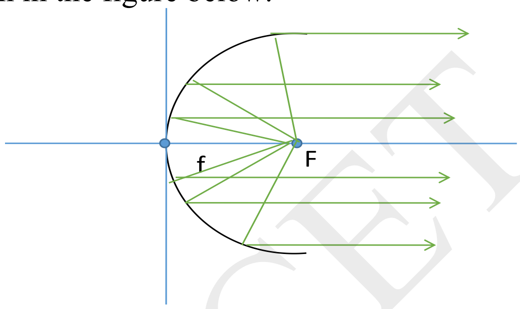


Fig: f/D = 1/4

- f over D ratio of parabolic reflector antenna is given by

$$\frac{f}{D} = \frac{1}{4}cot\left(\frac{\theta}{2}\right) = 0.25 \cot\left(\theta/2\right)$$

- The HPBW(Half Power Beam Width) of parabolic reflector with circular aperture is given by

$$HPBW = \frac{58\lambda}{D}\ degree$$

- The BWFN (Beam Width between First Nulls) of a parabolic reflector with circular aperture is given by

$$BWFN = \frac{140\lambda}{D}\ degree$$

- The directivity of a parabolic reflector with uniform illumination is given by

$$Directivity = 9.87\left(\frac{D}{\lambda}\right)^2$$

Where D represents the diameter of the circular aperture.

Feed Methods:

- Feed is nothing but a driven element. An ideal feed is one which illuminates the reflector uniformly.
- There are different types of methods for feeding the parabolic reflector antenna such as Dipole end fire feed, Horn feed, Cassegrain feed and Offset feed.
- The geometry of dipole end fire feed is shown in the following figure.

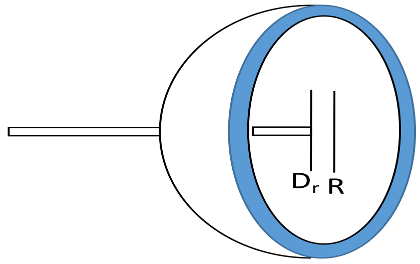


Fig: Dipole end fire feed

- In dipole end fire feed, the driven element is a two-element end fire array.
- With dipole feed, the driven element should be located at focus only.
- The most common feed method for paraboloid reflector is horn feed which is shown in the figure below.

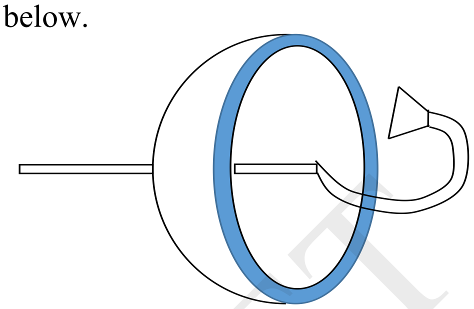


Fig: Horn feed

- In horn feed method, the horn antenna is used as the driven element.
- The horn antenna will be located at the focus such that, it can illuminate the reflector uniformly.
- The horn antenna will be located at the focus with the help of waveguide support.
- The drawback of horn feed is, the waveguide support will obstruct the waves which will affect the radiation pattern.
- In dipole feed and horn feed, the spillover will be present.
- To avoid the drawbacks of dipole feed and horn feed the cassegrain feed will be used
- The geometry of cassegrain feed is shown in the figure below.

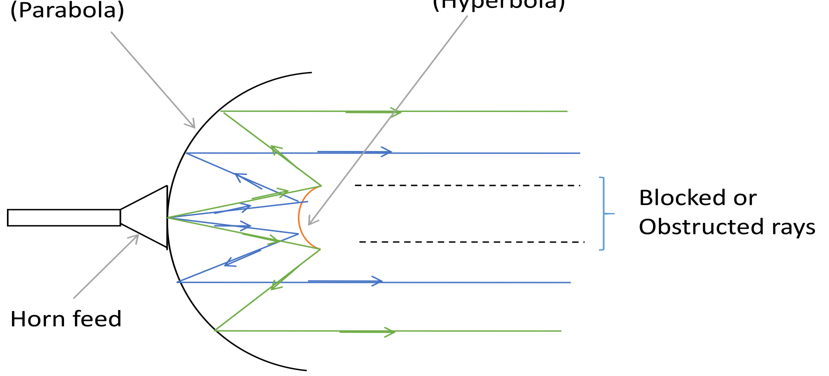


Fig: Geometry of cassegrain feed

- In cassegrain feeding, one more reflector called secondary reflector (preferably hyperbola) in addition to the primary reflector (Parabola) will be used.
- In cassegrain feed, the driven element can be located at our convenient location (preferably it will be located at vertex).
- The major advantages of cassegrain feed over the other methods are
  (i) Reduction in spill over and minor lobe radiation
  (ii) Ability to get an equivalent focal length much greater than the physical length.
  (iii) It allows us to place the feed at a convenient location
- The major drawback of cassegrain feed is, aperture blocking i.e. some of the area (aperture) of parabola is blocked by the secondary reflector hyperbola.
- To eliminate the drawback of cassegrain feeding, an offset feeding will be used.
- The structure of offset feed method is shown in figure below

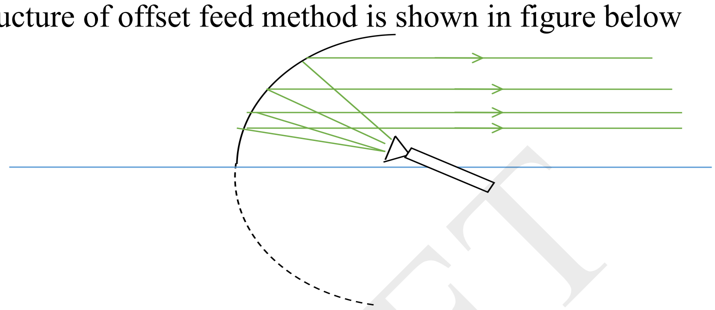
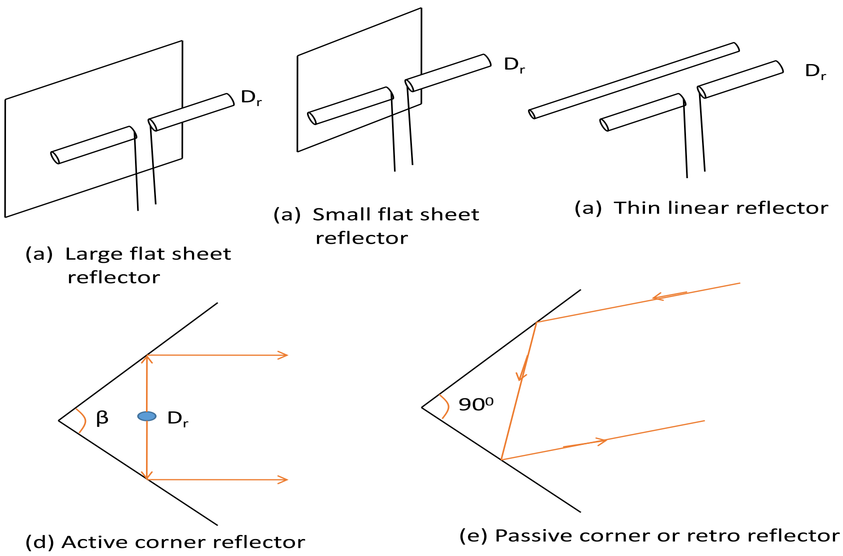


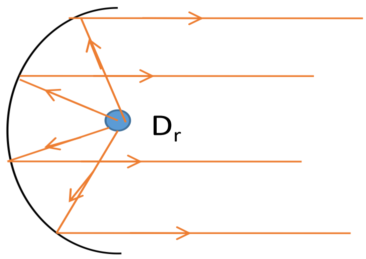
Fig: Offset feed

- In offset feeding, the horn antenna (driven element) will be arranged to illuminate only half of the parabola.

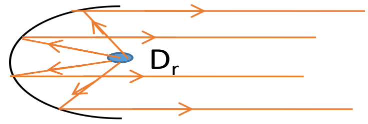
REFLECTOR TYPES - RELATED FEATURES:

- There are many types of reflectors such a flat sheet reflector, thin linear reflector, Corner reflector, parabolic reflector, elliptical reflector, circular reflector, etc.
- The structures of various types of reflectors are shown in the figure below.





(f) Parabolic reflector



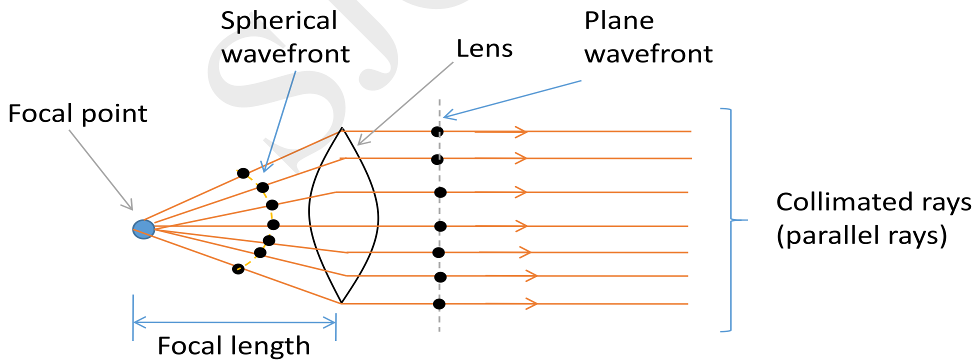
(g) Elliptical reflector

- The important features of reflector antennas are given by
  (i) Reflectors are made from good conductor.
  (ii) Reflectors are used to modify the radiation pattern of the antenna.
  (iii) The reflectors are also used to eliminate the back lobe radiation
  (iv) The reflectors are used to convert the bidirectional radiation pattern in to unidirectional radiation pattern.
  (v) The passive corner reflector called retro reflector will be used as target for the radar system.
  (vi) Parabolic reflectors will be used to convert spherical wavefront into plane wavefront.

LENS ANTENNAS

Basic Principle:

- Lens antennas are made from the lenses, preferably with concave and convex lenses.
- The collimating action of lens antenna can be understood from the following figure.

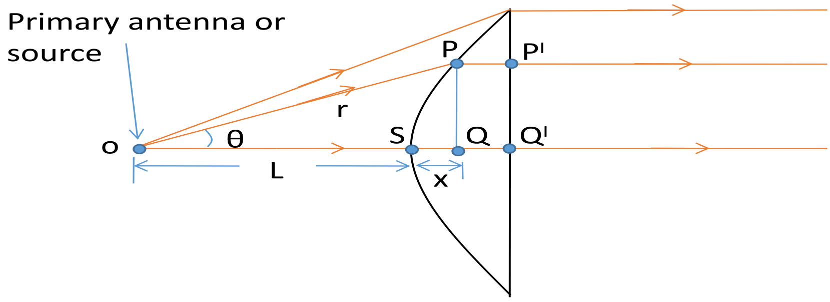


- Assuming the source or driven element at focal point at a distance of focal length, along the lens axis, it is seen that collimated or parallel rays are obtained on the right side of the lens.
- An optical lens operates by virtue of having a refractive index more than the unity.
- The principle of "equality of path length" called Fermat's principle will be applicable to then lens antennas.
- Lens medium can be used either to increase the speed of the waves or to decrease the speed of the waves.

Geometry of Non-metallic Dielectric Lenses:

- Basically there are two types of lens antennas such as Dielectric lens (or H-plane metal plate lens or Delay lens) and E-plane metal plate lens.

- The dielectric lens antennas are sub divided in to two types such as Non-metallic dielectric type and Metallic or Artificial dielectric type of lens.
- Non-metallic dielectric lens antenna will be made from Plano-concave lens.
- The geometry or constructional features of non-metallic dielectric lens antenna is shown in the figure below.



- The non-metallic dielectric lens will reduce the velocity of the waves such that equality of path length will be satisfied.
- The equation for the contour of the lens will be obtained as follows:
  From figure,

$$OP + PP' = OS + SQ' = OS + SQ + QQ' \quad \text{but } PP' = QQ'$$

Then

$$OP = OS + SQ$$

$$\frac{r}{c} = \frac{L}{c} + \frac{x}{v}$$

$$r = L + \left(\frac{c}{v}\right)x = L + \mu x$$

But

$$x = r\cos\theta - L$$

$$r = L + \mu(r\cos\theta - L)$$

$$r = \frac{L(\mu - 1)}{\mu\cos\theta - 1}$$

Where $\mu = \frac{c}{v}$ is called as refractive index of the lens medium.

The above equation represents the equation of hyperbola whose focal length is L and radius of curvature (R). Where $R = L(\mu-1)$.

Zoning:

- The process of reducing the size by removing certain portions of the lens antenna is called lens zoning.
- At low frequencies, the size of the lens antenna becomes bulky, because the size of the lens is inversely proportional to the frequency.
- The structures of zoned or stepped lens are shown in the figure below.

Fig: Zoned or stepped lens dielectric lenses

- The thickness of stepped or zoned lens is given by

$$t = \frac{\lambda}{\mu - 1}$$

- The zoned lens antenna is depends upon frequency. The zoned lens antennas can be used at low frequencies also.

Tolerances:

- Two important parameters to be considered while designing the lens antenna are thickness of the lens and refractive index of the lens medium.
- If there is any deviation in thickness from the ideal contour and any variations in the refractive index of the lens, there will be different path length of the waves passing through the lens. As a result equality of path length will not be satisfied.
- Therefore, there must be allowable variation to both thickness and refractive index.ie. Tolerances on thickness and refractive index should be considered.
- Tolerance on thickness(Δt) is given by

$$\Delta t = \frac{\lambda_0}{32(\mu - 1)} = \frac{0.03\lambda_0}{\mu - 1}$$

- The tolerance on the index of refraction or refractive index (μ) is given by

$$\Delta\mu = \frac{0.03}{t_\lambda}$$

Where t_λ is the thickness of lens in free-space wavelength.

- Following table gives the tolerances on thickness and refractive index of various lens antennas:

S.No	Type of Antenna	Type of tolerance	Amount of tolerance(rms)
1	Dielectric lens(unzoned)	Thickness	$\frac{0.03\lambda_0}{\mu - 1}$
		Index of refraction	$\frac{3}{\mu t_\lambda}\%$
2	Dielectric lens(zoned)	Thickness	3%
		Index of refraction	$\frac{3(\mu - 1)}{\mu}\%$
3	E-plane metal plate lens(unzoned)	Thickness	$\frac{0.03\lambda_0}{1 - \mu}$
		Plate spacing	$\frac{3\mu}{1 - \mu^2} t_\lambda\%$
4	E-plane metal plate lens(unzoned)	Thickness	3%
		Plate spacing	$\frac{3\mu}{1 + \mu}\%$

Applications of Lens antennas:

The applications of Lens antennas are given by

(i) These are suitable for above 3 GHz frequency.

(ii) Used like the wideband antenna.

(iii) These are used mainly for microwave frequency applications.

(iv) This antenna's converging properties can be used to develop a high range of antennas called parabolic reflector antennas, so these are extensively used within satellite communications.
(v) These are utilized as collimating elements within high-gain microwave systems like radio telescopes, millimeter wave radar & satellite antennas.

Solved Problems

1. **Calculate the directivity of 20 turn helix having $\alpha = 12^0$, circumference equal to one wavelength**.

Sol:

Given data:

No.of turns (N) = 20
Pitch angel (α) = 12^0
Circumference (C) = 1 λ

$$\alpha = \tan^{-1}\left(\frac{S}{C}\right)$$

$$S = C.\tan\alpha = \lambda \ \tan(12^0) = 0.2126\,\lambda$$

The directivity of helical antenna is given by

$$D = \frac{15\,N\,S\,C^2}{\lambda^3} = \frac{15 \times 20 \times 0.2126\lambda \times \lambda^2}{\lambda^3} = \frac{300 \times 0.2126\lambda^3}{\lambda^3} = 63.78$$

$$D = 10\ \log(63.78) = 18\ dB$$

2. Design a helical antenna to produce a circularly polarized waves for the following parameters of the helix

Circumference of helix = 2λ
No,of turns = 15

Sol:

Given data:

Circumference of helix(C) = 2λ
No,of turns (N) = 15
Axial Ratio (AR) = 1 for circular polarization

$$AR = 1 = \frac{2S\lambda}{\pi^2 D^2} = \frac{2S\lambda}{C^2}$$

$$2S\lambda = C^2$$

$$S = \frac{C^2}{2\lambda} = \frac{(2\lambda)^2}{2\lambda} = 2\lambda$$

$$C = \pi D$$

$$D = \frac{C}{\pi} = \frac{2\lambda}{\pi} = 0.636\,\lambda$$

$$Length\ of\ the\ helix\ (A) = N.S = 15 \times 2\lambda = 30\lambda$$

$$Length\ of\ each\ turn\ (L) = \sqrt{S^2 + C^2} = \sqrt{(2\lambda)^2 + (2\lambda)^2} = 2.828\,\lambda$$

3. Design a helical antenna for the frequency of 320 MHz to produce circularly polarized waves for the following parameters of helix

Diameter of helix = 0.56 m
No.of turns = 20

Sol:

Given data:

Frequency (f) = 320 MHz

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{320 \times 10^6} = 0.9375\,m$$

Diameter of helix = 0.56 m

No.of turns = 20

Axial Ratio (AR) = 1 for circular polarization

$$C = \pi D = \pi \times 0.56 = 1.76\, m$$

$$AR = 1 = \frac{2S\lambda}{\pi^2 D^2} = \frac{2S\lambda}{C^2}$$

$$2S\lambda = C^2$$

$$S = \frac{C^2}{2\lambda} = \frac{(1.76)^2}{2 \times 0.9375} = 1.65\, m$$

$$\alpha = \tan^{-1}\left(\frac{S}{C}\right) = \tan^{-1}\left(\frac{1.65}{1.76}\right) = 43.15^0$$

$$Length\ of\ the\ helix\ (A) = N.S = 20 \times 1.65 = 33\, m$$

$$Length\ of\ each\ turn\ (L) = \sqrt{S^2 + C^2} = \sqrt{(1.65)^2 + (1.76)^2} = 2.412\, m$$

4. Design helical antenna to produce the elliptically polarized waves for the following parameters of helix

Axial Ratio	**=**	**1.5**
No.of turns	**=**	**18**
Diameter of the helix	**=**	**0.5 m**
Frequency	**=**	**10 MHz.**

Sol:

Given data:

Axial Ratio	=	1.5
No.of turns	=	18
Diameter of the helix	=	0.5 m
Frequency	=	10 MHz.

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^6} = 30\, m$$

$$C = \pi D = \pi \times 0.5 = 1.57\, m$$

$$AR = 1.5 = \frac{2S\lambda}{\pi^2 D^2} = \frac{2S\lambda}{C^2}$$

$$S = \frac{1.5 \times C^2}{2\lambda} = \frac{1.5(1.57)^2}{2 \times 30} = 0.061\, m$$

$$Length\ of\ each\ turn\ (L) = \sqrt{S^2 + C^2} = \sqrt{(0.061)^2 + (1.57)^2} = 1.57\, m$$

$$Length\ of\ the\ helix\ (A) = N.S = 18 \times 0.061 = 1.098\, m$$

$$\alpha = \tan^{-1}\left(\frac{S}{C}\right) = \tan^{-1}\left(\frac{0.061}{1.57}\right) = 2.22^0$$

5. Find the values of D, S, α, L of helical antenna if the frequency is 312 MHz, circumference is 1.72 m and length of the helix is 35 m having 20 turns.

Sol:

Given data:

Frequency (f) = 312 MHz

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{312 \times 10^6} = 0.96\, m$$

Circumference of helix(C) = 1.72 m

Length of helix (A) = 35 m

No.of turns (N) = 20

$$C = \pi D$$

$$D = \frac{C}{\pi} = \frac{1.72}{\pi} = 0.547\, m$$

$$A = NS$$
$$S = \frac{A}{N} = \frac{35}{20} = 1.75\ m$$
$$\alpha = \tan^{-1}\left(\frac{S}{C}\right) = \tan^{-1}\left(\frac{1.75}{1.72}\right) = 45.5^0$$
$$Length\ of\ each\ turn\ (L) = \sqrt{S^2 + C^2} = \sqrt{(1.75)^2 + (1.72)^2} = 2.453\ m$$

6. Find the values of C, D, S, A, L of helical antenna if the frequency is 1.5 GHz, pitch angel α = 44.3⁰ and no.of turns is 25 with AR = 1

Sol:

Given data:

Frequency (f) = 1.5 GHz

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1.5 \times 10^9} = 0.2\ m$$

Pitch angel (α) = 44.3^0

No.of turns (N) = 25

Axial Ratio (AR) = 1

$$AR = 1 = \frac{2S\lambda}{\pi^2 D^2} = \frac{2S\lambda}{C^2}$$
$$2S\lambda = C^2$$
$$S = \frac{C^2}{2\lambda}$$
$$\alpha = 44.3^0 = \tan^{-1}\left(\frac{S}{C}\right) = \tan^{-1}\left(\frac{C^2}{2\lambda\, C}\right) = \tan^{-1}\left(\frac{C}{2\lambda}\right)$$
$$44.3 = \tan^{-1}\left(\frac{C}{2 \times 0.2}\right)$$
$$C = 0.4 \tan(44.3) = 0.39$$
$$D = \frac{C}{\pi} = \frac{0.39}{\pi} = 0.12\ m$$
$$S = \frac{C^2}{2\lambda} = \frac{(0.39)^2}{2 \times 0.2} = 0.38\ m$$
$$A = N.S = 25 \times 0.38 = 9.5\ m$$
$$L = \sqrt{S^2 + C^2} = \sqrt{(0.38)^2 + (0.39)^2} = 0.54\ m$$

7. Calculate the power gain of an optimum horn antenna approximately with a square aperture of 10λ on a side

Sol:

Give data:

Side of the horn antenna = 10 λ

The power gain of horn antenna is given by

$$G_P = \frac{4.5\ A}{\lambda^2} = \frac{4.5 \times 10\lambda \times 10\lambda}{\lambda^2} = 450$$
$$G_P = 10 \log(450) = 16.53\ dB$$

8. Find out the length L, width W and flare angles $\boldsymbol{\theta_E}$ and $\boldsymbol{\theta_H}$ of a pyramidal horn antenna for which the mouth height h = 10λ. The horn is fed by a rectangular waveguide with TE_{10} mode.

Sol:

Given data:

$$Mouth\ height(h) = 10\lambda$$

Let

$$\theta_E = 0.25\ \lambda\ \ and\ \theta_H = 0.4\lambda$$
$$L = \frac{h^2}{8\delta_E} = \frac{(10\lambda)^2}{8 \times 0.25\lambda} = 50\lambda$$
$$L = \frac{w^2}{8\delta_H}$$
$$w = \sqrt{8\delta_H L} = \sqrt{8 \times 0.4\lambda \times 50\lambda} = 12.65\ \lambda$$
$$\theta_E = 2\tan^{-1}\left(\frac{h}{2L}\right) = 2\tan^{-1}\left(\frac{10\lambda}{2 \times 50\lambda}\right) = 2 \times 5.71^0 = 11.42^0$$
$$\theta_E = 2\ \tan^{-1}\left(\frac{w}{2L}\right) = 2\ \tan^{-1}\left(\frac{12.65\lambda}{2 \times 50\lambda}\right) = 2 \times 7.2^0 = 14.4^0$$

9. (a) Determine the length L, H-plane aperture and flare angles θ_E and θ_H (in the E and H planes respectively) of a pyramidal horn for which E-plane aperture a_E = 10λ. The horn is fed by a rectangular waveguide with TE_{10} mode. Let δ = 0.2 λ in the E-plane and 0.375 λ in the H-plane (b) What are the beamwidths (c) What is the directivity

Sol:

Given data:

E-plane aperture (a_E = h) = 10λ

$$\delta_E = 0.2\lambda\ \ and\ \ \delta_H = 0.375\lambda$$

(a)

$$L = \frac{h^2}{8\delta_E} = \frac{(10\lambda)^2}{8 \times 0.2\lambda} = 62.5\lambda$$
$$L = \frac{w^2}{8\delta_H}$$
$$w = \sqrt{8\delta_H L} = \sqrt{8 \times 0.375\lambda \times 62.5\lambda} = 13.7\ \lambda$$
$$\theta_E = 2\tan^{-1}\left(\frac{h}{2L}\right) = 2\tan^{-1}\left(\frac{10\lambda}{2 \times 62.5\lambda}\right) = 2 \times 4.57^0 = 9.1^0$$
$$\theta_E = 2\ \tan^{-1}\left(\frac{w}{2L}\right) = 2\ \tan^{-1}\left(\frac{13.7\lambda}{2 \times 62.5\lambda}\right) = 2 \times 6.25^0 = 12.5^0$$

(b)

$$\theta_{E(HPBW)} = \frac{56\lambda}{h}\ degree = \frac{56\lambda}{10\lambda} = 5.6^0$$
$$\theta_{H(HPBW)} = \frac{67\lambda}{w}\ degree = \frac{67\lambda}{13.7\ \lambda} = 4.9^0$$

(c)

Directivity

$$D = \frac{7.5\ h.w}{\lambda^2} = \frac{7.5 \times 10\lambda \times 13.7\ \lambda}{\lambda^2} = 1027.5$$
$$D = 10\log(1027.5) = 30.1\ dB$$

10. Find out the power gain in dB of a paraboloidal reflector of open mouth aperture 10λ

Ans:

Given

Mouth aperture (D) = 10λ

Assume driven element is dipole (η_{ap} = 0.65)

$$G_p = G = \frac{4\pi A_e}{\lambda^2}$$

$$But \qquad A_e = \eta_{ap} A_p = 0.65\left(\frac{\pi D^2}{4}\right)$$

$$G = \frac{4\pi}{\lambda^2} \times 0.65\left(\frac{\pi D^2}{4}\right) = 6.4\left(\frac{D}{\lambda}\right)^2$$

$$G_p = 6.4\left(\frac{10\lambda}{\lambda}\right)^2 = 640$$

$$G_p = 10\log(640) = 28\ dB$$

11. Find out the beam width between first nulls and power gain of a 2 m paraboloid reflector operating at 6000 MHz.

Ans:

Given

Diameter (D) = 2 m

Frequency (f) = 6000 MHz

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{6000 \times 10^6} = 0.05\ m$$

$$BWFN = \frac{140\ \lambda}{D} = \frac{140 \times 0.05}{2} = 3.5\ degree$$

Assume driven element is dipole (η_{ap} = 0.65)

$$G_p = G = \frac{4\pi A_e}{\lambda^2}$$

$$But \qquad A_e = \eta_{ap} A_p = 0.65\left(\frac{\pi D^2}{4}\right)$$

$$G = \frac{4\pi}{\lambda^2} \times 0.65\left(\frac{\pi D^2}{4}\right) = 6.4\left(\frac{D}{\lambda}\right)^2$$

$$G_p = 6.4\left(\frac{2}{0.05}\right)^2 = 10240$$

$$G_p = 10\log(10240) = 40\ dB$$

12. A parabolic antenna having a circular mouth is to have a power gain of 1000 at λ = 10 cm. Estimate the diameter of the mouth and half power beam width of the antenna

Ans:

Given

Power gain (G_p) = 1000

$$\lambda = 10\ cm = 0.1\ m$$

Assume driven element is dipole (η_{ap} = 0.65)

$$G_p = G = \frac{4\pi A_e}{\lambda^2}$$

$$But \qquad A_e = \eta_{ap} A_p = 0.65\left(\frac{\pi D^2}{4}\right)$$

$$G_p = \frac{4\pi}{\lambda^2} \times 0.65\left(\frac{\pi D^2}{4}\right) = 6.4\left(\frac{D}{\lambda}\right)^2$$

$$1000 = 6.4\left(\frac{D^2}{0.1^2}\right)$$

$$D = \sqrt{\frac{1000 \times 0.1^2}{6.4}} = 1.25\ m$$

$$HPBW = \frac{58\lambda}{D} = \frac{58 \times 0.1}{1.25} = 4.64\ degree$$

13. A parabolic dish provides a gain of 75 dB at a frequency 15 GHz. Calculate the capture area of the antenna, its 3 dB and null beam widths

Ans:

Given

$$Gain(G) = 75\ dB$$

$$75 = 10 \log G$$

$$\frac{75}{10} = \log G$$

$$G = 10^{7.5} = 31622776.6$$

$$Frequency(f) = 15\ GHz$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{15 \times 10^9} = 0.02\ m$$

Assume driven element is dipole ($\eta_{ap} = 0.65$)

$$G_p = G = \frac{4\pi A_e}{\lambda^2}$$

$$But \qquad A_e = \eta_{ap} A_p = 0.65 \left(\frac{\pi D^2}{4}\right)$$

$$G = \frac{4\pi}{\lambda^2} \times 0.65 \left(\frac{\pi D^2}{4}\right) = 6.4\left(\frac{D}{\lambda}\right)^2$$

$$31622776.6 = 6.4\left(\frac{D}{\lambda}\right)^2 = 6.4\frac{D^2}{\lambda^2}$$

$$D = \sqrt{\frac{31622776.6 \times 0.02^2}{6.4}} = 44.45\ m$$

$$HPBW = \frac{58\lambda}{D} = \frac{58 \times 0.02}{44.45} = 0.02\ degree$$

$$BWFN = \frac{140\ \lambda}{D} = \frac{140 \times 0.02}{44.45} = 0.062\ degree$$

14. A 64 meter diameter paraboloid reflector is operated at 1430 MHz and is fed by non directional antenna. Estimate beam width between half power points(HPBW) and between nulls(BWFN) and power gain w.r.t half wave dipole.

Ans:

Given

Diameter (D) = 64 m

$$Frequency(f) = 1430\ MHz$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1430 \times 10^6} = 0.21\ m$$

$$HPBW = \frac{58\lambda}{D} = \frac{58 \times 0.21}{64} = 0.19\ degree$$

$$BWFN = \frac{140\ \lambda}{D} = \frac{140 \times 0.21}{64} = 0.46\ degree$$

For half wave dipole ($\eta_{ap} = 0.65$)

$$G_p = G = \frac{4\pi A_e}{\lambda^2}$$

$$But \qquad A_e = \eta_{ap} A_p = 0.65\left(\frac{\pi D^2}{4}\right)$$

$$G = \frac{4\pi}{\lambda^2} \times 0.65\left(\frac{\pi D^2}{4}\right) = 6.4\left(\frac{D}{\lambda}\right)^2$$

$$G_p = 6.4\left(\frac{64}{0.21}\right)^2 = 594430.8$$

$$G_p = 10\log(594430.8) = 57.7\ dB$$

15. A paraboloid reflector antenna with diameter 20 m is designed to operate at a frequency of 6 GHz and illumination efficiency of 0.54. Calculate antenna gain in decibles.

Ans:

Given

Diameter (D) = 20 m

$$Frequency(f) = 6\ GHz$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{6 \times 10^9} = 0.05\ m$$

Illumination efficiency (η_{ap}) = 0.54

$$G_p = G = \frac{4\pi A_e}{\lambda^2}$$

$$But \qquad A_e = \eta_{ap} A_p = 0.54\left(\frac{\pi D^2}{4}\right)$$

$$G = \frac{4\pi}{\lambda^2} \times 0.54\left(\frac{\pi D^2}{4}\right) = 5.33\left(\frac{D}{\lambda}\right)^2$$

$$G_p = 5.33\left(\frac{20}{0.05}\right)^2 = 852800$$

$$G_p = 10\log(852800) = 59.3\ dB$$

16. Calculate beam width between first nulls of a 2.5 m paraboloid reflector used at 6 GHz. What will be its gain in decibels?

Ans:

Given

Diameter (D) = 2.5 m

$$Frequency(f) = 6\ GHz$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{6 \times 10^9} = 0.05\ m$$

Assume driven element is dipole (η_{ap} = 0.65)

$$G_p = G = \frac{4\pi A_e}{\lambda^2}$$

$$But \qquad A_e = \eta_{ap} A_p = 0.65\left(\frac{\pi D^2}{4}\right)$$

$$G = \frac{4\pi}{\lambda^2} \times 0.65\left(\frac{\pi D^2}{4}\right) = 6.4\left(\frac{D}{\lambda}\right)^2$$

$$G_p = 6.4\left(\frac{2.5}{0.05}\right)^2 = 16000$$

$$G_p = 10\log(16000) = 42\ dB$$

17. Calculate the angular aperture for paraboloid reflector antenna for which the aperture number is (i) 0.25 (ii) 0.5 (iii) 0.6 Given that diameter of the reflector mouth is 10 m, calculate the position of the focal point with reference to the reflector mouth in each case.

Ans:

(i)

$$\frac{f}{D} = 0.25$$
$$\frac{f}{D} = \frac{1}{4}\cot\left(\frac{\theta}{2}\right)$$

$$0.25 = \frac{1}{4}\cot\left(\frac{\theta}{2}\right)$$
$$\theta = 90^0$$
$$Angular\ aperture = 2\theta = 2 \times 90^0 = 180^0$$
$$Given\ (D) = 10\ m$$
$$\frac{f}{D} = 0.25$$
$$f = 0.25 \times D = 0.25 \times 10 = 2.5\ m$$

(ii)

$$\frac{f}{D} = 0.5$$
$$\frac{f}{D} = \frac{1}{4}\cot\left(\frac{\theta}{2}\right)$$

$$0.5 = \frac{1}{4}\cot\left(\frac{\theta}{2}\right)$$
$$\theta = 26.54^0$$
$$Angular\ aperture = 2\theta = 2 \times 26.54^0 = 53.08^0$$
$$Given\ (D) = 10\ m$$
$$\frac{f}{D} = 0.5$$
$$f = 0.5 \times D = 0.5 \times 10 = 5\ m$$

(iii)

$$\frac{f}{D} = 0.6$$
$$\frac{f}{D} = \frac{1}{4}\cot\left(\frac{\theta}{2}\right)$$

$$0.6 = \frac{1}{4}\cot\left(\frac{\theta}{2}\right)$$
$$\theta = 45.22^0$$
$$Angular\ aperture = 2\theta = 2 \times 45.22^0 = 90.44^0$$
$$Given\ (D) = 10\ m$$
$$\frac{f}{D} = 0.6$$
$$f = 0.6 \times D = 0.6 \times 10 = 6\ m$$

18. Estimate the diameter and the effective aperture of a paraboloid reflector antenna required to produce a nulls width of 10^0 at 3 GHz.

Ans:

Given

$$BWFN = 10^0$$

$$Frequency(f) = 3\ GHz$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1\ m$$

$$BWFN = \frac{140\ \lambda}{D}\ degree$$

$$10 = \frac{140\ \times 0.1}{D}$$

$$D = \frac{140\ \times 0.1}{10} = 1.4\ m$$

UNIT-3 (ANTENNA ARRAYS AND PROPAGATION, WAVEGUIDES)

Syllabus: Antenna Arrays and propagation: Arrays of 2 Isotropic sources- Different cases, Principle of Pattern Multiplication, Uniform Linear Arrays – Broadside Arrays, End fire Arrays, EFA with Increased Directivity, Derivation of their characteristics and comparison, Bionomial Arrays, Different modes of wave propagation, Ground wave propagation Space wave propagation - Sky wave propagation **(Qualitative treatment).**

Waveguides: Introduction, Rectangular waveguides, Field expressions for TE and TM modes, Wave propagation in the guide, Phase and group velocities, Power transmission and attenuation, Waveguide current and mode excitation, Circular waveguide – TE and TM modes, Wave propagation, waveguide resonators.

ARRAYS OF 2 ISOTROPIC SOURCES- DIFFERENT CASES

Introduction:

Array is defined as method of combining the radiations from the group or array of elements (antenna) with involving wave interference. The total field at a distance point 'P' due to the antenna array is the vector sum of the fields produced by the individual antennas of the array system. Array is said to be linear, when all the elements are equally spaced along straight line. Further a array is said to be uniform linear array, if all the elements in the array are fed with currents of equal amplitudes and uniform progressive phase shift along the line. There are different types of antennas arrays such as Broad side array, End fire array, Collinear array and parasitic array. The structure of 7 element broad side and end fire array is shown in figure below.

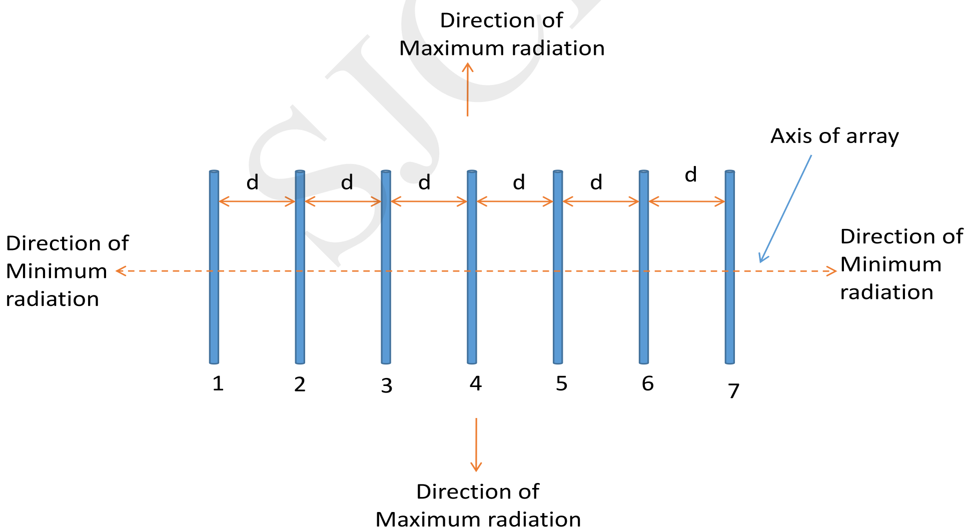


Fig: Broad side array arrangement

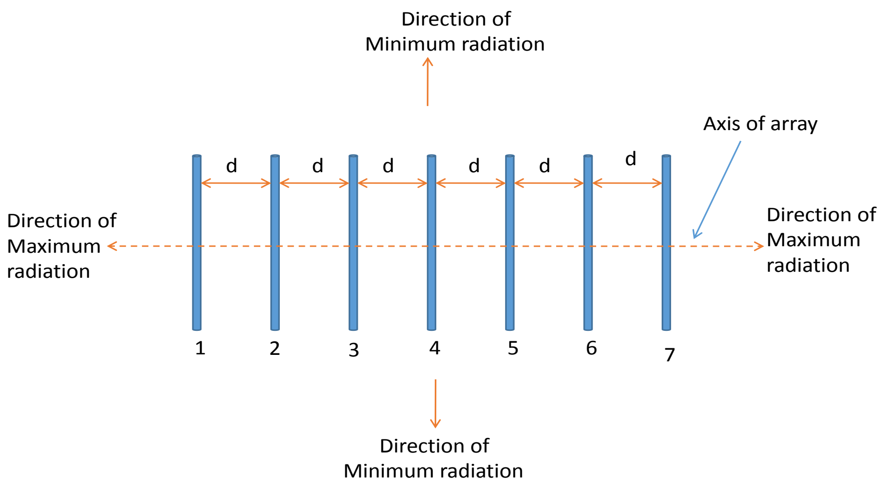
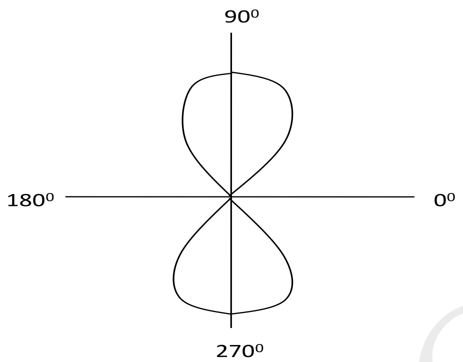
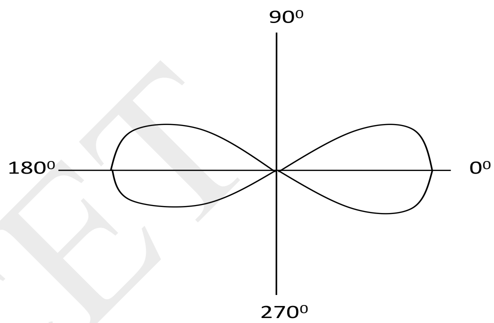


Fig: End fire array arrangement



Fig(a): Radiation pattern of Broad side array



Fig(b): Radiation pattern of End fire array

<u>Two-element Broad side array (Equal amplitudes and same phase):</u>

When all the elements of array supplied with currents of equal amplitude and same phase, then it fires the maximum radiation in perpendicular direction of the array axis and minimum radiation along the direction of the array axis. The structure of two element broad side array is shown in the figure below. The total field strength of two element broadside array is the vector sum of the fields of two individual elements.

The path difference between the weaves due to the two antennas is given by

$$P.d = dcos\theta \quad m$$

$$P.d = \frac{d}{\lambda}\cos\theta$$

Phase angle due to path difference is given by

$$\Psi = \beta dcos\theta \qquad -1$$

Total electric field at the interesting point (P) is given by

$$E = E_1 e^{-j\Psi/2} + E_2 e^{+j\Psi/2}$$

Where E_1 and E_2 are the magnitudes of electric filed strengths due to element 1 and 2 respectively.

Let $E_1 = E_2 = E_0$

$$E = E_0(e^{-j\Psi/2} + e^{+j\Psi/2})$$

$$E = E_0(e^{+j\Psi/2} + e^{-j\Psi/2})$$

Multiply R.H.S with 2/2

$$E = \frac{2}{2} E_0 (e^{+j\Psi/2} + e^{-j\Psi/2})$$

$$E = 2E_0 \left(\frac{e^{+j\Psi/2} + e^{-j\Psi/2}}{2} \right) = 2E_0 \cos(\Psi/2) \qquad -2$$

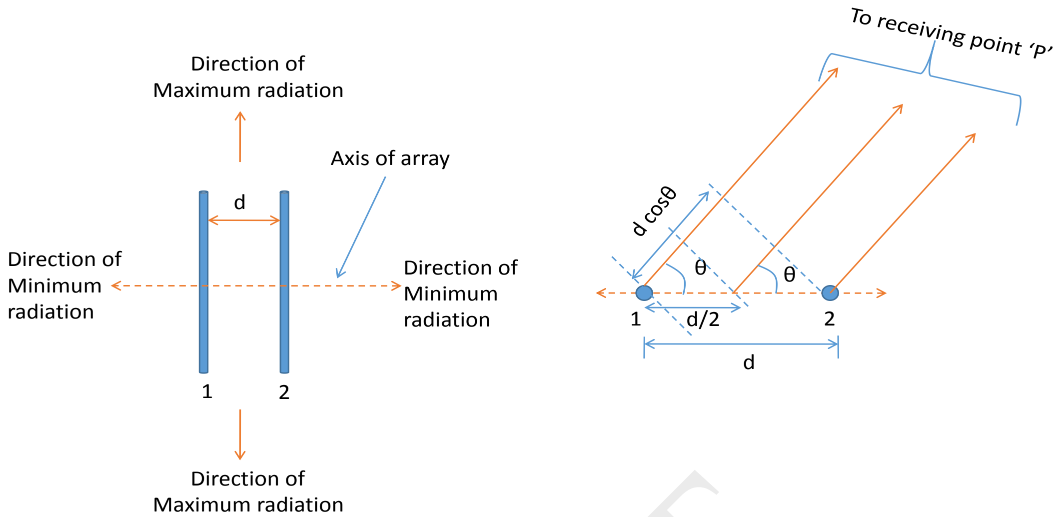


Fig: Two-element broad side array

Substitute equation 1 in equation 2,

$$E = 2E_0 cos(\Psi/2) = 2E_0 cos(\beta d\ cos\theta/2) \qquad -3$$

The above equation represents the total electric field strength at the receiving point due to two element broad side array. In above equation the term ‘2E$_0$’ represent the magnitude and cos(βd cosθ) represents the phase factor or pattern factor or array factor.

Direction of maximum radiation:

The direction of maximum radiation of major lobe can be obtained as follows:

We have

$$E = 2E_0 cos(\beta d\ cos\theta/2)$$

To have the maximum value for E, the pattern factor must be maximum. i.e.

$$\cos\left(\frac{\beta d\ cos\theta}{2}\right) = 1$$

$$\cos\left(\frac{2\pi}{\lambda}\frac{d\ cos\theta}{2}\right) = 1$$

$$\cos\left(\frac{\pi d\ cos\theta}{\lambda}\right) = 1$$

To satisfy the above equation,

$$\frac{\pi d\ cos\theta}{\lambda} = \pm\ n\pi$$

Where n = 0,1,2,3,…

$$\cos\theta = \pm\frac{n\pi\lambda}{\pi d} = \pm\frac{n\lambda}{d}$$

$$\theta = \theta_{max} = cos^{-1}\left(\pm\frac{n\lambda}{d}\right) \qquad -4$$

Direction of minimum radiation:

The direction of minimum radiation of major lobe can be obtained as follows:

We have

$$E = 2E_0 cos(\beta d\ cos\theta/2)$$

To have the minimum value for E, the pattern factor must be minimum. i.e.

$$\cos\left(\frac{\beta d\ cos\theta}{2}\right) = 0$$

$$\cos\left(\frac{2\pi}{\lambda}\frac{d\ cos\theta}{2}\right) = 0$$

$$\cos\left(\frac{\pi d\ cos\theta}{\lambda}\right) = 0$$

To satisfy the above equation,

$$\frac{\pi d\ cos\theta}{\lambda} = \pm\ (2n+1)\pi/2$$

Where n = 0,1,2,3,…

$$\cos\theta = \pm\frac{(2n+1)\pi\lambda}{2\pi d} = \pm\frac{(2n+1)\lambda}{2d}$$

$$\theta = \theta_{min} = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2d}\right) \qquad -5$$

Direction of half power points:

The direction of half power points of major lobe can be obtained as follows:
We have

$$E = 2E_0 cos(\beta d\ cos\theta/2)$$

At half power points, E must be 1/√2 times of maximum value

$$\cos\left(\frac{\beta d\ cos\theta}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{2\pi}{\lambda}\frac{d\ cos\theta}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{\pi d\ cos\theta}{\lambda}\right) = \frac{1}{\sqrt{2}}$$

To satisfy the above equation,

$$\frac{\pi d\ cos\theta}{\lambda} = \pm\ (2n+1)\pi/4$$

Where n = 0,1,2,3,…

$$\cos\theta = \pm\frac{(2n+1)\pi\lambda}{4\pi d} = \pm\frac{(2n+1)\lambda}{4d}$$

$$\theta = \theta_{HPP} = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{4d}\right) \qquad -6$$

The radiation pattern of 2-element broad side array with λ/2 spacing by using the above relations can be obtained as follows:

$$\theta_{max} = cos^{-1}\left(\pm\frac{n\lambda}{d}\right) = cos^{-1}\left(\pm\frac{n\lambda}{\frac{\lambda}{2}}\right) = cos^{-1}(\pm 2n)$$

When n = 0, $\theta_{max} = cos^{-1}(\pm 0) = 90^0\ \&\ 270^0$

When n = 1, $\theta_{max} = cos^{-1}(\pm 2) = function\ not\ satisfied$

Similarly, $\theta_{min} = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2d}\right) = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2\frac{\lambda}{2}}\right)$

$$\theta_{min} = cos^{-1}\big(\pm\ (2n+1)\big)$$

When n = 0, $\theta_{min} = cos^{-1}(\pm 1) = 0^0\ \&\ 180^0$

When n = 1, $\theta_{min} = cos^{-1}(\pm 3) = not\ satisfied$

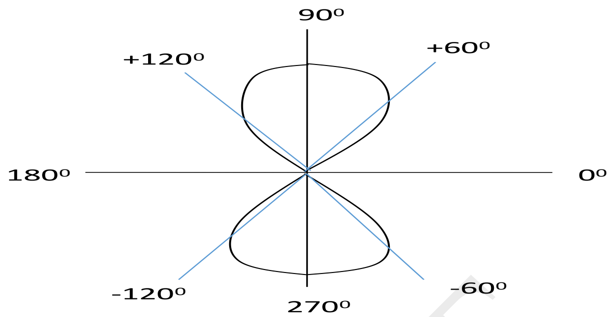
Similarly $$\theta_{HPP} = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{4d}\right) = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{4\frac{\lambda}{2}}\right)$$

$$\theta_{HPP} = cos^{-1}\left(\pm\frac{(2n+1)}{2}\right)$$

When n = 0, $\theta_{HPP} = cos^{-1}\left(\pm\frac{1}{2}\right) = \pm 60^0\ \& \pm 120^0 = +60^0, -60^0, +120^0\ \& -120^0$

When n = 1, $\theta_{HPP} = cos^{-1}\left(\pm\frac{3}{2}\right) = not\ satisfied$

The resultant radiation pattern of 2 element broad side array with spacing λ/2 is shown in figure below.



Fig(a): Radiation pattern of Broad side array

Two-element end fire array (Equal amplitudes and opposite phase):

When all the elements of array supplied with currents of equal amplitude and opposite phase, then it fires the maximum radiation along the direction of the array axis and minimum radiation in perpendicular direction of the array axis. The structure of two element end-fire array is shown in the figure below. The total field strength of two element broadside array is the vector sum of the fields of two individual elements.

The path difference between the weaves due to the two antennas is given by

$$P.d = dcos\theta \quad m$$

$$P.d = \frac{d}{\lambda}\cos\theta$$

Phase angle due to path difference is given by

$$\Psi = \beta dcos\theta \qquad -1$$

Total electric field at the interesting point (P) is given by

$$E = -E_1 e^{-j\Psi/2} + E_2 e^{+j\Psi/2}$$

Where E_1 and E_2 are the magnitudes of electric filed strengths due to element 1 and 2 respectively. In above equation negative sign to E_1 represents the current of element 1 is opposite to the current of element 2.

Let $E_1 = E_2 = E_0$

$$E = E_0(-e^{-j\Psi/2} + e^{+j\Psi/2})$$

$$E = E_0(e^{+\frac{j\Psi}{2}} - e^{-j\Psi/2})$$

Multiply R.H.S with 2j/2j

$$E = \frac{2j}{2j}E_0(e^{+j\Psi/2} + e^{-j\Psi/2})$$

$$E = 2jE_0\left(\frac{e^{+j\Psi/2} + e^{-j\Psi/2}}{2j}\right) = 2jE_0 \sin(\Psi/2) \qquad -2$$

Substitute equation 1 in equation 2,

$$E = 2jE_0 sin(\Psi/2) = 2jE_0 sin(\beta d\ cos\theta/2) \qquad -3$$

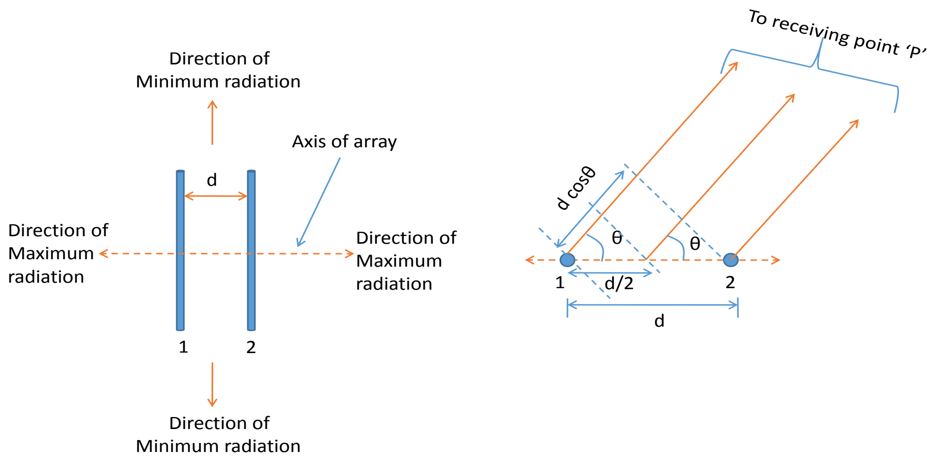


Fig: Two-element end fire array

The above equation represents the total electric field strength at the receiving point due to two element end fire array. In above equation the term '2E$_0$' represent the magnitude and sin(βd cosθ) represents the phase factor or pattern factor or array factor.

Direction of maximum radiation:

The direction of maximum radiation of major lobe can be obtained as follows:

We have

$$E = 2jE_0 sin(\beta d\, cos\theta/2)$$

To have the maximum value for E, the pattern factor must be maximum. i.e.

$$\sin\left(\frac{\beta d\; cos\theta}{2}\right) = 1$$

$$\sin\left(\frac{2\pi}{\lambda}\frac{d\; cos\theta}{2}\right) = 1$$

$$\sin\left(\frac{\pi d\; cos\theta}{\lambda}\right) = 1$$

To satisfy the above equation,

$$\frac{\pi d\; cos\theta}{\lambda} = \pm\; (2n+1)\pi/2$$

Where n = 0,1,2,3,…

$$\cos\theta = \pm\frac{(2n+1)\pi\lambda}{2\pi d} = \pm\frac{(2n+1)\lambda}{2d}$$

$$\theta = \theta_{max} = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2d}\right) \qquad -4$$

Direction of minimum radiation:

The direction of minimum radiation of major lobe can be obtained as follows:

We have

$$E = 2jE_0 sin(\beta d\, cos\theta/2)$$

To have the minimum value for E, the pattern factor must be minimum. i.e.

$$\sin\left(\frac{\beta d\; cos\theta}{2}\right) = 0$$

$$\sin\left(\frac{2\pi}{\lambda}\frac{d\; cos\theta}{2}\right) = 0$$

$$\sin\left(\frac{\pi d\; cos\theta}{\lambda}\right) = 0$$

To satisfy the above equation,

$$\frac{\pi d \, cos\theta}{\lambda} = \pm \; n\pi$$

Where n = 0,1,2,3,…

$$\cos\theta = \pm \frac{n\pi\lambda}{\pi d} = \pm \frac{n\lambda}{d}$$

$$\theta = \theta_{min} = cos^{-1}\left(\pm \frac{n\lambda}{d}\right) \qquad -5$$

Direction of half power points:

The direction of half power points of major lobe can be obtained as follows:
We have

$$E = 2jE_0 sin(\beta d \, cos\theta/2)$$

At half power points, E must be 1/√2 times of maximum value

$$\sin\left(\frac{\beta d \, cos\theta}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\sin\left(\frac{2\pi}{\lambda}\frac{d \, cos\theta}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\sin\left(\frac{\pi d \, cos\theta}{\lambda}\right) = \frac{1}{\sqrt{2}}$$

To satisfy the above equation,

$$\frac{\pi d \, cos\theta}{\lambda} = \pm \; (2n+1)\pi/4$$

Where n = 0,1,2,3,…

$$\cos\theta = \pm \frac{(2n+1)\pi\lambda}{4\pi d} = \pm \frac{(2n+1)\lambda}{4d}$$

$$\theta = \theta_{HPP} = cos^{-1}\left(\pm \frac{(2n+1)\lambda}{4d}\right) \qquad -6$$

The radiation pattern of 2-element end fire array with λ/2 spacing by using the above relations can be obtained as follows:

$$\theta_{max} = cos^{-1}\left(\pm \frac{(2n+1)\lambda}{2d}\right) = cos^{-1}\left(\pm \frac{(2n+1)\lambda}{2\frac{\lambda}{2}}\right) = cos^{-1}(\pm (2n+1))$$

When n = 0, $\theta_{max} = cos^{-1}(\pm 1) = 0^0 \; \& \; 180^0$

When n = 1, $\theta_{max} = cos^{-1}(\pm 3) = not \; satisfied$

Similarly, $\theta_{min} = cos^{-1}\left(\pm \frac{n\lambda}{d}\right) = cos^{-1}\left(\pm \frac{n\lambda}{\frac{\lambda}{2}}\right) = cos^{-1}(\pm 2n)$

When n = 0, $\theta_{min} = cos^{-1}(\pm 0) = \; 90^0 \; \& \; 270^0$

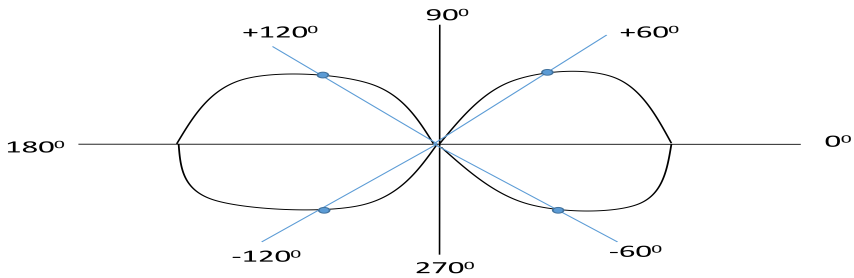
When n = 1, $\theta_{min} = cos^{-1}(\pm 2) = \; function \; not \; satisfied$

Similarly, $\theta_{HPP} = cos^{-1}\left(\pm \frac{(2n+1)\lambda}{4d}\right) = cos^{-1}\left(\pm \frac{(2n+1)\lambda}{4\frac{\lambda}{2}}\right) = cos^{-1}\left(\pm \frac{(2n+1)}{2}\right)$

When n = 0, $\theta_{HPP} = cos^{-1}\left(\pm \frac{1}{2}\right) = \pm 60^0 \; \& \pm 120^0 = +\, 60^0, \; -60^0, \; +120^0 \; \& -120^0$

When n = 1, $\theta_{HPP} = cos^{-1}\left(\pm \frac{3}{2}\right) = not \; satisfied$

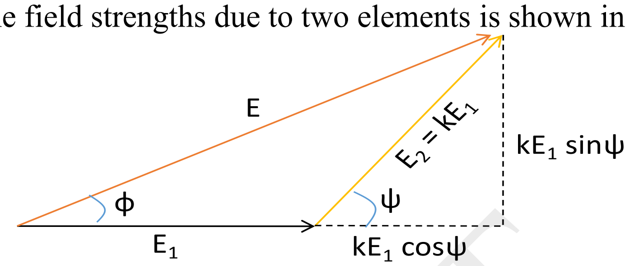
The resultant radiation pattern of 2 element end fire array with spacing λ/2 is shown in figure below.



Fig(b): Radiation pattern of End fire array

Two-element array with unequal amplitudes and any phase:

To find out the total electric field strength at any receiving point due to two-element array with unequal amplitudes and any phase, the vector addition is used. The vector diagram to add the field strengths due to two elements is shown in the figure below.



The total phase difference between waves due to element 1 and 2 is given by
Ψ = Phase angle due to path difference + phase angle difference between the input currents.

$$\psi = \beta d \cos\theta + \alpha$$

Total electric field strength at interesting point with reference to element 1 is given by

$$E = E_1 e^{j(0)} + E_2 e^{j\psi} = E_1 + E_2 e^{j\psi}$$

$$E = E_1\left(1 + \frac{E_2}{E_1} e^{j\psi}\right) = E_1\left(1 + k e^{j\psi}\right)$$

Where $k = \frac{E_2}{E_1}$

$$E = E_1\big(1 + k(\cos\psi + j\sin\psi)\big)$$

$$E = E_1(1 + k\cos\psi + jk\sin\psi) = E_1\big((1 + k\cos\psi) + jk\sin\psi\big)$$

$$|E| = E_1\sqrt{(1 + k\cos\psi)^2 + (jk\sin\psi)^2}$$

The phase angle ϕ is given by

$$\phi = \tan^{-1}\left(\frac{kE_1\sin\psi}{E_1 + kE_1\cos\psi}\right) = \tan^{-1}\left(\frac{k\sin\psi}{1 + k\cos\psi}\right)$$

UNIFORM LINEAR ARRAYS

Consider the uniform linear array contains N-no. of elements shown in the figure below.

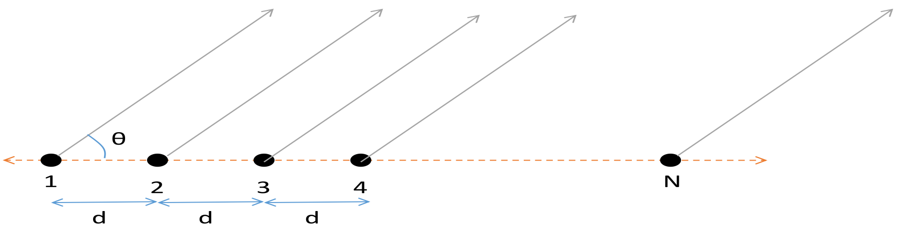


Fig: N-element linear array

Let us derive the general equation for the total electric field strength due to N-element linear array.

The total electric field strength (w.r.t element 1) at a point P is given by

$$E_t = E_1 e^{j0} + E_2 e^{j\Psi} + E_3 e^{j2\Psi} + E_4 e^{j3\Psi} + \ldots\ldots\ldots\ldots + E_N e^{j(N-1)\Psi} \qquad -1$$

Assume $E_1 = E_2 = E_3 = E_4 = \ldots\ldots\ldots E_N = E_0$

Then equation 1 becomes

$$E_t = E_0\left(1 + e^{j\Psi} + e^{j2\Psi} + e^{j3\Psi} + \ldots\ldots\ + e^{j(N-1)\Psi}\right) \qquad -2$$

Multiply both side with $e^{j\Psi}$

$$E_t e^{j\Psi} = e^{j\Psi} E_0\left(1 + e^{j\Psi} + e^{j2\Psi} + e^{j3\Psi} + \ldots\ldots\ + e^{j(N-1)\Psi}\right)$$

$$E_t e^{j\Psi} = E_0\left(e^{j\Psi} + e^{j2\Psi} + e^{j3\Psi} + e^{j4\Psi} \ldots\ldots\ + e^{jN\Psi}\right) \qquad -3$$

Subtract equation 3 from 2,

$$E_t(1 - e^{j\Psi}) = E_0(1 - e^{jN\Psi})$$

$$E_t = \frac{E_0(1 - e^{jN\Psi})}{(1 - e^{j\Psi})}$$

$$E_t = \frac{E_0\left(e^{jN\psi/2}.e^{-jN\psi/2} - e^{jN\psi/2}.e^{jN\psi/2}\right)}{e^{j\psi/2}.e^{-j\psi/2} - e^{j\psi/2}.e^{j\psi/2}} = \frac{-E_0 e^{jN\psi/2}\left(-e^{-jN\psi/2} + e^{jN\psi/2}\right)}{-e^{j\psi/2}\left(-e^{-j\psi/2} + e^{j\psi/2}\right)}$$

$$E_t = \frac{E_0 e^{jN\psi/2}\left(e^{jN\psi/2} - e^{-jN\psi/2}\right)}{e^{j\psi/2}\left(e^{j\psi/2} - e^{-j\psi/2}\right)} \qquad -4$$

We know that,

$$\sin(N\psi/2) = \frac{e^{jN\psi/2} - e^{-jN\psi/2}}{2j}$$

$$e^{jN\psi/2} - e^{-jN\psi/2} = 2j\sin(N\psi/2) \qquad -5$$

Similarly $\sin(\psi/2) = \frac{e^{j\psi/2} - e^{-j\psi/2}}{2j}$

$$e^{j\psi/2} - e^{-j\psi/2} = 2j\sin(\psi/2) \qquad -6$$

Substitute equations 5 and 6 in equation 4

$$E_t = \frac{E_0 e^{jN\psi/2}(2j\sin(N\psi/2))}{e^{j\psi/2}(2j\sin(\psi/2))} = E_0 \frac{\sin(N\Psi/2)}{\sin(\Psi/2)} e^{j\emptyset}$$

Wher $\emptyset = (N-1)\Psi/2$

In above equation the factor $\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$ is called array factor or pattern factor.

N-Element Broadside Arrays:

When all the elements of array supplied with currents of equal amplitude and same phase, then it fires the maximum radiation in perpendicular direction of the array axis and minimum radiation along the direction of the array axis.

Direction of maximum radiation of minor lobes or pattern maxima:

The total electric field strength due to N-element broad side array is given by

$$E_t = E_0 \frac{\sin(N\Psi/2)}{\sin(\Psi/2)} e^{j\emptyset}$$

To have the maximum value for E, the pattern factor must be maximum. That is

$$\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$$

must be maximum. To have maximum value for $\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$ its numerator must be maximum.

$$\sin\left(\frac{N\Psi}{2}\right) = 1$$

To satisfy the above relation

$$\frac{N\psi}{2} = \pm\ (2n+1)\pi/2$$

Where n = 1,2,3,…

n = 0 for major lobe

$$N\psi = \pm\ (2n+1)\pi$$

$$\psi = \pm \frac{(2n+1)\pi}{N}$$

But $$\psi = \beta d \cos\theta + \alpha$$

$$\beta d \cos\theta + \alpha = \pm \frac{(2n+1)\pi}{N}$$

But α = 0 for broad side array

$$\beta d \cos\theta = \pm \frac{(2n+1)\pi}{N}$$

$$\cos\theta = \pm \frac{(2n+1)\pi}{\beta d N} = \pm \frac{(2n+1)\pi}{\frac{2\pi}{\lambda} dN}$$

$$\cos\theta = \pm \frac{(2n+1)\lambda}{2Nd}$$

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm \frac{(2n+1)\lambda}{2Nd}\right) \qquad -7$$

Direction of minimum radiation of minor lobes or pattern minima:

The total electric field strength due to N-element broad side array is given by

$$E_t = E_0 \frac{\sin(N\Psi/2)}{\sin(\Psi/2)} e^{j\phi}$$

To have the minimum value for E, the pattern factor must be minimum. That is

$$\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$$

must be minimum. To have minimum value for $\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$ its numerator must be minimum.

$$\sin\left(\frac{N\Psi}{2}\right) = 0$$

To satisfy the above relation

$$\frac{N\psi}{2} = \pm\ n\pi$$

Where n = 1,2,3,… n = 0 for major lobe

$$N\psi = \pm\ 2n\pi$$

$$\psi = \pm \frac{2n\pi}{N}$$

But $$\psi = \beta d \cos\theta + \alpha$$

$$\beta d \cos\theta + \alpha = \pm \frac{(2n+1)\pi}{N}$$

But α = 0 for broad side array

$$\beta d \cos\theta = \pm \frac{2n\pi}{N}$$

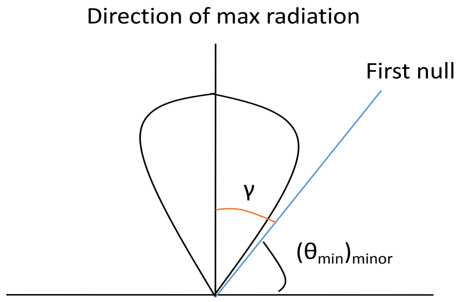
$$\cos\theta = \pm \frac{2n\pi}{\beta d N} = \pm \frac{2n\pi}{\frac{2\pi}{\lambda} dN}$$

$$\cos\theta = \pm \frac{n\lambda}{Nd}$$

$$(\theta_{min})_{minor} = \cos^{-1}\left(\pm\frac{n\lambda}{Nd}\right)$$

Beam width of major lobe:

Beam width or BWFN of a major lobe is defined as the angle between the first nulls or twice the
angle between first null and major lobe maximum radiation direction as shown in figure below.



Let the angle between first null and major lobe maximum radiation direction is γ

From figure, $(\theta_{min})_{minor} = 90 - \gamma$

$$90 - \gamma = \cos^{-1}\left(\frac{n\lambda}{Nd}\right)$$

$$\cos(90 - \gamma) = \frac{n\lambda}{Nd}$$

$$\sin\gamma \cong \gamma = \frac{n\lambda}{Nd}$$

Replace n with 1, because first null occurs when n = 1

$$\gamma = \frac{\lambda}{Nd}$$

$$Beam\ width = BWFN = 2\gamma = \frac{2\lambda}{Nd}\quad in\ radians$$

$$Beam\ width = BWFN = \frac{2\lambda}{Nd} \times 57.3^0 = \frac{114.6\,\lambda}{Nd}\quad in\ degrees$$

Half Power Beam Width of N-element Broad side array is given by

$$\text{HPBW} = \frac{BWFN}{\mathbf{2}} = \frac{57.3\,\lambda}{\text{Nd}}\ \text{degree}$$

<u>End-fire Arrays:</u>

When all the elements of array supplied with currents of equal amplitude and opposite phase, then it fires the maximum radiation in the direction of the array axis and minimum radiation in perpendicular direction of the array axis.

Let us get small equation for α.

To have a maximum value of electric field strength at any direction, the total phase angle ψ must be zero. In case of end fire array, the maximum radiation will be at 0^0 and 180^0 directions.

Therefore

$$\psi = \beta d\cos\theta + \alpha = 0$$

Let maximum radiation direction is 0^0. i.e $\theta = 0^0$

$$\beta d\cos(0) + \alpha = 0$$

$$\beta d + \alpha = 0$$

$$\alpha = -\beta d \qquad -1$$

Direction of maximum radiation of minor lobes or pattern maxima:

The total electric field strength due to N-element broad side array is given by

$$E_t = E_0 \frac{\sin(N\Psi/2)}{\sin(\Psi/2)} e^{j\emptyset}$$

To have the maximum value for E, the pattern factor must be maximum. That is

$$\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$$

must be maximum. To have maximum value for $\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$ its numerator must be maximum.

$$\sin\left(\frac{N\Psi}{2}\right) = 1$$

To satisfy the above relation $\frac{N\psi}{2} = \pm\ (2n+1)\pi/2$

Where n = 1,2,3,… n = 0 for major lobe

$$N\psi = \pm\ (2n+1)\pi$$

$$\psi = \pm \frac{(2n+1)\pi}{N}$$

But $\psi = \beta d \cos\theta + \alpha$

$$\beta d \cos\theta + \alpha = \pm \frac{(2n+1)\pi}{N} \qquad -2$$

Substitute equation 1 in equation2

$$\beta d \cos\theta - \beta d = \pm \frac{(2n+1)\pi}{N}$$

$$\beta d(\cos\theta - 1) = \pm \frac{(2n+1)\pi}{N} =$$

$$\cos\theta - 1 = \pm \frac{(2n+1)\pi}{\beta d N} = \pm \frac{(2n+1)\pi}{\frac{2\pi}{\lambda} dN}$$

$$\cos\theta - 1 = \pm \frac{(2n+1)\lambda}{2Nd}$$

$$\cos\theta = \pm \frac{(2n+1)\lambda}{2Nd} + 1$$

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm \frac{(2n+1)\lambda}{2Nd} + 1\right) \qquad -3$$

Direction of minimum radiation of minor lobes or pattern minima:

The total electric field strength due to N-element broad side array is given by

$$E_t = E_0 \frac{\sin(N\Psi/2)}{\sin(\Psi/2)} e^{j\emptyset}$$

To have the minimum value for E, the pattern factor must be minimum. That is

$$\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$$

must be minimum. To have minimum value for $\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$ its numerator must be minimum.

$$\sin\left(\frac{N\Psi}{2}\right) = 0$$

To satisfy the above relation $\frac{N\psi}{2} = \pm\ n\pi$

Where n = 1,2,3,… n = 0 for major lobe

$$N\psi = \pm\ 2n\pi$$

$$\psi = \pm \frac{2n\pi}{N}$$

But $$\psi = \beta d\cos\theta + \alpha$$

$$\beta d\cos\theta + \alpha = \pm\frac{(2n+1)\pi}{N} \qquad -4$$

Substitute equation 1 in equation4

$$\beta d\cos\theta - \beta d = \pm\frac{2n\pi}{N}$$

$$\beta d(\cos\theta - 1) = \pm\frac{2n\pi}{N}$$

$$\cos\theta - 1 = \pm\frac{2n\pi}{\beta dN} = \pm\frac{2n\pi}{\frac{2\pi}{\lambda}dN}$$

$$\cos\theta - 1 = \pm\frac{n\lambda}{Nd}$$

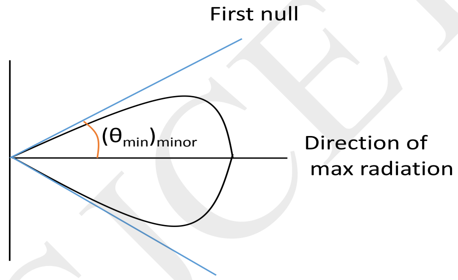
But $$\cos\theta = 1 - 2sin^2\theta/2$$

$$1 - 2sin^2\theta/2 - 1 = \pm\frac{n\lambda}{Nd}$$

$$(\theta_{min})_{minor} = 2\sin^{-1}\left(\pm\sqrt{\frac{n\lambda}{2Nd}}\right) \qquad -5$$

Beam width of major lobe:

The beam width can be obtained from the following figure



$$\text{Beam width (BWFN)} = 2\text{ X }(\theta_{min})_{minor} \qquad -6$$

$$(\theta_{min})_{minor} = 2\sin^{-1}\left(\sqrt{\frac{n\lambda}{2Nd}}\right) = \sin^{-1}\left(\sqrt{\frac{n\lambda}{2Nd}}\right)$$

$$\sin\left(\frac{(\theta_{min})_{minor}}{2}\right) = \left(\sqrt{\frac{n\lambda}{2Nd}}\right)$$

When $(\theta_{min})_{minor}$ is small, then

$$\sin\left(\frac{(\theta_{min})_{minor}}{2}\right) \cong \left(\frac{(\theta_{min})_{minor}}{2}\right) = \left(\sqrt{\frac{n\lambda}{2Nd}}\right)$$

$$(\theta_{min})_{minor} = 2\sqrt{\frac{n\lambda}{2Nd}} = \sqrt{\frac{2n\lambda}{Nd}} \qquad -7$$

Substitute equation 7 in equation 6

$$BWFN = 2\sqrt{\frac{2n\lambda}{Nd}}$$

Replace n with 1, because first null occurs when n = 1

Beam Width between First Nulls (BWFN) of N-element end-fire array is given by

$$\mathrm{BWFN} = 2\sqrt{\frac{2\lambda}{\mathrm{Nd}}}\,\mathrm{radian} = 2\sqrt{\frac{2\lambda}{\mathrm{Nd}}} \times 57.3\ \mathrm{degree}$$

Half Power Beam Width of N-element end-fire array is given by

$$\mathrm{HPBW} = \sqrt{\frac{2\lambda}{\mathrm{Nd}}}\,\mathrm{radian} = \sqrt{\frac{2\lambda}{\mathrm{Nd}}} \times 57.3\ \mathrm{degree}$$

COMPARISON OF DIFFERENT ARRAY:

Comparison of various arrays is given in the table below:

S.No	Parameter	Broad side array	Endfire array	EFA with increased directivity
1	HPBW	$\frac{57.3\,\lambda}{\mathrm{Nd}}$ degree	$\sqrt{\frac{2\lambda}{\mathrm{Nd}}} \times 57.3$ degree	$\sqrt{\frac{2\lambda}{\mathrm{Nd}}} \times 57.3$ degree
2	BWFN	$\frac{114.6\,\lambda}{\mathrm{Nd}}$ degree	$2\sqrt{\frac{2\lambda}{\mathrm{Nd}}} \times 57.3$ degree	$2\sqrt{\frac{2\lambda}{\mathrm{Nd}}} \times 57.3$ degree
3	Direction of minor lobes maxima	$\cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2Nd}\right)$	$\cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2Nd} + 1\right)$	$\cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2Nd} + 1\right)$
4	Direction of minor lobes minima	$\cos^{-1}\left(\pm\frac{n\lambda}{Nd}\right)$	$2\cos^{-1}\left(\pm\sqrt{\frac{n\lambda}{2Nd}}\right)$	$2\cos^{-1}\left(\pm\sqrt{\frac{n\lambda}{2Nd}}\right)$
5	Directivity	$\mathrm{D} = \frac{2\mathrm{Nd}}{\lambda}$	$\mathrm{D} = \frac{4\mathrm{Nd}}{\lambda}$	$\mathrm{D} = 1.789\left(\frac{4\mathrm{Nd}}{\lambda}\right)$

EFA with Increased Directivity:

The condition for producing the maximum radiation in the direction of array axis and minimum radiation in perpendicular direction is, all the elements should be supplied with currents of equal amplitudes and opposite phase. That is the phase angel (progressive phase shift) $\alpha = \pm\,\beta d$ $(\,\alpha = -\,\beta d\ for\ \theta = 0^0\ \ and\ \alpha = +\,\beta d\ for\ \theta = 180^0)$. This particular condition may give the maximum bandwidth but not the directivity.

In order to increase the directivity of the endfire array without destroying any other characteristics, Hansen and Woodyard proposed some other conditions. These conditions are given by

$$\alpha = -\left(\beta d + \frac{\pi}{N}\right) \quad for\ maximum\ in\ \theta = 0^0$$

And

$$\alpha = \left(\beta d + \frac{\pi}{N}\right) \quad for\ maximum\ in\ \theta = 180^0$$

Where N is the no.of elements in the aray.
In order to realize the increased directivity due to the Hansen-Woodyard conditions, it is necessary that besides the conditions given above, the value of the phase difference (Ψ) assumes the values as follows:

(i) For maximum radiation along $\theta = 0^0$

$$|\Psi| = |\beta d \cos\theta + \alpha|_{\theta=0^0} = \frac{\pi}{N}$$
$$|\Psi| = |\beta d \cos\theta + \alpha|_{\theta=180^0} = \pi$$

(ii) For maximum radiation along $\theta = 180^0$

$$|\Psi| = |\beta d \cos\theta + \alpha|_{\theta=0^0} = \pi$$
$$|\Psi| = |\beta d \cos\theta + \alpha|_{\theta=180^0} = \frac{\pi}{N}$$

BINOMIAL ARRAYS:

The major disadvantage of linear arrays such as broad side or endfire array is, the number of minor lobes will be increases when the distance between the elements (d) or number of elements (N) increases. This drawback can be avoided by using the binomial array .i.e. number of minor lobes can be reduced by using the binomial array. In binomial array, non uniform amplitudes will be applied to the individual elements. In this array, the amplitudes of the radiating sources are arranged according to the coefficients of successive terms of the following binomial series and hence the name.

$$(a+b)^{n-1} = a^{n-1} + \frac{(n-1)}{1!}a^{n-2}b + \frac{(n-1)(n-2)}{2!}a^{n-3}b^2 + \frac{(n-1)(n-2)(n-3)}{3!}a^{n-4}b^3 + \ .....$$

Where n is the number of elements in the array.
The following two conditions were satisfied in binomial array to reduce the number of minor lobes.

(i) Spacing between the two consecutive radiating sources does not exceed λ/2.
(ii) The current amplitudes in radiating sources are proportional to the coefficients of the successive terms of the binomial series.

The coefficients of successive terms can be obtained either from the binomial series or from the Pascal's triangle shown in the figure below.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1

Fig: Pascal's Triangle

Advantages of Binomial array:

(i) Less number of minor lobes as compared with linear arrays
(ii) Beam width increases

Disadvantages of Binomial array:

(i) Beam width increases and hence the directivity will be decreases

(ii) For design of large array, larger amplitude ratio of sources is required.

PRINCIPLE OF PATTERN MULTIPLICATION

The principle of pattern multiplication or multiplication of patterns is stated as follows: "The total field pattern of array of non-isotropic but similar sources is the multiplication of the individual source pattern and the pattern of array of isotropic point sources each located at the phase center of individual source, and having the relative amplitude and phase, where as the total phase pattern is the addition of the phase pattern of the individual sources and that of the array of isotropic point sources". This statement can be expressed as

$$E = \{E_i(\theta,\varphi) \times E_a(\theta,\varphi)\} \times \{E_{pi}(\theta,\varphi) \times E_{pa}(\theta,\varphi)\}$$

Where $E_i(\theta,\varphi)$ is the field or magnitude pattern of individual source

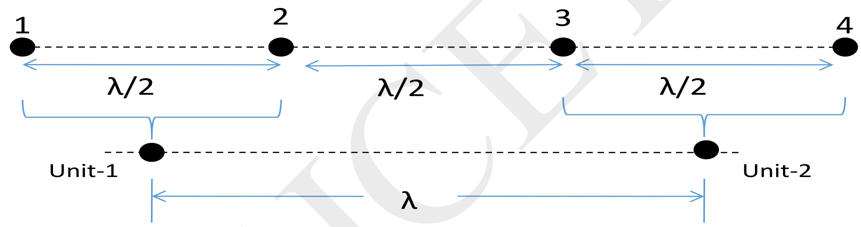
$E_a(\theta,\varphi)$ is field or magnitude pattern of array of isotropic point sources

$E_{pi}(\theta,\varphi)$ is the phase pattern of individual source

$E_{pa}(\theta,\varphi)$ is the phase pattern of array of isotropic point sources.

The pattern multiplication has great advantage that, it makes possible to sketch rapidly, almost by inspection, the pattern of complicated arrays.

Let us explain the concept of pattern multiplication by considering the 4 element broadside array with spacing λ/2 as shown in figure below.



The pattern of elements 1 and 2 operating as a unit, that is two antennas spaced λ/2 and fed in phase (broad side array). Also antennas 2 and 3 will be considered as another unit with the same pattern of figure of eight shape. Now the 4-element array has been reduced to 2-element (units) array with spacing λ as shown figure. The radiation pattern of two element broad side array with spacing λ can be obtained by using the two element array analysis and is shown in the figure below. This pattern is called group pattern.

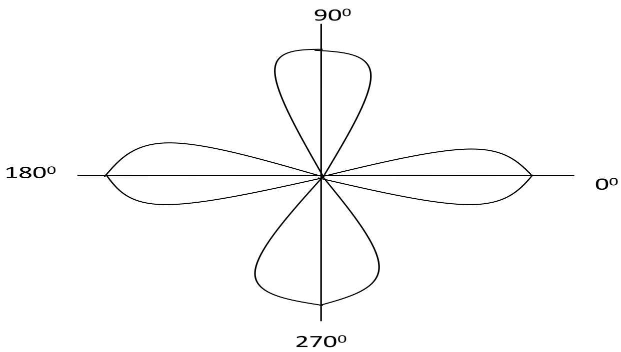
From the two element array analysis, we know that

$$\theta_{max} = \cos^{-1}\left(\pm\frac{n\lambda}{d}\right) = \cos^{-1}\left(\pm\frac{n\lambda}{\lambda}\right) = \cos^{-1}(\pm n)$$

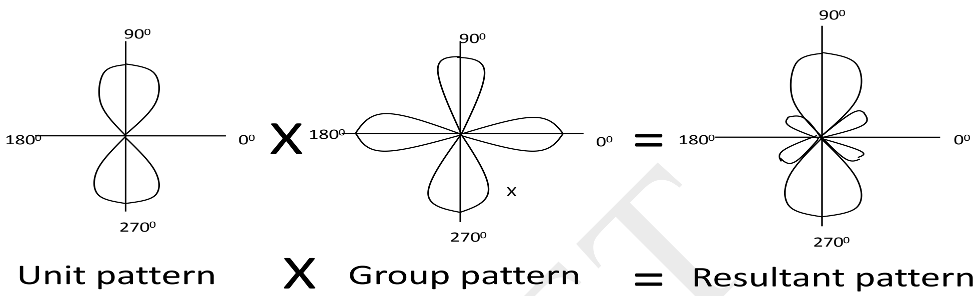
When n = 0, $\theta_{max} = \cos^{-1}(0) = 90^0 \;\&\; 270^0$

When n = 1, $\theta_{max} = \cos^{-1}(1) = 0^0 \;\&\; 180^0$

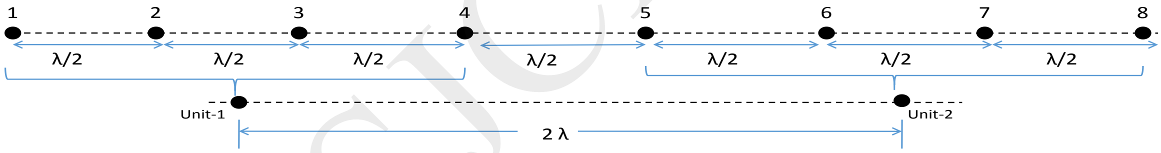
When n = 2, $\theta_{max} = \cos^{-1}(2) = not\ satisfied$

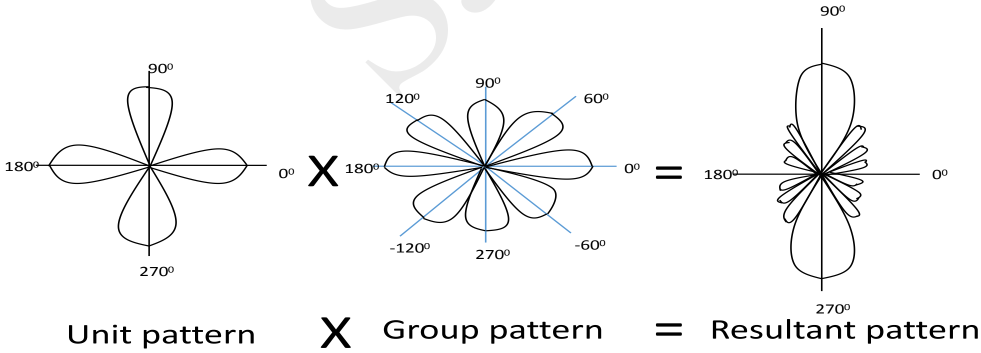


The resultant pattern for the original 4-element array is obtained by multiplying the unit pattern with group pattern. The above procedure can be represented in the following figure below.



Similarly the radiation pattern of 8 element broad side array with spacing λ/2 is shown in the figure below.



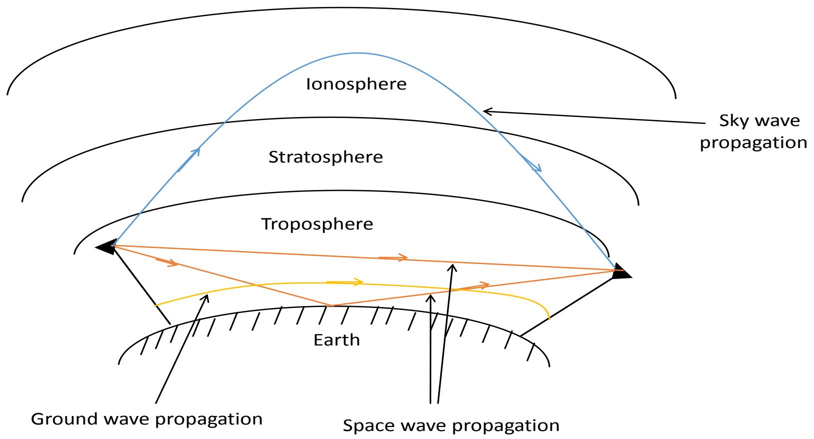


DIFFERENT MODES OF WAVE PROPAGATION

There are three modes of wave propagation such as

(i) Ground wave or Surface wave propagation
(ii) Space wave or Tropospheric wave propagation
(iii)Sky wave or Ionospheric wave propagation

These three modes are represented in the figure shown below



Ground wave or Surface wave propagation: When the waves propagate along the surface of the earth, then it is known as ground wave or surface wave propagation. The ground wave propagation will be used for the frequencies less than 2 MHz. The ground wave propagation covers more distance as compared with the space wave propagation. The ground wave field strength dies out after traveling long distance due to wavefront tilting and earth attenuation.

Space wave or Tropospheric wave propagation: When the waves propagate through the tropospheric layer of atmosphere then it is known as space wave or tropospheric wave propagation. This propagation will be used for the frequency greater than the 30 MHz. This propagation is also called as LOS (Line Of Sight) propagation. It covers less distance as compared with ground wave and sky wave propagation. Space waves do not follow the earth's curvature and hence they cannot travel more distance.

Sky wave or Ionospheric wave propagation: When the waves propagate through the Ionospheric layer of atmosphere then it is known as sky wave or ionospheric wave propagation. This propagation will be used for the frequency between 2 MHz to 30 MHz. It covers very long distance as compared with ground wave and sky wave propagation.

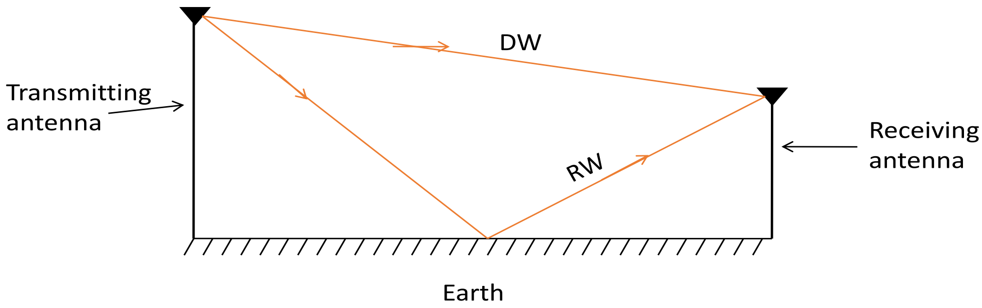
GROUND WAVE PROPAGATION

Introduction:

When the waves propagate along the surface of the earth, then it is known as ground wave or surface wave propagation. The ground wave propagation will be used for the frequencies less than 2 MHz. The ground wave propagation covers more distance as compared with the space wave propagation. The ground wave field strength dies out after traveling long distance due to wavefront tilting and earth attenuation.

Plane earth reflections:

When a wave is reflected from the flat earth or plane earth, then it is known as plane earth reflections. When the distance between the transmitting and receiving antenna is small, then the earth can be imagined as flat or plane surface. The plane earth reflections are shown in the figure below.



From the above figure it can be observed that, the signal strength at the receiving point is the combination of direct wave and reflected wave. The magnitude of the reflected signal depends upon the type of earth surface i.e smooth surface or rough surface. The roughness of the earth
can be calculated based on the following empherical formula.

$$R = 4\pi\sigma sin\theta/\lambda$$

Where

σ is the standard deviation of the surface irregularities

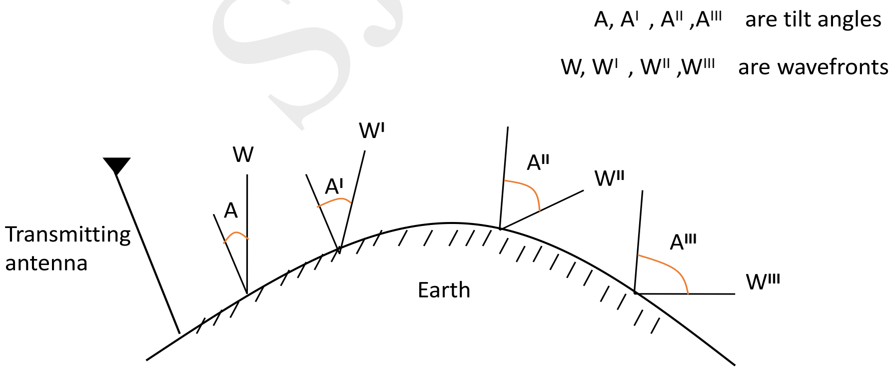
θ is the angle of incidence with respect to the normal to the earth surface

λ is the wavelength.

When the value of R is less than 0.1, then the earth surface can be considered as smooth and when it is greater than 10, then it is considered as rough surface. When the angle θ is zero, then the surface is smooth. When the signal is reflected from the smooth surface, then the amplitude of the reflected signal will be equal to the incident signal where as if it reflected from the rough surface, the amplitude of the reflected signal will be less than the incident signal because of scattering of signal by the rough surface.

Wave tilt:

The following figure represents the basic principle involved in ground wave propagation.

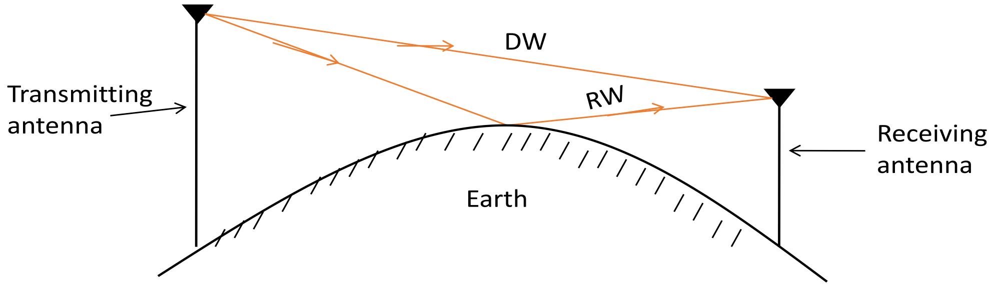


In ground wave propagation, the waves glides over the earth surface i.e the waves travels along the surface of the earth. In this propagation, the lower tip of the electric filed strength vector will be in touch with the earth surface. Initially if we assume the orientation of electric vector is vertical, later it becomes horizontal after traveling long distance due to the tilting of wavefront. Due to the surface irregularities of the earth, the wavefront of the waves will tilt as shown in the figure. Therefore, the ground wave field strength will be dies out after traveling certain distance due to shorting out the horizontal component by the earth(earth is assumed as good conductor).

Curved earth reflections:

When a wave is reflected from the curved earth or plane earth, then it is known as curved earth reflections. When the distance between the transmitting and receiving

antenna is large, then the earth can be imagined as curved earth. The curved earth reflections are shown in the figure below.
Due to the curvature of the earth, the space wave signal will be affected more as compared with the surface wave signal. Form the above figure it can be observed that, the physical heights of the antennas will be more than the effective heights. The major difference between the plane earth reflections and curved earth reflections is, the path difference between the direct wave and reflected wave will be more in plane earth reflections as compared with curved earth reflections.



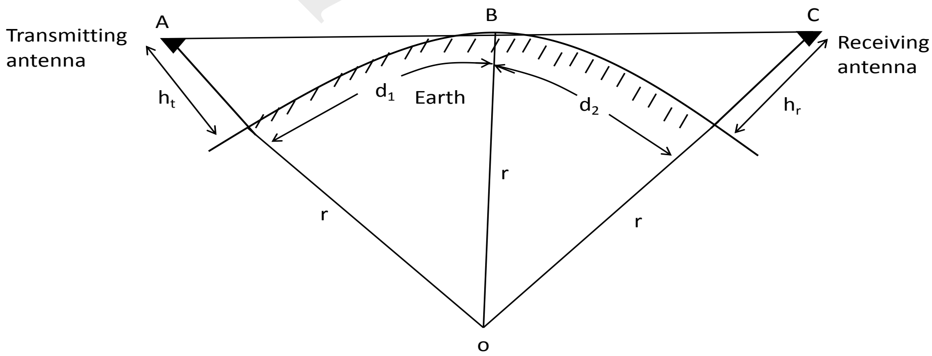
SPACE WAVE PROPAGATION

Introduction:

When the waves propagate through the tropospheric layer of atmosphere then it is known as space wave or tropospheric wave propagation. This propagation will be used for the frequency greater than the 30 MHz. This propagation is also called as LOS (Line Of Sight) propagation. It covers less distance as compared with ground wave and sky wave propagation. Space waves do not follow the earth's curvature and hence they cannot travel more distance.

LOS range or coverage distance:

The coverage range or LOS distance is defined as the maximum distance at which the space wave signal would be received with receiving antenna. The LOS range can be calculated from the figure shown below.



From triangle OAB,

$$(r + h_t)^2 = r^2 + d_1^2$$
$$d_1^2 = (r + h_t)^2 - r^2$$
$$d_1 = \sqrt{(r + h_t)^2 - r^2}$$
$$d_1 = \sqrt{r^2 + 2rh_t + h_t^2 - r^2}$$
$$d_1 = \sqrt{2rh_t} \qquad -1$$

h_t^2 is neglected as compared with 2Rh$_t$

Similarly from triangle OBC,

$$(r + h_r)^2 = r^2 + d_2^2$$
$$d_2^2 = (r + h_r)^2 - r^2$$
$$d_2 = \sqrt{(r + h_r)^2 - r^2}$$
$$d_2 = \sqrt{r^2 + 2rh_r + h_r^2 - r^2}$$
$$d_1 = \sqrt{2rh_r} \qquad - 2$$

h_r^2 is neglected as compared with 2Rh$_r$

$$LOS\ range\ (d) = d_1 + d_2 \qquad - 3$$

Substitute equations 1 and 2 in equation 3

Then,

$$d = \sqrt{2rh_t} + \sqrt{2rh_r}$$
$$d = \sqrt{2r}\left(\sqrt{h_t} + \sqrt{h_r}\right) \quad in\ meters$$

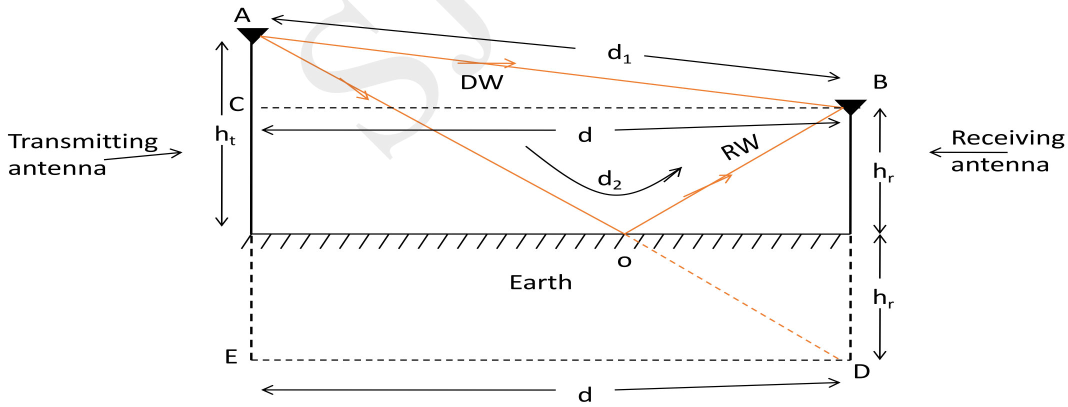
In above equation the letter ‘r’ represents the earth radius which will be equal to 6370 km.

The above equation can be expressed in kilometers as

$$d = \sqrt{2 \times 6370\ km}\left(\sqrt{h_t} + \sqrt{h_r}\right)$$
$$d = \sqrt{2 \times 6370 \times 10^3}\left(\sqrt{h_t} + \sqrt{h_r}\right)$$
$$d = \sqrt{2 \times 6.37 \times 10^3 \times 10^3}\left(\sqrt{h_t} + \sqrt{h_r}\right)$$
$$d = \sqrt{2 \times 6.37}\left(\sqrt{h_t} + \sqrt{h_r}\right) \times 10^3$$
$$d = 3.57\left(\sqrt{h_t} + \sqrt{h_r}\right) \quad km$$

Field strength of space wave propagation:

The equation for the space wave field strength is derived as follows:



The path difference between the direct wave and reflected wave is given by

$$P.d = d_2 - d_1 \qquad - 1$$

Form the Δ OAB shown in above figure,

$$d_1^2 = d^2 + (h_t - h_r)^2$$
$$d_1^2 = d^2\left(1 + \left(\frac{h_t - h_r}{d}\right)^2\right)$$
$$d_1 = d\sqrt{1 + \left(\frac{h_t - h_r}{d}\right)^2} = d\left[1 + \left(\frac{h_t - h_r}{d}\right)^2\right]^{1/2}$$

Expand by using binomial expansion

$$d_2 = d\left[1 + \frac{1}{2}\left(\frac{h_t - h_r}{d}\right)^2\right]$$

$$d_1 = d + \frac{(h_t - h_r)^2}{2d} \qquad - 2$$

Similarly from ΔOBC,

$$d_2^2 = d^2 + (h_t + h_r)^2$$

$$d_2^2 = d^2\left(1 + \left(\frac{h_t + h_r}{d}\right)^2\right)$$

$$d_2 = d\sqrt{1 + \left(\frac{h_t + h_r}{d}\right)^2} = d\left[1 + \left(\frac{h_t + h_r}{d}\right)^2\right]^{1/2}$$

Expand by using binomial expansion

$$d_2 = d\left[1 + \frac{1}{2}\left(\frac{h_t + h_r}{d}\right)^2\right]$$

$$d_2 = d + \frac{(h_t + h_r)^2}{2d} \qquad - 3$$

Substitute equations 2 and 3 in equation 1

$$p.d = d + \frac{(h_t + h_r)^2}{2d} - \left(d + \frac{(h_t - h_r)^2}{2d}\right)$$

$$p.d = d + \frac{(h_t + h_r)^2}{2d} - d - \frac{(h_t - h_r)^2}{2d}$$

$$p.d = \frac{(h_t + h_r)^2 - (h_t - h_r)^2}{2d} = \frac{4h_t h_r}{2d}$$

Phase angel due to path difference is given by

$$\alpha = \frac{2\pi}{\lambda}(p.d) = \frac{2\pi}{\lambda}\frac{4h_t h_r}{2d} = \frac{4\pi h_t h_r}{\lambda d} \qquad - 4$$

Total phase difference between the direct wave and reflected wave is given by

$$\theta = \alpha + \beta$$

Where β is called the phase of the reflected signal due to reflection by the earth which is equal to 180^0.

$$\theta = \alpha + 180^0 \qquad - 5$$

The total electric field strength at the receiving point is given by

$$E_R = E_0 e^{j0} + kE_0 e^{-j\theta} = E_0\left(1 + ke^{-j\theta}\right)$$

$$E_R = E_0\left(1 + k(cos\theta - jsin\theta)\right)$$

$$E_R = E_0\left((1 + kcos\theta) - jksin\theta\right)$$

$$|E_R| = E_0\sqrt{(1 + kcos\theta)^2 + k^2 sin^2\theta}$$

$$|E_R| = E_0\sqrt{1 + k^2cos^2\theta + 2kcos\theta + k^2sin^2\theta} = E_0\sqrt{1 + k^2 + 2kcos\theta}$$

But

$$cos\theta = 2cos^2\frac{\theta}{2} - 1$$

$$|E_R| = E_0\sqrt{1 + k^2 + 2k\left(2cos^2\frac{\theta}{2} - 1\right)}$$

But for good conductor the reflection coefficient k equal to unity.

$$|E_R| = E_0\sqrt{1+1+2\left(2cos^2\frac{\theta}{2}-1\right)} = E_0\sqrt{2+4cos^2\frac{\theta}{2}-2}$$

$$|E_R| = 2E_0 cos\frac{\theta}{2} \qquad -6$$

Substitute equation 5 in equation 6

$$|E_R| = 2E_0 cos\left(\frac{\alpha+180^0}{2}\right) = 2E_0 sin\left(\frac{\alpha}{2}\right) \qquad -7$$

Substitute equation 4 in equation 7

$$|E_R| = 2E_0 sin\left(\frac{4\pi h_t h_r}{2\lambda d}\right)$$

$$|E_R| \cong 2E_0\left(\frac{4\pi h_t h_r}{2\lambda d}\right) = E_0\frac{4\pi h_t h_r}{\lambda d}$$

But

$$E_0 = \frac{7\sqrt{P}}{d}$$

Where P is the power transmitted.

$$|E_R| = \frac{7\sqrt{P}}{d}\frac{4\pi h_t h_r}{\lambda d}$$

$$|E_R| = \frac{88\sqrt{P}\, h_t h_r}{\lambda d^2}$$

Where

P is the power transmitted

h_t is the height of the transmitting antenna

h_r is the height of the receiving antenna

λ is the wavelength

d is the distance between the transmitting and receiving antennas.

It can be observed that, the filed strength and distance are inversely proportional that is the field strength is decreases with increase in distance. Also it is observed that, the space wave field strength is a function of heights of the transmitting and receiving antenna. To increases the field strength, either the height of the transmitting antenna or receiving antenna or both can be increased.

SKY WAVE PROPAGATION

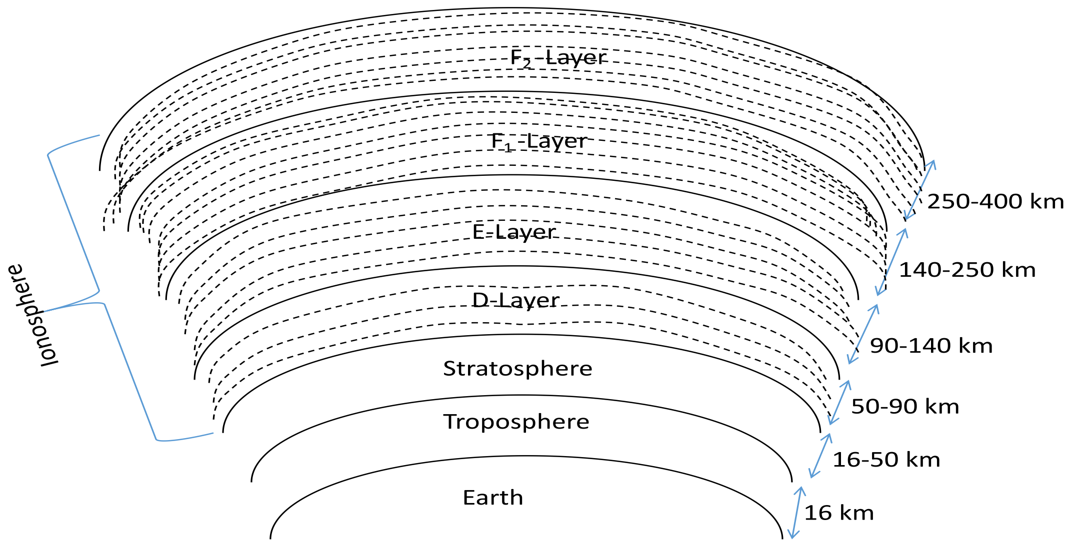
Introduction:

When the waves propagate through the Ionospheric layer of atmosphere then it is known as sky wave or ionospheric wave propagation. This propagation will be used for the frequency between 2 MHz to 30 MHz. It covers very long distance as compared with ground wave and sky wave propagation.

Structure of ionosphere:

The ionosphere is a region or layer above the earth and is composed of ionized layers. It is the highest layer of the atmosphere. The ionization is appreciable in this layer and hence the name ionosphere arises. The structure of ionosphere is shown in the figure below. The ionosphere consists of three layers such as D-layer, E-Layer and F-layer.

D-layer: It exists between the heights 50 to 90 km above the earth's surface. D-Layer is also known as Kennelly Heaviside layer. This layer will exist during the day time only where as in night times it disappears. The ionization in this layer is less because, it receives lee amount of solar energy. This layer will absorb the signal in the LF and MF ranges.



E-layer: This layer exists at a height of 90 to 140 km above the earth's surface with maximum density at 110 km. This layer is also known as Appleton layer. This almost constant with little diurnal and seasonal variations. It permits medium distance communication in LF and HF frequency bands. In E-layer there is one more layer called sporadic E-layer which exists occasionally. This layer results of an anomalous phenomena and falls under the category of irregular variations. The main reason for the existence of this layer is infiltration of charge carriers from the upper layers. This layer may not exist at all places. Also it do not present at all seasons.

F-Layer: It exists at heights of 150 to 400 km above the earth's surface. It is subdivided in to two layers such as F_1 layer and F_2 layer. F_1 layer will present at a height of 150 to 250 km where as F_2 layer will present at a height of 250 to 400 km. This F-layer will be used for long distance communication. During the night time both F_1 and F_2 layers combine and form as a single layer because of less solar energy during the night times.

Refraction and reflection of sky waves by ionosphere:

The basic principle of ionosphereic propagation is reflection but the phenomenon that takes place is refraction. To understand the refraction and reflection of sky waves, it is better to derive the equation for the refractive index of the ionosphere.

Let $E = E_m \sin\omega t$ be the electric field strength that enter in to the ionosphere. Due to this electric field, there is a force on the electrons present in the ionosphere. This force is given by

$$F_e = -eE \qquad -1$$

Where e is the charge of electron and F_e is the electric force.

Also from Newton's law the force is given by

$$F = ma = m\frac{dv}{dt} \qquad -2$$

Where 'm' is the mass of electron, 'v' is the velocity of electron and 'a' is the acceleration.

Equate equations 1 and 2

$$m\frac{dv}{dt} = -eE$$

$$dv = -\frac{eE}{m}dt$$

Take integration on both sides,

$$v = -\int \frac{eE}{m}dt = -\frac{e}{m}\int E_m \sin\omega t$$

$$v = \frac{eE_m cos\omega t}{m\omega}$$

$$v = \left(\frac{e}{m\omega}\right) E_m cos\omega t \qquad -3$$

If 'N' be the number of electrons per cubic meter, then the instantaneous electric current is given by

$$i_e = -Nev \qquad -4$$

Substitute equation 3 in 4

$$i_e = -Ne\left(\frac{e}{m\omega}\right) E_m cos\omega t = -\frac{Ne^2}{m\omega} E_m cos\omega t \qquad -5$$

The above current is called as inductive current. Beside this inductive current , there is additional current called capacitive current or displacement current due to the displacement of charge carriers. This current is known as capacitive current and is given by

$$i_c = \frac{dD}{dt} = \frac{d}{dt}(\varepsilon_0 E) = \frac{d}{dt}(\varepsilon_0 E_m \sin\omega t)$$

$$i_c = \varepsilon_0 E_m cos\omega t\omega \qquad -6$$

Total current is given by

$$i = i_c + i_e \qquad -7$$

Substitute equations 6 and 5 in equation 7

$$i = \varepsilon_0 E_m cos\omega t\omega - \frac{Ne^2}{m\omega} E_m cos\omega t$$

$$i = E_m cos\omega t\omega \left[\varepsilon_0 - \frac{Ne^2}{m\omega^2}\right] \qquad -8$$

By comparing equations 6 and 8 we can say that the term $\left[\varepsilon_0 - \frac{Ne^2}{m\omega^2}\right]$ is called effective dielectric constant (ε).

$$\varepsilon = \left[\varepsilon_0 - \frac{Ne^2}{m\omega^2}\right]$$

$$\varepsilon = \varepsilon_0\left[1 - \frac{Ne^2}{m\omega^2\varepsilon_0}\right]$$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = 1 - \frac{Ne^2}{m\omega^2\varepsilon_0}$$

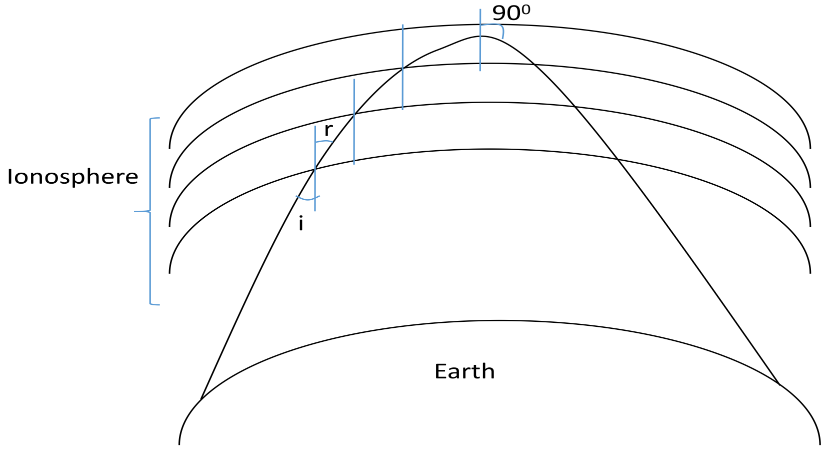
The refractive index is given by

$$\mu = \sqrt{\varepsilon_r} = \sqrt{1 - \frac{Ne^2}{m\omega^2\varepsilon_0}}$$

Substitute the values m = 9.107 X10^{-31} kg, e = 1.602 X 10^{-19} coulombs, ε_0 = 8.854 X 10^{-12} and ω = 2πf, Then

$$\mu = \sqrt{1 - \frac{81\,N}{f^2}}$$

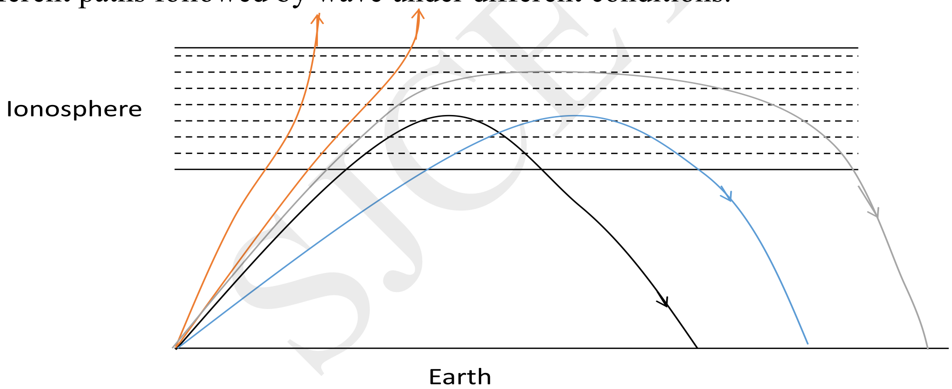
The above equation represents the refractive index of the ionosphere. The following figure represents the principle of refraction and reflection of sky wave by the ionosphere.



When the EM wave enters in the ionosphere, it undergoes refraction due to the variation of refractive index. The refractive index of upper layers will be less than the lower layers. Due to this variation of refractive index, the wave will bend as shown in the figure. The angel of refraction is go on increases as the wave travels through the layers of ionosphere. At a peak point the angle of refraction becomes 90^0, then onwards the wave travel in downward direction and finally it reaches the receiving point on the earth surface.

Ray path:

The path followed by the wave is termed as ray path. The following figure represents the different paths followed by wave under different conditions.



When the angle of incidence (i) is large, then the wave will reflected by the ionosphere and returned to the earth as indicated by ray 1.When the angle of incidence decreases, the penetration of wave in to the layer will increases as shown by the rays 2, 3 and 4. When the angle of incidence is very small, the waves will enter in to the outer region and they will not return to the earth surface as shown by the rays 5 and 6.

Critical frequency:

The critical frequency (f_c) is defined as the highest frequency that returns from the ionosphere layer at vertical incidence for that particular layer. Different layers will posses different values of critical frequency. The equation for the critical frequency is derived as follows:

$$\mu = \frac{\sin i}{\sin r} = \sqrt{1 - \frac{81\,N}{f^2}}$$

But the critical frequency is defined for vertical incidence, that is i = 0^0, then the above equation becomes

$$0 = \sqrt{1 - \frac{81\,N}{f^2}}$$
$$0 = 1 - \frac{81\,N}{f^2}$$
$$\frac{81\,N}{f^2} = 1$$
$$f^2 = 81\,N$$
$$f = f_c = \sqrt{81\,N} = 9\sqrt{N} = 9\sqrt{N_{max}}$$

Where N_{max} is the maximum electron density of the ionospheric layer.

MUF (Maximum Usable Frequency):

MUF is abbreviated as Maximum Usable Frequency and is defined as the highest frequency that returns from the ionosphere layer at some angle of incidence other that the vertical incidence. Major difference between the critical frequency and MUF is, the critical frequency will be defined with respect to vertical incidence where as MUF will be defined with respect to some angle of incidence other than the vertical incidence. The equation for the MUF is derived as follows:

$$\mu = \frac{\sin i}{\sin r} = \sqrt{1 - \frac{81\,N}{f^2}}$$

To have reflection from the ionosphere the angel of refraction r must be equal to 90^0.

$$\frac{\text{Sin}\, i}{\sin 90^0} = \sqrt{1 - \frac{81\,N}{f^2}}$$
$$\sin i = \sqrt{1 - \frac{81\,N}{f^2}}$$
$$sin^2 i = 1 - \frac{81\,N}{f^2}$$
$$\frac{81\,N}{f^2} = 1 - sin^2 i = cos^2 i$$
$$f^2 = \frac{81\,N}{cos^2 i} = \frac{f_c^2}{cos^2 i}$$
$$f = f_{MUF} = \frac{f_c}{\cos i} = f_c \sec i$$

The above equation represents the MUF. This equation is specifically called as secant law because it is in terms of secant.

LUF (Lowest Usable Frequency):

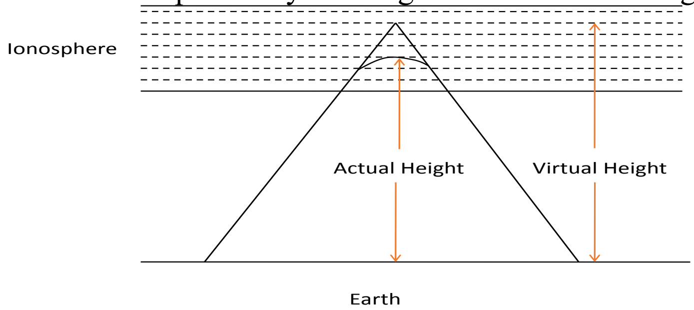
Lowest usable Frequency (LUF) is defined as the frequency below which the entire signal strength will be absorbed by the ionospheric layer. MUF gives the upper limit on the usable frequency where as LUF gives the lower limit on the usable frequency.

OF (Optimum Frequency):

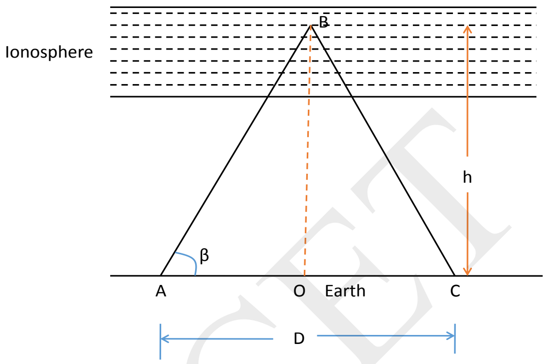
MUF and LUF give the upper and lower limit on the usable frequency but they may not be used to select the practical operating frequency. The reason for this is the MUF and LUF will change with heights of the layers. Therefore it is required to define on more frequency called optimum frequency (OF) or optimum working frequency (OWF) for the selection operating frequency. The Optimum Frequency (OF) is 85% of MUF.

Virtual height and skip distance:

Virtual Height: It may be defined as "the height to which a short pulse of energy sent vertically upward and traveling with the speed of light would reach taking the same two-way travel time as does the actual pulse reflected from the ionospheric layer". The following figure represents the virtual height ant physical height (Actual height). The virtual height of the ionospheric layer is larger than the actual height.



The virtual height of the flat earth shown in figure below



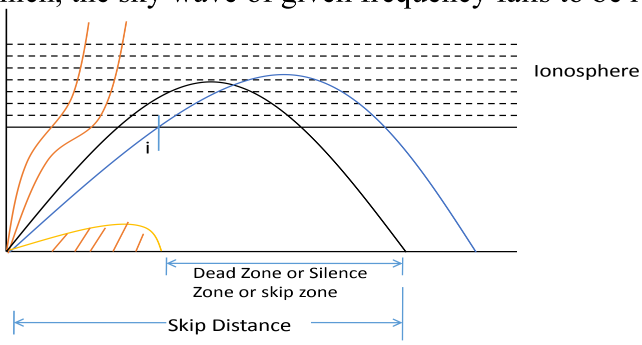
The virtual height of the flat earth shown is given by

$$h = \frac{D\, tan\beta}{2}$$

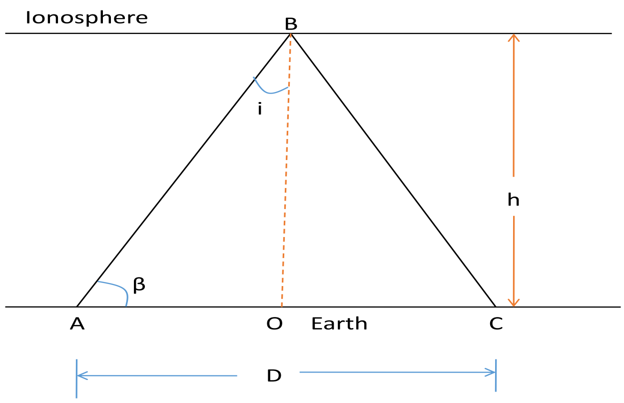
where D is the skip distance, β is the elevation angle. The virtual height of the curved earth is given by

$$h = \frac{cT}{2}$$

where c is the velocity of light and T is the two-way transit time of pulse.
Skip distance: The following figure represents the skip distance. The skip distance is defined as the minimum distance from the transmitter at which sky wave of given frequency is returned to earth by the ionosphere. It is also defined as minimum distance within which, the sky wave of given frequency fails to be reflected back.



<u>Relation between muf and skip distance:</u>
For flat earth, the relation in between the MU=F (f_{MUF}) and skip distance (D) is derived from the following figure.



From Δ OAB,

$$\cos i = \frac{h}{\sqrt{h^2 + \frac{D^2}{4}}} \qquad -1$$

$$\mu = \frac{\sin i}{\sin r} = \sqrt{1 - \frac{81\,N}{f^2}} = \sqrt{1 - \frac{81\,N_{max}}{f_{MUF}^2}}$$

But

$$81\,N_{max} = f_c^2$$

And

Angel of refraction r = 90^0 to have the reflection by the ionosphere

$$\frac{\sin i}{\sin(90)} = \sqrt{1 - \frac{f_c^2}{f_{MUF}^2}}$$

$$\sin i = \sqrt{1 - \frac{f_c^2}{f_{MUF}^2}}$$

$$sin^2 i = 1 - \frac{f_c^2}{f_{MUF}^2}$$

$$\frac{f_c^2}{f_{MUF}^2} = 1 - sin^2 i = cos^2 i \qquad -2$$

Substitute equation 1 in equation 2

$$\frac{f_c^2}{f_{MUF}^2} = \left(\frac{h}{\sqrt{h^2 + \frac{D^2}{4}}}\right)^2$$

$$\frac{f_{MUF}^2}{f_c^2} = \frac{4h^2 + D^2}{4h^2}$$

$$\frac{f_{MUF}}{f_c} = \sqrt{\frac{4h^2 + D^2}{4h^2}}$$

$$f_{MUF} = f_c\sqrt{\frac{4h^2 + D^2}{4h^2}} = f_c\sqrt{1 + \left(\frac{D}{2h}\right)^2}$$

Skip distance is given by

$$D = 2h\sqrt{\frac{f_{MUF}^2}{f_c^2} - 1}$$

The above equation represents the relation between the MUF and skip distance for flat earth or plane earth.

WAVEGUIDES

RECTANGULAR WAVEGUIDES:

Solution of wave equation in rectangular coordinates

A **rectangular waveguide** is a hollow metallic tube with a **rectangular** cross section. The conducting walls of the **waveguide** confine the electromagnetic fields and thereby guide the electromagnetic wave.

The Maxwell's equation are given by

$$\nabla \cdot B = 0 \qquad \text{-(1)}$$

$$\nabla \cdot D = \rho_v \qquad \text{- (2)}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} = -j\omega\mu H \qquad \text{- (3)}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

But for dielectric medium J = 0, then

$$\nabla \times H = \frac{\partial D}{\partial t} = j\omega\varepsilon E \qquad \text{-(4)}$$

Express equation 3 in rectangular coordinate system

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu\left(H_x a_x + H_y a_y + H_z a_z\right)$$

$$a_x\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) - a_y\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) + a_z\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = -j\omega\mu\left(H_x a_x + H_y a_y + H_z a_z\right)$$

Equate individual components on both sides

$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) = -j\omega\mu H_x \qquad \text{- (5)}$$

$$\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) = j\omega\mu H_y \qquad \text{- (6)}$$

$$\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = -j\omega\mu H_z \qquad \text{- (7)}$$

Similarly express equation 4 in rectangular coordinate system

$$\begin{vmatrix} a_x & a_y & a_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon\left(E_x a_x + E_y a_y + E_z a_z\right)$$

$$a_x\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - a_y\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + a_z\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = j\omega\varepsilon\left(E_x a_x + E_y a_y + E_z a_z\right)$$

Equate individual components on both sides

$$\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) = j\omega\varepsilon E_x \qquad \textbf{-}\ (8)$$

$$\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) = -j\omega\varepsilon E_y \qquad -\ (9)$$

$$\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = j\omega\varepsilon E_z \qquad \textbf{-}\ (10)$$

When the wave is traveling along the z-direction, then we can have

$$E_x = E_{x0} e^{-\gamma z}$$

Differentiate above equation with respect to z

$$\frac{\partial E_x}{\partial z} = -\gamma E_{x0} e^{-\gamma z} = -\gamma E_x ----(11)$$

Similarly,

$$\frac{\partial E_y}{\partial z} = -\gamma E_{y0} e^{-\gamma z} = -\gamma E_y ----(12)$$

$$\frac{\partial H_x}{\partial z} = -\gamma H_{x0} e^{-\gamma z} = -\gamma H_x ----(13)$$

$$\frac{\partial H_y}{\partial z} = -\gamma H_{y0} e^{-\gamma z} = -\gamma H_z ----(14)$$

Substitute equation 11 in equation 6

$$\left(\frac{\partial E_z}{\partial x} + \gamma E_x\right) = j\omega\mu H_y \qquad \textbf{- (15)}$$

Substitute equation 12 in equation 5

$$\left(\frac{\partial E_z}{\partial y} + \gamma E_y\right) = -j\omega\mu H_x ----(16)$$

Substitute equation 13 in equation 9

$$\left(\frac{\partial H_z}{\partial x} + \gamma H_x\right) = -j\omega\varepsilon E_y ----(17)$$

Substitute equation 14 in equation 8

$$\left(\frac{\partial H_z}{\partial y} + \gamma H_y\right) = j\omega\varepsilon E_x ----(18)$$

From equation 17, we can have

$$\frac{\partial H_z}{\partial x} + \gamma H_x = -j\omega\varepsilon E_y$$

$$\gamma H_x = -j\omega\varepsilon E_y - \frac{\partial H_z}{\partial x}$$

$$H_x = \frac{1}{\gamma}\left(-j\omega\varepsilon E_y - \frac{\partial H_z}{\partial x}\right) ----(19)$$

Substitute equation 19 in equation 16

$$\left(\frac{\partial E_z}{\partial y} + \gamma E_y\right) = -j\omega\mu\left(\frac{1}{\gamma}\left(-j\omega\varepsilon E_y - \frac{\partial H_z}{\partial x}\right)\right)$$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = \frac{j^2\omega^2\mu\varepsilon E_y}{\gamma} + \frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial x}$$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -\frac{\omega^2\mu\varepsilon E_y}{\gamma} + \frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial x}$$

$$\gamma E_y + \frac{\omega^2\mu\varepsilon E_y}{\gamma} = \frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial x} - \frac{\partial E_z}{\partial y}$$

$$\left(\gamma + \frac{\omega^2\mu\varepsilon}{\gamma}\right)E_y = \frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial x} - \frac{\partial E_z}{\partial y}$$

$$\left(\frac{\gamma^2 + \omega^2\mu\varepsilon}{\gamma}\right)E_y = \frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial x} - \frac{\partial E_z}{\partial y}$$

$$\left(\frac{k^2}{\gamma}\right)E_y = \frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial x} - \frac{\partial E_z}{\partial y}$$

Where,

$$k^2 = \gamma^2 + \omega^2\mu\varepsilon$$

Known as characteristic equation.

$$E_y = \frac{\gamma}{k^2}\left(\frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial x} - \frac{\partial E_z}{\partial y}\right)$$

$$E_y = \frac{\gamma}{k^2}\frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial x} - \frac{\gamma}{k^2}\frac{\partial E_z}{\partial y}$$

$$E_y = \frac{j\omega\mu}{k^2}\frac{\partial H_z}{\partial x} - \frac{\gamma}{k^2}\frac{\partial E_z}{\partial y} ----(20)$$

From equation 18, we can have

$$\gamma H_y = j\omega\varepsilon E_x - \frac{\partial H_z}{\partial y}$$

$$H_y = \frac{1}{\gamma}\left(j\omega\varepsilon E_x - \frac{\partial H_z}{\partial y}\right) \qquad - (21)$$

Substitute equation 21 in equation in 15

$$\left(\frac{\partial E_z}{\partial x} + \gamma E_x\right) = \frac{j\omega\mu}{\gamma}\left(j\omega\varepsilon E_x - \frac{\partial H_z}{\partial y}\right)$$

$$\frac{\partial E_z}{\partial x} + \gamma E_x = \frac{j^2\omega^2\mu\varepsilon E_x}{\gamma} - \frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial y}$$

$$\frac{\partial E_z}{\partial x} + \gamma E_x = -\frac{\omega^2 \mu\varepsilon E_x}{\gamma} - \frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial y}$$

$$\gamma E_x + \frac{\omega^2 \mu\varepsilon E_x}{\gamma} = -\frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial y} - \frac{\partial E_z}{\partial x}$$

$$E_x\left(\gamma + \frac{\omega^2 \mu\varepsilon}{\gamma}\right) = -\frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial y} - \frac{\partial E_z}{\partial x}$$

$$E_x\left(\frac{\gamma^2 + \omega^2 \mu\varepsilon}{\gamma}\right) = -\frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial y} - \frac{\partial E_z}{\partial x}$$

$$E_x\left(\frac{k^2}{\gamma}\right) = -\frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial y} - \frac{\partial E_z}{\partial x}$$

$$E_x = \frac{\gamma}{k^2}\left(-\frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial y} - \frac{\partial E_z}{\partial x}\right)$$

$$E_x = -\frac{\gamma}{k^2}\frac{j\omega\mu}{\gamma}\frac{\partial H_z}{\partial y} - \frac{\gamma}{k^2}\frac{\partial E_z}{\partial x}$$

$$E_x = -\frac{j\omega\mu}{k^2}\frac{\partial H_z}{\partial y} - \frac{\gamma}{k^2}\frac{\partial E_z}{\partial x} ----(22)$$

Substitute equation 20 in 19

$$H_x = \frac{1}{\gamma}\left(-j\omega\varepsilon\left(\frac{j\omega\mu}{k^2}\frac{\partial H_z}{\partial x} - \frac{\gamma}{k^2}\frac{\partial E_z}{\partial y}\right) - \frac{\partial H_z}{\partial x}\right)$$

$$H_x = \frac{1}{\gamma}\left(\frac{-j^2\omega^2\mu\varepsilon}{k^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon\gamma}{k^2}\frac{\partial E_z}{\partial y} - \frac{\partial H_z}{\partial x}\right)$$

$$H_x = \frac{1}{\gamma}\left(\frac{\omega^2\mu\varepsilon}{k^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon\gamma}{k^2}\frac{\partial E_z}{\partial y} - \frac{\partial H_z}{\partial x}\right)$$

$$H_x = \frac{1}{\gamma}\left(\left(\frac{\omega^2\mu\varepsilon}{k^2} - 1\right)\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon\gamma}{k^2}\frac{\partial E_z}{\partial y}\right)$$

$$H_x = \frac{1}{\gamma}\left(\left(\frac{\omega^2\mu\varepsilon - k^2}{k^2}\right)\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon\gamma}{k^2}\frac{\partial E_z}{\partial y}\right) ----(23)$$

But

$$k^2 = \gamma^2 + \omega^2\mu\varepsilon$$

$$-\gamma^2 = \omega^2\mu\varepsilon - k^2 ----(24)$$

Substitute equation 24 in equation 23

$$H_x = \frac{1}{\gamma}\left(\left(\frac{-\gamma^2}{k^2}\right)\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon\gamma}{k^2}\frac{\partial E_z}{\partial y}\right)$$

$$H_x = -\frac{1}{\gamma}\frac{\gamma^2}{k^2}\frac{\partial H_z}{\partial x} + \frac{1}{\gamma}\frac{j\omega\varepsilon\gamma}{k^2}\frac{\partial E_z}{\partial y}$$

$$H_x = -\frac{\gamma}{k^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{k^2}\frac{\partial E_z}{\partial y} ----(25)$$

Substitute equation 22 in equation 21

$$H_y = \frac{1}{\gamma}\left(j\omega\varepsilon\left(-\frac{j\omega\mu}{k^2}\frac{\partial H_z}{\partial y} - \frac{\gamma}{k^2}\frac{\partial E_z}{\partial x} \right) - \frac{\partial H_z}{\partial y} \right)$$

$$H_y = \frac{1}{\gamma}\left(-\frac{j^2\omega^2\mu\varepsilon}{k^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon\gamma}{k^2}\frac{\partial E_z}{\partial x} - \frac{\partial H_z}{\partial y} \right)$$

$$H_y = \frac{1}{\gamma}\left(\frac{\omega^2\mu\varepsilon}{k^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon\gamma}{k^2}\frac{\partial E_z}{\partial x} - \frac{\partial H_z}{\partial y} \right)$$

$$H_y = \frac{1}{\gamma}\left(\left(\frac{\omega^2\mu\varepsilon}{k^2} - 1 \right)\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon\gamma}{k^2}\frac{\partial E_z}{\partial x} \right)$$

$$H_y = \frac{1}{\gamma}\left(\left(\frac{\omega^2\mu\varepsilon - k^2}{k^2} \right)\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon\gamma}{k^2}\frac{\partial E_z}{\partial x} \right) ----(26)$$

Substitute equation 24 in equation 26

$$H_y = \frac{1}{\gamma}\left(\left(\frac{-\gamma^2}{k^2} \right)\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon\gamma}{k^2}\frac{\partial E_z}{\partial x} \right)$$

$$H_y = \frac{-\gamma^2}{\gamma k^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon\gamma}{\gamma k^2}\frac{\partial E_z}{\partial x}$$

$$H_y = \frac{-\gamma}{k^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{k^2}\frac{\partial E_z}{\partial x} ----(27)$$

Finally, the basic equations are given by

$$E_x = -\frac{j\omega\mu}{k^2}\frac{\partial H_z}{\partial y} - \frac{\gamma}{k^2}\frac{\partial E_z}{\partial x} ----(28)$$

$$E_y = \frac{j\omega\mu}{k^2}\frac{\partial H_z}{\partial x} - \frac{\gamma}{k^2}\frac{\partial E_z}{\partial y} ----(29)$$

$$H_x = -\frac{\gamma}{k^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{k^2}\frac{\partial E_z}{\partial y} ----(30)$$

$$H_y = \frac{-\gamma}{k^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{k^2}\frac{\partial E_z}{\partial x} ----(31)$$

<u>TM mode analysis, Expressions for fields:</u>

The structure of rectangular waveguide is shown in the following figure

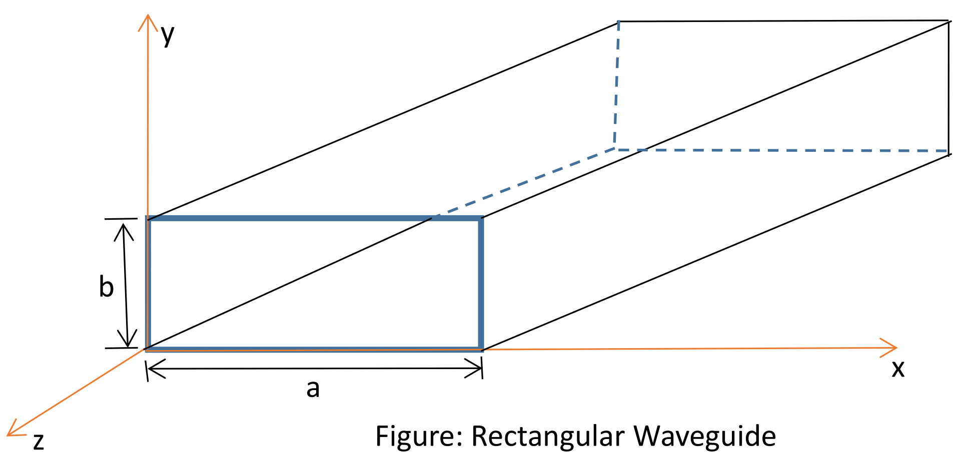


Figure: Rectangular Waveguide

TM(Transverse Magnetic) mode is defined as the mode of propagation in which there is no component of magnetic field in the dirction of wave propgtion. For example, if the wav propagating along the z-direction, then z-component of H will be zero.
The basic equation are given by

$$E_x = -\frac{j\omega\mu}{k^2}\frac{\partial H_z}{\partial y} - \frac{\gamma}{k^2}\frac{\partial E_z}{\partial x} ----(1)$$

$$E_y = \frac{j\omega\mu}{k^2}\frac{\partial H_z}{\partial x} - \frac{\gamma}{k^2}\frac{\partial E_z}{\partial y} ----(2)$$

$$H_x = -\frac{\gamma}{k^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{k^2}\frac{\partial E_z}{\partial y} ----(3)$$

$$H_y = \frac{-\gamma}{k^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{k^2}\frac{\partial E_z}{\partial x} ----(4)$$

For TM mode $H_z = 0$, then above four eqauitons becomes

$$E_x = -\frac{\gamma}{k^2}\frac{\partial E_z}{\partial x} ----(5)$$

$$E_y = -\frac{\gamma}{k^2}\frac{\partial E_z}{\partial y} ----(6)$$

$$H_x = \frac{j\omega\varepsilon}{k^2}\frac{\partial E_z}{\partial y} ----(7)$$

$$H_y = -\frac{j\omega\varepsilon}{k^2}\frac{\partial E_z}{\partial x} ----(8)$$

We know that the wave equation interms of E is

$$\nabla^2 E = \gamma^2 E \qquad -(9)$$

Where E is Electric field intensity and γ propagation constant and is given by

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

Then equation 9 becomes

$$\nabla^2 E = j\omega\mu(\sigma + j\omega\varepsilon)E \qquad -(10)$$

But, for dielectric medium, the conductivity(σ) = 0, then above equation becomes

$$\nabla^2 E = j^2\omega^2\mu\varepsilon E = -\omega^2\mu\varepsilon E$$

Express equation above equation in rectangular coordinate system

$$\nabla^2\left(E_x a_x + E_y a_y + E_z a_z\right) = -\omega^2\mu\varepsilon\left(E_x a_x + E_y a_y + E_z a_z\right)$$

But

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\left(E_x a_x + E_y a_y + E_z a_z\right) = -\omega^2\mu\varepsilon\left(E_x a_x + E_y a_y + E_z a_z\right)$$

Equate individual components on both sides

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = -\omega^2\mu\varepsilon E_x ----(11)$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = -\omega^2\mu\varepsilon E_y ----(12)$$

$$\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+\frac{\partial^2 E_z}{\partial z^2}=-\omega^2\mu\varepsilon E_z----(13)$$

When the wave is traveling along the z-direction, then we can have

$$E_z=E_{z0}e^{-\gamma z}$$

Differentiate above equation with respect to z

$$\frac{\partial E_z}{\partial z}=-\gamma E_{z0}e^{-\gamma z}$$

Again differentiate with respect to z

$$\frac{\partial^2 E_z}{\partial z^2}=-\gamma E_{z0}e^{-\gamma z}(-\gamma)=\gamma^2 E_{z0}e^{-\gamma z}$$

$$\frac{\partial^2 E_z}{\partial z^2}=\gamma^2 E_z----(14)$$

Substitute equation 14 in equation 13

$$\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+\gamma^2 E_z=-\omega^2\mu\varepsilon E_z$$

$$\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+\gamma^2 E_z+\omega^2\mu\varepsilon E_z=0$$

$$\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+\left(\gamma^2+\omega^2\mu\varepsilon\right)E_z=0$$

But

$$\gamma^2+\omega^2\mu\varepsilon=k^2$$

$$\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+k^2 E_z=0$$

But

$$E_z=E_{z0}e^{-\gamma z}$$

$$\frac{\partial^2 E_{z0}e^{-\gamma z}}{\partial x^2}+\frac{\partial^2 E_{z0}e^{-\gamma z}}{\partial y^2}+k^2 E_{z0}e^{-\gamma z}=0$$

$$e^{-\gamma z}\left(\frac{\partial^2 E_{z0}}{\partial x^2}+\frac{\partial^2 E_{z0}}{\partial y^2}+k^2 E_{z0}\right)=0$$

$$\frac{\partial^2 E_{z0}}{\partial x^2}+\frac{\partial^2 E_{z0}}{\partial y^2}+k^2 E_{z0}=0----(15)$$

The above partial differential equation can be solved by using method of variable separable. Let us assume the solution of product form

$$E_{z0}=XY----(16)$$

Where X is the function of ‘x’ only and Y is the function of ‘y’ only.

Take

$$\frac{\partial^2}{\partial x^2}$$

On both sides

$$\frac{\partial^2 E_{z0}}{\partial x^2} = Y \cdot \frac{\partial^2 X}{\partial x^2} ----(17)$$

Similarly,

$$\frac{\partial^2 E_{z0}}{\partial y^2} = X \cdot \frac{\partial^2 Y}{\partial y^2} ----(18)$$

Substitute equations 16, 17 and 18 in eqaution 15

$$Y \cdot \frac{\partial^2 X}{\partial x^2} + X \cdot \frac{\partial^2 Y}{\partial y^2} + k^2 XY = 0$$

Dividing throughout by XY, we get

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + k^2 = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k^2$$

But,

$$k^2 = k_x^2 + k_y^2$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k_x^2 - k_y^2$$

Then,

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -k_x^2$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + k_x^2 = 0 ----(19)$$

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k_y^2$$

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + k_y^2 = 0 ----(20)$$

The solution differential equations 19 and 20 are given by

$$X = A_1 \cos k_x x + A_2 \sin k_x x ----(21)$$

$$Y = A_3 \cos k_y y + A_4 \sin k_y y ----(22)$$

Substitute equations 21 and 22 in equation 16

$$E_{z0} = \left(A_1 \cos k_x x + A_2 \sin k_x x\right)\left(A_3 \cos k_y y + A_4 \sin k_y y\right) ----(23)$$

From the figure of rectangular waveguide, we can have the following boundary conditions.

At $x = 0, \quad E_{z0} = 0$ - (24)

At $x = a, \quad E_{z0} = 0$ - (25)

At $y = 0, \quad E_{z0} = 0$ - (26)

At $y = b, \quad E_{z0} = 0$ - (27)

Apply equation 24 to equation 23

$$0=\left(A_1\cos k_x(0)+A_2\sin k_x(0)\right)\left(A_3\cos k_y y+A_4\sin k_y y\right)$$

$$0=\left(A_1+0\right)\left(A_3\cos k_y y+A_4\sin k_y y\right)$$

$$0=A_1\left(A_3\cos k_y y+A_4\sin k_y y\right)$$

To satisfy above relation,

$$A_1=0 \qquad -(27)$$

Substitute equation 27 in equation 23

$$E_{z0}=\left(0+A_2\sin k_x x\right)\left(A_3\cos k_y y+A_4\sin k_y y\right)$$

$$E_{z0}=A_2\sin k_x x\left(A_3\cos k_y y+A_4\sin k_y y\right)----(28)$$

Apply equation 26 to equation 28

$$0=A_2\sin k_x x\left(A_3\cos k_y(0)+A_4\sin k_y(0)\right)$$

$$0=A_2\sin k_x x\left(A_3\right)$$

$$0=A_2A_3\sin k_x x$$

To satisfy the above relation,

$$A_2=0$$

Or

$$A_3=0----(29)$$

But we need to consider $A_3 = 0$, because if $A_2 = 0$ is substituted in equation 28, entire E_{z0} will be zero.

Substitute equation 29 in equation 28

$$E_{z0}=A_2\sin k_x x\left((0)\cos k_y y+A_4\sin k_y y\right)$$

$$E_{z0}=A_2\sin k_x x\left(A_4\sin k_y y\right)$$

$$E_{z0}=A_2A_4\sin k_x x\cdot\sin k_y y----(30)$$

Apply equation 25 to equation 30

$$0=A_2A_4\sin k_x a\cdot\sin k_y y$$

$$\sin k_x a=0$$

To satisfy the above relation, we can have

$$k_x a=\pm m\pi$$

Where m = 0,1,2,3,4,……

$$k_x=\frac{m\pi}{a}----(31)$$

Substitute equation 31 in equation 30

$$E_{z0}=A_2A_4\sin\left(\frac{m\pi x}{a}\right)\cdot\sin k_y y----(32)$$

Apply equation 27 to equation 32

$$0=A_2A_4\sin\left(\frac{m\pi x}{a}\right)\cdot\sin k_y b$$

$$\sin k_y b=0$$

To satisfy the above relation, we can have

$$k_y b=\pm n\pi$$

Where n = 0,1,2,3,4,……

$$k_y = \frac{n\pi}{b} ----(33)$$

Substitute equation 33 in equation 32

$$E_{z0} = A_2 A_4 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) ----(33)$$

Let A_2A_4 = A, Then

$$E_{z0} = A\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) ----(34)$$

Or

$$E_{z0} = A\sin k_x x \sin k_y y ----(35)$$

Differentiate above equation with respect to x

$$\frac{\partial E_{z0}}{\partial x} = Ak_x \cos k_x x \cdot \sin k_y y ----(36)$$

Similarly differentiate equation 35 with respect to y

$$\frac{\partial E_{z0}}{\partial y} = Ak_y \sin k_x x \cdot \cos k_y y ----(37)$$

Equations 5,6,7 and 8 can be written as

$$E_{x0} = -\frac{\gamma}{k^2}\frac{\partial E_{z0}}{\partial x} ----(38)$$

$$E_{y0} = -\frac{\gamma}{k^2}\frac{\partial E_{z0}}{\partial y} ----(39)$$

$$H_{x0} = \frac{j\omega\varepsilon}{k^2}\frac{\partial E_{z0}}{\partial y} ----(40)$$

$$H_{y0} = -\frac{j\omega\varepsilon}{k^2}\frac{\partial E_{z0}}{\partial x} ----(41)$$

Substitute equation 36 in 38

$$E_{x0} = -\frac{\gamma}{k^2}\left(Ak_x \cos k_x x \cdot \sin k_y y\right)$$

$$E_{x0} = -\frac{\gamma A k_x}{k^2}\cos k_x x \cdot \sin k_y y ----(42)$$

But

$$E_x = E_{x0}e^{-\gamma z} ----(43)$$

Substitute equation 42 in 43

$$E_x = -\frac{\gamma A k_x}{k^2}\cos k_x x \cdot \sin k_y y e^{-\gamma z} ----(44)$$

For lossless dielectric medium, the propagation constant will be

$$\gamma = j\beta ----(45)$$

Substitute equation 45 in 44

$$E_x = -\frac{j\beta A k_x}{k^2}\cos k_x x \cdot \sin k_y y e^{-j\beta z} ----(46)$$

Substitute equation 37 in equation 39

$$E_{y0} = -\frac{\gamma}{k^2} A k_y \sin k_x x \cdot \cos k_y y ----(47)$$

But

$$E_y = E_{y0} e^{-\gamma z} ----(48)$$

Substitute equation 47 in 48

$$E_y = -\frac{\gamma}{k^2} A k_y \sin k_x x \cdot \cos k_y y e^{-\gamma z} ----(49)$$

For lossless dielectric medium, the propagation constant will be

$$\gamma = j\beta ----(50)$$

Substitute equation 50 in 49

$$E_y = -\frac{j\beta}{k^2} A k_y \sin k_x x \cdot \cos k_y y e^{-j\beta z} ----(51)$$

Similarly substitute equation 37 in equation 40

$$H_{x0} = \frac{j\omega\varepsilon}{k^2} A k_y \sin k_x x \cdot \cos k_y y ----(52)$$

But

$$H_x = H_{x0} e^{-\gamma z} ----(53)$$

Substitute equation 52 in 53

$$H_x = \frac{j\omega\varepsilon}{k^2} A k_y \sin k_x x \cdot \cos k_y y e^{-\gamma z} ----(54)$$

For lossless dielectric medium, the propagation constant will be

$$\gamma = j\beta ----(55)$$

Substitute equation 55 in 54

$$H_x = \frac{j\omega\varepsilon}{k^2} A k_y \sin k_x x \cdot \cos k_y y e^{-j\beta z} ----(56)$$

Similarly substitute equation 36 in equation 41

$$H_{y0} = -\frac{j\omega\varepsilon}{k^2} A k_x \cos k_x x \cdot \sin k_y y ----(57)$$

But

$$H_y = H_{y0} e^{-\gamma z} ----(58)$$

Substitute equation 57 in 58

$$H_y = -\frac{j\omega\varepsilon}{k^2} A k_x \cos k_x x \cdot \sin k_y y e^{-\gamma z} ----(59)$$

For lossless dielectric medium, the propagation constant will be

$$\gamma = j\beta ----(60)$$

Substitute equation 60 in 59

$$H_y = -\frac{j\omega\varepsilon}{k^2} A k_x \cos k_x x \cdot \sin k_y y e^{-j\beta z} ----(61)$$

But

$$k_x = \frac{m\pi}{a} \textit{ and}$$

$$k_y = \frac{n\pi}{b}$$

Then finally, the equations for E_x, E_y,E_z, H_x,and H_y are given by

$$E_x = -\frac{j\beta Am\pi}{k^2 a}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} ----(62)$$

$$E_y = -\frac{j\beta An\pi}{k^2 b}\sin\left(\frac{m\pi x}{a}\right)\cdot\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z} ----(63)$$

$$E_z = A\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} ----(34)$$

$$H_x = \frac{j\omega\varepsilon An\pi}{k^2 b}\sin\left(\frac{m\pi x}{a}\right)\cdot\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z} ----(64)$$

$$H_y = -\frac{j\omega\varepsilon Am\pi}{k^2 a}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} ----(65)$$

TE mode analysis, Expressions for fields:

TE(Transverse Electric) mode is defined as the mode of propagation in which there is no component of electric field in the dirction of wave propgtion. For example, if the wav propagating along the z-direction, then z-component of E will be zero.

The basic equation are given by

$$E_x = -\frac{j\omega\mu}{k^2}\frac{\partial H_z}{\partial y} - \frac{\gamma}{k^2}\frac{\partial E_z}{\partial x} ----(1)$$

$$E_y = \frac{j\omega\mu}{k^2}\frac{\partial H_z}{\partial x} - \frac{\gamma}{k^2}\frac{\partial E_z}{\partial y} ----(2)$$

$$H_x = -\frac{\gamma}{k^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{k^2}\frac{\partial E_z}{\partial y} ----(3)$$

$$H_y = \frac{-\gamma}{k^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{k^2}\frac{\partial E_z}{\partial x} ----(4)$$

For TE mode $E_z = 0$, then above four eqauitons becomes

$$E_x = -\frac{j\omega\mu}{k^2}\frac{\partial H_z}{\partial y} ----(5)$$

$$E_y = \frac{j\omega\mu}{k^2}\frac{\partial H_z}{\partial x} ----(6)$$

$$H_x = -\frac{\gamma}{k^2}\frac{\partial H_z}{\partial x} ----(7)$$

$$H_y = \frac{-\gamma}{k^2}\frac{\partial H_z}{\partial y} ----(8)$$

We know that the wave equation interms of H is

$$\nabla^2 H = \gamma^2 H \qquad -(9)$$

Where H is Magnetic field intensity and γ propagation constant and is given by

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

Then equation 9 becomes

$$\nabla^2 H = j\omega\mu(\sigma + j\omega\varepsilon)H \qquad - (10)$$

But, for dielectric medium, the conductivity(σ) = 0, then above equation becomes

$$\nabla^2 H = j^2\omega^2\mu\varepsilon H = -\omega^2\mu\varepsilon H$$

Express equation above equation in rectangular coordinate system

$$\nabla^2\left(H_x a_x + H_y a_y + H_z a_z\right) = -\omega^2\mu\varepsilon\left(H_x a_x + H_y a_y + H_z a_z\right)$$

But

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\left(H_x a_x + H_y a_y + H_z a_z\right) = -\omega^2\mu\varepsilon\left(H_x a_x + H_y a_y + H_z a_z\right)$$

Equate individual components on both sides

$$\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} = -\omega^2\mu\varepsilon H_x \; ----(11)$$

$$\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + \frac{\partial^2 H_y}{\partial z^2} = -\omega^2\mu\varepsilon H_y \; ----(12)$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2\mu\varepsilon H_z \; ----(13)$$

When the wave is traveling along the z-direction, then we can have

$$H_z = H_{z0}e^{-\gamma z}$$

Differentiate above equation with respect to z

$$\frac{\partial H_z}{\partial z} = -\gamma H_{z0}e^{-\gamma z}$$

Again differentiate with respect to z

$$\frac{\partial^2 H_z}{\partial z^2} = -\gamma H_{z0}e^{-\gamma z}\left(-\gamma\right) = \gamma^2 H_{z0}e^{-\gamma z}$$

$$\frac{\partial^2 H_z}{\partial z^2} = \gamma^2 H_z \; ----(14)$$

Substitute equation 14 in equation 13

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z = -\omega^2\mu\varepsilon H_z$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z + \omega^2\mu\varepsilon H_z = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \left(\gamma^2 + \omega^2\mu\varepsilon\right)H_z = 0$$

But

$$\gamma^2 + \omega^2\mu\varepsilon = k^2$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + k^2 H_z = 0$$

But

$$H_z = H_{z0}e^{-\gamma z}$$

$$\frac{\partial^2 H_{z0}e^{-\gamma z}}{\partial x^2} + \frac{\partial^2 H_{z0}e^{-\gamma z}}{\partial y^2} + k^2 H_{z0}e^{-\gamma z} = 0$$

$$e^{-\gamma z}\left(\frac{\partial^2 H_{z0}}{\partial x^2} + \frac{\partial^2 H_{z0}}{\partial y^2} + k^2 H_{z0}\right) = 0$$

$$\frac{\partial^2 H_{z0}}{\partial x^2}+\frac{\partial^2 H_{z0}}{\partial y^2}+k^2 H_{z0}=0----(15)$$

The above partial differential equation can be solved by using method of variable separable. Let us assume the solution of product form

$$H_{z0}=XY----(16)$$

Where X is the function of ‘x’ only and Y is the function of ‘y’ only.
Take

$$\frac{\partial^2}{\partial x^2}$$

On both sides

$$\frac{\partial^2 H_{z0}}{\partial x^2}=Y\cdot\frac{\partial^2 X}{\partial x^2}----(17)$$

Similarly,

$$\frac{\partial^2 H_{z0}}{\partial y^2}=X\cdot\frac{\partial^2 Y}{\partial y^2}----(18)$$

Substitute equations 16, 17 and 18 in eqaution 15

$$Y\cdot\frac{\partial^2 X}{\partial x^2}+X\cdot\frac{\partial^2 Y}{\partial y^2}+k^2XY=0$$

Dividing throughout by XY, we get

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2}+\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}+k^2=0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2}+\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}=-k^2$$

But,

$$k^2=k_x^2+k_y^2$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2}+\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}=-k_x^2-k_y^2$$

Then,

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2}=-k_x^2$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2}+k_x^2=0----(19)$$

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}=-k_y^2$$

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}+k_y^2=0----(20)$$

The solution differential equations 19 and 20 are given by

$$X=A_1\cos k_x x+A_2\sin k_x x----(21)$$

$$Y = A_3 \cos k_y y + A_4 \sin k_y y ----(22)$$

Substitute equations 21 and 22 in equation 16

$$H_{z0} = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y) ----(23)$$

From the figure of rectangular waveguide, we can have the following boundary conditions.

$$\text{At} \quad x = 0, \quad \frac{\partial H_{z0}}{\partial x} = 0 \qquad - (24)$$

$$\text{At} \quad x = a, \quad \frac{\partial H_{z0}}{\partial x} = 0 \qquad - (25)$$

$$\text{At} \quad y = 0, \quad \frac{\partial H_{z0}}{\partial y} = 0 \qquad - (26)$$

$$\text{At} \quad y = b, \quad \frac{\partial H_{z0}}{\partial y} = 0 \qquad - (27)$$

Differentiate equation 23 with respect to x, then

$$\frac{\partial H_{z0}}{\partial x} = \frac{\partial}{\partial x}\left((A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y)\right)$$

$$\frac{\partial H_{z0}}{\partial x} = (-A_1 k_x \sin k_x x + A_2 k_x \cos k_x x)(A_3 \cos k_y y + A_4 \sin k_y y) \qquad - (28)$$

Apply equation 24 to equation 28

$$0 = (-A_1 k_x \sin k_x (0) + A_2 k_x \cos k_x (0))(A_3 \cos k_y y + A_4 \sin k_y y)$$

$$0 = (A_2 k_x)(A_3 \cos k_y y + A_4 \sin k_y y)$$

To satisfy above relation,

$$A_2 = 0 \qquad - (29)$$

Substitute equation 29 in equation 23

$$H_{z0} = (A_1 \cos k_x x)(A_3 \cos k_y y + A_4 \sin k_y y)$$

$$H_{z0} = A_1 \cos k_x x (A_3 \cos k_y y + A_4 \sin k_y y) ----(30)$$

Differentiate above equation with respect to y

$$\frac{\partial H_{z0}}{\partial y} = \frac{\partial}{\partial y}\left(A_1 \cos k_x x (A_3 \cos k_y y + A_4 \sin k_y y)\right)$$

$$\frac{\partial H_{z0}}{\partial y} = A_1 \cos k_x x (-A_3 k_y \sin k_y y + A_4 k_y \cos k_y y) ----(31)$$

Apply equation 26 to equation 31

$$0 = A_1 \cos k_x x (-A_3 k_y \sin k_y (0) + A_4 k_y \cos k_y (0))$$

$$0 = A_1 \cos k_x x (A_4 k_y)$$

$$0 = A_1 A_4 k_y \cos k_x x$$

To satisfy the above relation,

$$A_1 = 0$$

Or

$$A_4 = 0 ----(32)$$

But we need to consider A$_4$ = 0, because if A$_1$ = 0 is substituted in equation 30, entire H$_{z0}$ will be zero.

Substitute equation 32 in equation 30

$$H_{z0} = A_1 \cos k_x x\left(A_3 \cos k_y y\right)$$

$$H_{z0} = A_1 A_3 \cos k_x x \cos k_y y ----(33)$$

Differentiate above equation with respect to x

$$\frac{H_{z0}}{\partial x} = \frac{\partial}{\partial x}\left(A_1 A_3 \cos k_x x \cos k_y y\right)$$

$$\frac{H_{z0}}{\partial x} = -A_1 A_2 k_x \sin k_x x \cdot \cos k_y y ----(34)$$

Apply equation 25 to equation 34

$$0 = -A_1 A_2 k_x \sin k_x a \cdot \cos k_y y$$

$$\sin k_x a = 0$$

To satisfy the above relation, we can have

$$k_x a = \pm m\pi$$

Where m = 0,1,2,3,4,……

$$k_x = \frac{m\pi}{a} ----(35)$$

Substitute equation 35 in equation 33

$$H_{z0} = A_1 A_3 \cos\left(\frac{m\pi x}{a}\right)\cos k_y y ----(36)$$

Differentiate above equation with respect to y

$$\frac{\partial H_{z0}}{\partial y} = \frac{\partial}{\partial y}\left(A_1 A_3 \cos\left(\frac{m\pi x}{a}\right)\cos k_y y\right)$$

$$\frac{\partial H_{z0}}{\partial y} = A_1 A_3 \cos\left(\frac{m\pi x}{a}\right)\cdot\left(-k_y \sin k_y y\right)$$

$$\frac{\partial H_{z0}}{\partial y} = -A_1 A_3 \cos\left(\frac{m\pi x}{a}\right)\cdot k_y \sin k_y y ----(37)$$

Apply equation 27 to equation 37

$$0 = -A_1 A_3 \cos\left(\frac{m\pi x}{a}\right)\cdot k_y \sin k_y b$$

$$\sin k_y b = 0$$

To satisfy the above relation, we can have

$$k_y b = \pm n\pi$$

Where n = 0,1,2,3,4,……

$$k_y = \frac{n\pi}{b} ----(38)$$

Substitute equation 38 in equation 36

$$H_{z0} = A_1 A_3 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

Let $A_1A_3 = A'$ Then

$$H_{z0} = A'\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

Or

$$H_{z0} = A'\cos k_x x \cdot \cos k_y y ----(39)$$

Differentiate above equation with respect to x

$$\frac{\partial H_{z0}}{\partial x} = -A'k_x \sin k_x x \cdot \cos k_y y ----(40)$$

Similarly differentiate equation 39 with respect to y

$$\frac{\partial H_{z0}}{\partial y} = -A'k_y \cos k_x x \cdot \sin k_y y ----(41)$$

Equations 5,6,7 and 8 can be written as

$$E_{x0} = -\frac{j\omega\mu}{k^2}\frac{\partial H_{z0}}{\partial y} ----(42)$$

$$E_{y0} = \frac{j\omega\mu}{k^2}\frac{\partial H_{z0}}{\partial x} ----(43)$$

$$H_{x0} = -\frac{\gamma}{k^2}\frac{\partial H_{z0}}{\partial x} ----(44)$$

$$H_{y0} = \frac{-\gamma}{k^2}\frac{\partial H_{z0}}{\partial y} ----(45)$$

Substitute equation 41 in 42

$$E_{x0} = -\frac{j\omega\mu}{k^2}\left(-A'k_y \cos k_x x \cdot \sin k_y y\right)$$

$$E_{x0} = \frac{j\omega\mu A'k_y}{k^2}\cos k_x x \cdot \sin k_y y ----(46)$$

But

$$E_x = E_{x0}e^{-\gamma z} ----(47)$$

Substitute equation 46 in 47

$$E_x = \frac{j\omega\mu A'k_y}{k^2}\cos k_x x \cdot \sin k_y y e^{-\gamma z} ----(48)$$

For lossless dielectric medium, the propagation constant will be

$$\gamma = j\beta ----(49)$$

Substitute equation 49 in 48

$$E_x = \frac{j\omega\mu A'k_y}{k^2}\cos k_x x \cdot \sin k_y y e^{-j\beta z} ----(50)$$

Similarly ubstitute equation 40 in equation 43

$$E_{y0} = \frac{j\omega\mu}{k^2}\left(-A'k_x \sin k_x x \cdot \cos k_y y\right)$$

$$E_{y0} = -\frac{j\omega\mu A'k_x}{k^2}\sin k_x x \cdot \cos k_y y ----(51)$$

But

$$E_y = E_{y0}e^{-\gamma z} ----(52)$$

Substitute equation 51 in 52

$$E_y = -\frac{j\omega\mu A' k_x}{k^2} \sin k_x x \cdot \cos k_y y e^{-\gamma z} ----(53)$$

For lossless dielectric medium, the propagation constant will be

$$\gamma = j\beta ----(54)$$

Substitute equation 54 in 53

$$E_y = -\frac{j\omega\mu A' k_x}{k^2} \sin k_x x \cdot \cos k_y y e^{-j\beta z} ----(55)$$

Similarly substitute equation 40 in equation 44

$$H_{x0} = -\frac{\gamma}{k^2}\left(-A' k_x \sin k_x x \cdot \cos k_y y\right)$$

$$H_{x0} = \frac{\gamma A' k_x}{k^2} \sin k_x x \cdot \cos k_y y ----(56)$$

But

$$H_x = H_{x0} e^{-\gamma z} ----(57)$$

Substitute equation 56 in 57

$$H_x = \frac{\gamma A' k_x}{k^2} \sin k_x x \cdot \cos k_y y e^{-\gamma z} ----(58)$$

For lossless dielectric medium, the propagation constant will be

$$\gamma = j\beta ----(59)$$

Substitute equation 59 in 58

$$H_x = \frac{j\beta A' k_x}{k^2} \sin k_x x \cdot \cos k_y y e^{-j\beta z} ----(60)$$

Similarly substitute equation 41 in equation 45

$$H_{y0} = \frac{-\gamma}{k^2}\left(-A' k_y \cos k_x x \cdot \sin k_y y\right)$$

$$H_{y0} = \frac{\gamma A' k_y}{k^2} \cos k_x x \cdot \sin k_y y ----(61)$$

But

$$H_y = H_{y0} e^{-\gamma z} ----(62)$$

Substitute equation 62 in 61

$$H_y = \frac{\gamma A' k_y}{k^2} \cos k_x x \cdot \sin k_y y e^{-\gamma z} ----(63)$$

For lossless dielectric medium, the propagation constant will be

$$\gamma = j\beta ----(64)$$

Substitute equation 64 in 63

$$H_y = \frac{j\beta A' k_y}{k^2} \cos k_x x \cdot \sin k_y y e^{-j\beta z} ----(65)$$

But

$$k_x = \frac{m\pi}{a} \; and$$

$$k_y = \frac{n\pi}{b}$$

Then finally, the equations for E_x, E_y, H_x, H_y and H_z are given by

$$E_x = \frac{j\omega\mu A' n\pi}{k^2 b}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} \quad ----(66)$$

$$E_y = -\frac{j\omega\mu A' m\pi}{k^2 a}\sin\left(\frac{m\pi x}{a}\right)\cdot\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z} \quad ----(67)$$

$$H_x = \frac{j\beta A' m\pi}{k^2 a}\sin\left(\frac{m\pi x}{a}\right)\cdot\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z} \quad ----(68)$$

$$H_y = \frac{j\beta A' n\pi}{k^2 b}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} \quad ----(69)$$

$$H_z = A'\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z} \quad ----(70)$$

CHARACTERISTIC EQUATION AND CUTOFF FREQUENCIES

The characteristic equation of rectangular waveguide is given by

$$k^2 = \gamma^2 + \omega^2\mu\varepsilon$$

The characteristic equation will be used to find out certain important characteristics of rectangular waveguide such as cutoff or critical frequency, cutoff wavelength, Phase constant, attenuation constant, propagation constant, phase velocity, Group velocity, Guide wavelength, etc.

The cutoff frequency or critical frequency is defined as the frequency below which the wave propagation through the waveguide is not possible. The equation for the cutoff frequency will be derived as follows:

The characteristic equation of rectangular waveguide is given by

$$k^2 = \gamma^2 + \omega^2\mu\varepsilon \qquad [1]$$

Also

$$k^2 = k_x^2 + k_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \qquad [2]$$

Equate equations 1 and 2

$$\gamma^2 + \omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

At cutoff frequency,

$$\omega_c^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega_c^2 = \frac{1}{\mu\varepsilon}\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega_c = \sqrt{\frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right]}$$

$$2\pi f_c = \sqrt{\frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right]}$$

$$f_c = \frac{1}{2\pi}\sqrt{\frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right]}$$

$$f_c = \frac{1}{2\pi}\sqrt{\frac{\pi^2}{\mu\varepsilon}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{m}{b}\right)^2\right]}$$

$$f_c = \frac{\pi}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{m}{b}\right)^2\right]}$$

$$f_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{m}{b}\right)^2\right]}$$

But

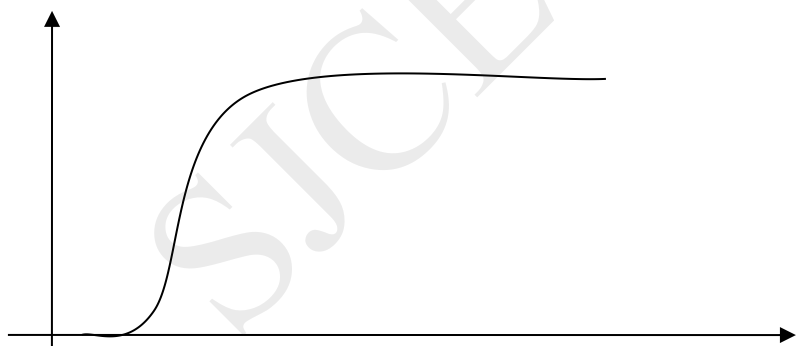
$$\frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$f_c = \frac{c}{2}\sqrt{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{m}{b}\right)^2\right]}$$

The above equation is called as the cutoff frequency.

FILTER CHARACTERISTICS

The waveguide act as a high pass filter i.e. it allows only high frequencies. The filter characteristic of waveguide is shown in the following figure.



The waveguide allows the signal when the signal frequency is greater than the cutoff frequency f_c as shown in the figure above. The cutoff frequency or critical frequency is defined as the frequency below which the wave propagation through the waveguide is not possible. That is below the cutoff frequency, only attenuation will be present. The cutoff frequency of rectangular waveguide is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

DOMINANT AND DEGENERATE MODES

A mode is defined as a field distribution or configuration within a waveguide. In general a mode is represented as TE_{mn}, TM_{mn}. A mode which is having lowest cutoff frequency or highest cutoff wavelength is called as the dominant mode. In case of rectangular waveguide the dominant mode is TE_{10} when a > b. Where a and b are the dimensions of the waveguide. A set of modes having the same cutoff frequency are known as the degenerate modes. In rectangular waveguide TE_{10} & TM_{10}, TE_{11} & TM_{11}, TE_{01} & TM_{01}, etc are called degenerate modes.

MODE CHARACTERISTICS- PHASE AND GROUP VELOCITIES

Phase velocity is defined as the speed or velocity with which the signal travels through the medium. The phase velocity is represented with v_p or with c in case of free space. The phase velocity is given by

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} \quad m/s$$

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$v_p = \frac{\omega}{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}$$

$$v_p = \frac{\omega}{\sqrt{\omega^2\mu\varepsilon\left(1-\frac{\omega_c^2}{\omega^2}\right)}}$$

$$v_p = \frac{\omega}{\omega\sqrt{\mu\varepsilon}\sqrt{1-\frac{\omega_c^2}{\omega^2}}}$$

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-\frac{\omega_c^2}{\omega^2}}}$$

But

$$\frac{1}{\sqrt{\mu\varepsilon}} = c, \omega_c = 2\pi f_c, \omega = 2\pi f$$

$$v_p = \frac{c}{\sqrt{1-\left(\frac{2\pi f_c}{2\pi f}\right)^2}}$$

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

Therefore, phase velocity in a rectangular waveguide is given by

$$v_p = \frac{c}{\sqrt{1-(f_c/f)^2}} = \frac{c}{\sqrt{1-(\lambda/\lambda_c)^2}}$$

Group velocity is defined as the rate at which the velocity changes. It is also defined as the speed at which the group of waves or wavefront travels through the medium. It represented with v_g. The group velocity is given by

$$v_g = \frac{d\omega}{d\beta}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$\frac{d\beta}{d\omega} = \frac{d}{d\omega}\left(\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}\right)$$

$$\frac{d\beta}{d\omega} = \frac{d}{d\omega}\left(\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon\right)^{1/2}$$

$$\frac{d\beta}{d\omega}=\frac{1}{2}\left(\omega^2\mu\varepsilon-\omega_c^2\mu\varepsilon\right)^{-1/2}\left(2\omega\mu\varepsilon\right)$$

$$\frac{d\beta}{d\omega}=\frac{1}{2}\left(\frac{1}{\sqrt{\omega^2\mu\varepsilon-\omega_c^2\mu\varepsilon}}\right)\left(2\omega\mu\varepsilon\right)$$

$$\frac{d\beta}{d\omega}=\left(\frac{1}{\sqrt{\omega^2\left(\mu\varepsilon-\frac{\omega_c^2}{\omega^2}\mu\varepsilon\right)}}\right)\left(\omega\mu\varepsilon\right)$$

$$\frac{d\beta}{d\omega}=\left(\frac{1}{\omega\sqrt{\left(\mu\varepsilon-\frac{\omega_c^2}{\omega^2}\mu\varepsilon\right)}}\right)\left(\omega\mu\varepsilon\right)$$

$$\frac{d\beta}{d\omega}=\left(\frac{\mu\varepsilon}{\sqrt{\left(\mu\varepsilon-\frac{\omega_c^2}{\omega^2}\mu\varepsilon\right)}}\right)$$

$$\frac{d\beta}{d\omega}=\left(\sqrt{\frac{(\mu\varepsilon)^2}{\mu\varepsilon-\frac{\omega_c^2}{\omega^2}\mu\varepsilon}}\right)$$

$$\frac{d\beta}{d\omega}=\left(\sqrt{\frac{(\mu\varepsilon)^2}{\mu\varepsilon\left(1-\frac{\omega_c^2}{\omega^2}\right)}}\right)$$

$$\frac{d\beta}{d\omega}=\left(\sqrt{\frac{\mu\varepsilon}{\left(1-\frac{\omega_c^2}{\omega^2}\right)}}\right)$$

$$\frac{d\beta}{d\omega}=\left(\frac{\sqrt{\mu\varepsilon}}{\sqrt{\left(1-\frac{\omega_c^2}{\omega^2}\right)}}\right)$$

$$v_g=\frac{d\omega}{d\beta}=\frac{\sqrt{\left(1-\frac{\omega_c^2}{\omega^2}\right)}}{\sqrt{\mu\varepsilon}}$$

But

$$\frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$v_g = c\sqrt{\left(1-\frac{\omega_c^2}{\omega^2}\right)} = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$v_p v_g = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \cdot c\sqrt{1-\left(\frac{f_c}{f}\right)^2} = c^2$$

<u>PHASE CONSTANT AND ATTENUATION CONSTANT</u>

The phase constant is derived as follows:

We have

$$\gamma^2 + \omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon$$

Above cut off frequency,

$$\gamma = j\beta,$$

But at cut off frequency,

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \omega_c^2\mu\varepsilon$$

$$(j\beta)^2 = \omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon$$

$$-\beta^2 = \omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon$$

$$\beta^2 = \omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon$$

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon\left(1-\frac{\omega_c^2}{\omega^2}\right)}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon\left(1-\frac{2\pi f_c^2}{2\pi f^2}\right)}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon\left(1-\left(\frac{f_c}{f}\right)^2\right)}$$

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{\left(1-\left(\frac{f_c}{f}\right)^2\right)}$$

The attenuation constant is derived as follows:

We have

$$\gamma^2 + \omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon$$

Below cut off frequency,

$$\gamma = \alpha,$$

But at cut off frequency,

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \omega_c^2\mu\varepsilon$$

$$\alpha^2 = \omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon$$

$$\alpha = \sqrt{\omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon}$$

$$\alpha = \sqrt{\omega^2\mu\varepsilon\left(\frac{\omega_c^2}{\omega^2} - 1\right)}$$

$$\alpha = \sqrt{\omega^2\mu\varepsilon\left(\left(\frac{f_c}{f}\right)^2 - 1\right)}$$

$$\alpha = \omega\sqrt{\mu\varepsilon}\sqrt{\left(\left(\frac{f_c}{f}\right)^2 - 1\right)}$$

WAVELENGTHS AND IMPEDANCE RELATIONS

(1) Cutoff wavelength and guide wavelength:

The cutoff wavelength or critical wavelength is defined as the wavelength below which the wave propagation through the waveguide is possible. It is represented with λ_c. The cutoff wavelength is inversely proportional to the cutoff frequency.

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\lambda_c = \frac{c}{f_c} = 2\Big/\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

For dominant mode $\lambda_c = 2a$

The guide wavelength (λ_g) is given by

$$\lambda_g = \frac{2\pi}{\beta}$$

But

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{\left(1 - \left(\frac{f_c}{f}\right)^2\right)}$$

$$\lambda_g = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}\sqrt{\left(1 - \left(\frac{f_c}{f}\right)^2\right)}}$$

$$\lambda_g = \frac{2\pi}{2\pi f\sqrt{\mu\varepsilon}\sqrt{\left(1-\left(\frac{f_c}{f}\right)^2\right)}}$$

$$\lambda_g = \frac{1}{f\sqrt{\mu\varepsilon}\sqrt{\left(1-\left(\frac{f_c}{f}\right)^2\right)}}$$

But

$$\frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\lambda_g = \frac{c}{f\sqrt{\left(1-\left(\frac{f_c}{f}\right)^2\right)}}$$

But

$$\frac{c}{f} = \lambda$$

$$\lambda_g = \frac{\lambda}{\sqrt{\left(1-\left(\frac{f_c}{f}\right)^2\right)}}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-(\lambda/\lambda_c)^2}}$$

$$\lambda_g^2 = \frac{\lambda^2}{1-\frac{\lambda^2}{\lambda_c^2}}$$

$$\lambda_g^2 = \frac{\lambda^2}{\frac{\lambda_c^2-\lambda^2}{\lambda_c^2}}$$

$$\lambda_g^2 = \frac{\lambda^2\lambda_c^2}{\lambda_c^2-\lambda^2}$$

$$\frac{1}{\lambda_g^2} = \frac{\lambda_c^2-\lambda^2}{\lambda^2\lambda_c^2}$$

$$\frac{1}{\lambda_g^2} = \frac{\lambda_c^2}{\lambda^2\lambda_c^2} - \frac{\lambda^2}{\lambda^2\lambda_c^2}$$

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}$$

Where λ is the signal wavelength and λ_c is the cutoff wavelength.

(2) Wave impedance:
The wave impedance is defined as the ratio between the electric field vector (E) and magnetic field vector (H). That is

$$Z_g = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$$

The wave impedance for TM waves is given by

$$Z_g = \eta_{TM} = \frac{E_x}{H_y} ----(1)$$

But for TM mode

$$E_x = -\frac{j\beta Am\pi}{k^2 a}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} ----(2)$$

$$H_y = -\frac{j\omega\varepsilon Am\pi}{k^2 a}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} ----(3)$$

Substitute equations 2 and 3 in equation 1

$$\eta_{TM} = \frac{-\frac{j\beta Am\pi}{k^2 a}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}}{-\frac{j\omega\varepsilon Am\pi}{k^2 a}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}}$$

$$\eta_{TM} = \frac{\beta}{\omega\varepsilon}$$

But $\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$
Therefore

$$\eta_{TM} = \frac{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}{\omega\varepsilon}$$

$$\eta_{TM} = \frac{\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{\omega\varepsilon}$$

$$\eta_{TM} = \frac{\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{\varepsilon}$$

$$\eta_{TM} = \frac{\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{\sqrt{\varepsilon^2}}$$

$$\eta_{TM} = \sqrt{\frac{\mu\varepsilon}{\varepsilon^2}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\eta_{TM} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

But

$$\sqrt{\frac{\mu}{\varepsilon}} = \eta_0$$

$$\eta_{TM} = \eta_0\sqrt{1 - \left(\frac{f_c}{f}\right)^2} \qquad - (4)$$

Similarly for TE waves the wave impedance is given by

$$Z_g = \eta_{TE} = \frac{E_x}{H_y} ----(5)$$

But for TE mode,

$$E_x = \frac{j\omega\mu A' n\pi}{k^2 b}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} ----(6)$$

$$H_y = \frac{j\beta A' n\pi}{k^2 b}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} ----(7)$$

Substitute equations 6 and 7 in equation 5

$$\eta_{TE} = \frac{\frac{j\omega\mu A' n\pi}{k^2 b}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}}{\frac{j\beta A' n\pi}{k^2 b}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}}$$

$$\eta_{TE} = \frac{\omega\mu}{\beta}$$

But $\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$

Therefore

$$\eta_{TE} = \frac{\omega\mu}{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}$$

$$\eta_{TE} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\eta_{TE} = \frac{\mu}{\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\eta_{TE} = \sqrt{\frac{\mu^2}{\mu\varepsilon}}\frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\eta_{TE} = \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\eta_{TE} = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

WAVE PROPAGATION IN THE GUIDE

For TE mode we have the filed expressions as

$$E_x = \frac{j\omega\mu A' n\pi}{k^2 b}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} ----(1)$$

$$E_y = -\frac{j\omega\mu A' m\pi}{k^2 a}\sin\left(\frac{m\pi x}{a}\right)\cdot\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z} ----(2)$$

$$H_x = \frac{j\beta A' m\pi}{k^2 a}\sin\left(\frac{m\pi x}{a}\right)\cdot\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z} ----(3)$$

$$H_y = \frac{j\beta A' n\pi}{k^2 b}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} ----(4)$$

$$H_z = A'\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z} ----(5)$$

Examination of above equations shows that the field components all involve the terms sine or cosine of $\frac{m\pi x}{a}$ and $\frac{n\pi y}{b}$

For TE_{10} mode the above equations becomes

$$E_x = \frac{j\omega\mu}{k^2}A'\frac{n\pi}{b}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} = 0$$

$$E_y = -\frac{j\omega\mu}{k^2}A'\frac{m\pi}{a}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z} = -\frac{j\omega\mu}{k^2}A'\frac{\pi}{a}\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

$$H_x = \frac{j\beta A' m\pi}{k^2 a}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z} = \frac{j\beta A'\pi}{k^2 a}\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

$$H_y = \frac{j\beta A' n\pi}{k^2 b}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} = 0$$

$$H_z = A'\cos\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

Let us consider y-component of E,

$$E_y = -\frac{j\omega\mu}{k^2}A'\frac{\pi}{a}\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z} \qquad -1$$

But

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Or

$$\sin\left(\frac{\pi x}{a}\right) = \frac{e^{j\frac{\pi x}{a}} - e^{-j\frac{\pi x}{a}}}{2j} \qquad -2$$

Substitute equation 2 in equation 1

$$E_y = -\frac{j\omega\mu}{k^2}A'\frac{\pi}{a}\left(\frac{e^{j\frac{\pi x}{a}} - e^{-j\frac{\pi x}{a}}}{2j}\right)e^{-j\beta z}$$

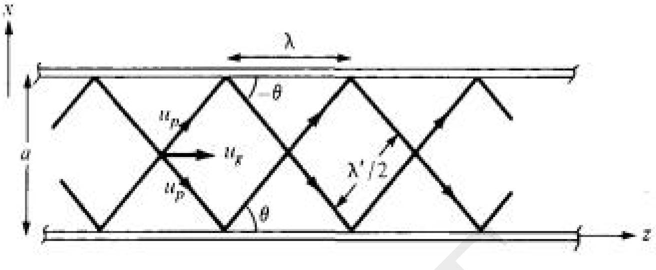
$$E_y = -\frac{\omega\mu}{2k^2}A'\frac{\pi}{a}\left(e^{j\frac{\pi x}{a}} - e^{-j\frac{\pi x}{a}}\right)e^{-j\beta z}$$

$$E_y =-\frac{\omega\mu}{2k^2}A^{'}\frac{\pi}{a}\left(e^{-j\beta\left(z+\frac{\pi x}{\beta a}\right)}-e^{-j\beta\left(z-\frac{\pi x}{\beta a}\right)}\right) \qquad -3$$

In the above equation, the first term represents a wave traveling in the positive z-direction at an angle

$$\theta = \tan^{-1}\left(\frac{\pi}{\beta a}\right)$$

With z-axis. The second term of eq. (3) represents a wave traveling in the positive z-direction at an angle $-\theta$. The field may be depicted as a sum of two plane TEM waves propagating along zigzag paths between the guide walls at $x = 0$ and $x = a$ as illustrated in Figure 1. The decomposition of the TE_{10} mode into two plane waves can be extended to any TE and TM mode. When n and m are both different from zero, four plane waves result from the decomposition.



(a)

Figure 1: Decomposition of TE10 mode into two plane waves

The wave component in the z-direction has a different wavelength from that of the plane waves. This wavelength along the axis of the guide is called the waveguide wavelength and is given by

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

Where

$$\lambda = \frac{u}{\mathrm{f}}$$

Known as signal wavelength

As a consequence of the zigzag paths, we have three types of velocity: the *medium veocity c,* the *phase velocity v_p,* and the *group velocity v_g.* The phase velocity is given by

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

This shows that $v_p > c$. *That is* v_p is greater than the speed of light in vacuum. Does this violate Einstein's relativity theory that messages cannot travel faster than the speed of light? Not really, because information (or energy) in a waveguide generally does not travel at the phase velocity. Information travels at the group velocity, which must be less than the speed of light. The group velocity *ug* is the velocity with which the resultant repeated reflected waves are traveling down the guide and is given by

$$v_g = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

POWER TRANSMISSION IN RECTANGULAR WAVEGUIDE

The power transmitted through the waveguide can be calculated by using the pointing theorem. The average power transmission is given by

$$P_{tr} = \int \frac{1}{2}(E \times H^*) \cdot ds$$

$$P_{tr} = \frac{1}{2Z_g}\int |E|^2 ds = \frac{Z_g}{2}\int |H|^2 ds$$

Where

$$Z_g = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$$

$$|E|^2 = |E_x|^2 + |E_y|^2$$

$$|H|^2 = |H_x|^2 + |H_y|^2$$

The wave impedance for TE_{mn} waves is given by

$$Z_g = \frac{\eta_0}{\sqrt{1-(f_c/f)^2}}$$

Therefore the power transmission for TE_{mn} waves is given by

$$p_{tr} = \frac{\sqrt{1-(f_c/f)^2}}{2\eta_0}\int_0^b\int_0^a \left(|E_x|^2 + |E_y|^2\right) dxdy$$

Similarly the wave impedance for TM_{mn} waves is given by

$$Z_g = \eta_0\sqrt{1-(f_c/f)^2}$$

Therefore the power transmission for TM_{mn} waves is given by

$$p_{tr} = \frac{1}{2\eta_0\sqrt{1-(f_c/f)^2}}\int_0^b\int_0^a \left(|E_x|^2 + |E_y|^2\right) dxdy$$

ATENUATION

There are two types of power losses in rectangular waveguide:

(i) Losses in the dielectric
(ii) Losses in the guide walls

The losses due to dielectric will arise due to finite conductivity of the dielectric. When the dielectric posses conductivity, there will be loss of power by the dielectric. The loss factor or attenuation factor (α) due to the any dielectric is given by

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta\sigma}{2} = \frac{\eta_g\sigma}{2}$$

The attenuation losses due to dielectric for TE waves and TM waves are given by

$$\alpha_g = \frac{\eta_{TE}\sigma}{2} = \frac{\sigma\eta_0}{2\sqrt{1-(f_c/f)^2}} \quad \text{fot TE mode}$$

$$\alpha_g = \frac{\eta_{TM}\sigma}{2} = \frac{\sigma\eta_0}{2}\sqrt{1-(f_c/f)^2} \quad \text{for TM mode}$$

The power losses due to guide walls will arise due to the presence of finite resistivity in the guide walls. The power loss due to the guide walls can be obtained as follows:

Let the EM waves traveling along the z-direction, then E and H can be written as

$$|E| = |E_{0z}|e^{-\alpha_g z}$$

$$|H| = |H_{0z}|e^{-\alpha_g z}$$

Where E_{oz} and H_{oz} are the field intensities at z = 0. The time average power flow decreases proportianally to $e^{-2\alpha_g z}$. Hence

$$P_{tr} = \left(P_{tr} + P_{loss}\right)e^{-2\alpha_g z}$$

$$P_{tr} = \left(P_{tr} + P_{loss}\right)\left(1 - 2\alpha_g z\right)$$

$$P_{tr} = P_{tr} - 2\alpha_g z P_{tr} + P_{loss} - 2\alpha_g z P_{loss}$$

But

$$P_{loss} << P_{tr}, 2\alpha_g z << 1$$

$$P_{tr} = P_{tr} - 2\alpha_g z P_{tr} + P_{loss}$$

Divide throughout with P_{tr}

$$1 = 1 - 2\alpha_g z + \frac{P_{loss}}{P_{tr}}$$

$$2\alpha_g z = \frac{P_{loss}}{P_{tr}}$$

$$\alpha_g = \frac{P_{loss}}{2zP_{tr}}$$

$$\alpha_g = \frac{P_L}{2P_{tr}} ----(1)$$

Where

$$P_L = \frac{P_{loss}}{z}$$

But

$$P_L = \frac{R_s}{2}\int |H_t|^2 ds \quad watts/unit\ length \qquad -(2)$$

$$P_{tr} = \frac{Z_g}{2}\int |H|^2 ds \qquad -(3)$$

Substitute equations 2 and 3 in equation1

$$\alpha_g = \frac{\frac{R_s}{2}\int |H_t|^2 ds}{2\left(\frac{Z_g}{2}\int |H|^2 ds\right)}$$

The attenuation due to the guide walls is given by

$$\alpha_g = \frac{R_s \int |H_t|^2 ds}{2Z_g \int |H|^2 ds}$$

WAVEGUIDE CURRENT AND MODE EXCITATION

For either TM or TE modes, the surface current density K on the walls of the waveguide may be found using

$$K = a_n \times H \qquad - 1$$

where a_n is the unit outward normal to the wall and H is the field intensity evaluated on the wall. The current flow on the guide walls for TE_{10} mode propagation can be found using eq. (1). The result is sketched in Figure 1.

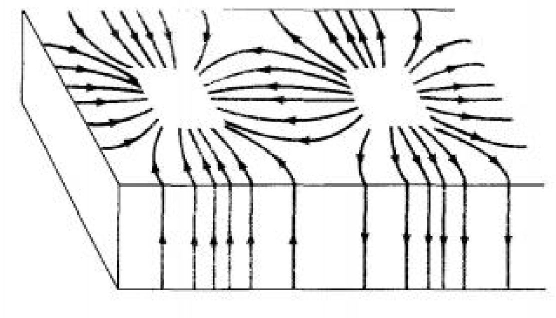


Figure 1: Surface current on guide walls for TE10 mode.

The surface charge density p_s on the walls is given by

$$p_s = a_n \bullet D = a_n \bullet eE$$

where E is the electric field intensity evaluated on the guide wall

A waveguide is usually fed or excited by a coaxial line or another waveguide. Most often, a probe (central conductor of a coaxial line) is used to establish the field intensities of the desired mode and achieve a maximum power transfer. The probe is located so as to produce E and H fields that are roughly parallel to the lines of E and H fields of the desired mode. To excite the TE_{10} mode, for example,

$$E_y = -\frac{j\omega\mu}{k^2} A' \frac{\pi}{a} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$

we know that E_y has maximum value at $x = a/2$. Hence, the probe is located at $x = a/2$ to excite the TE_{10} mode as shown in Figure 2(a). Similarly, the TM_{11} mode is launched by placing the probe along the z-direction as in Figure 2(b).

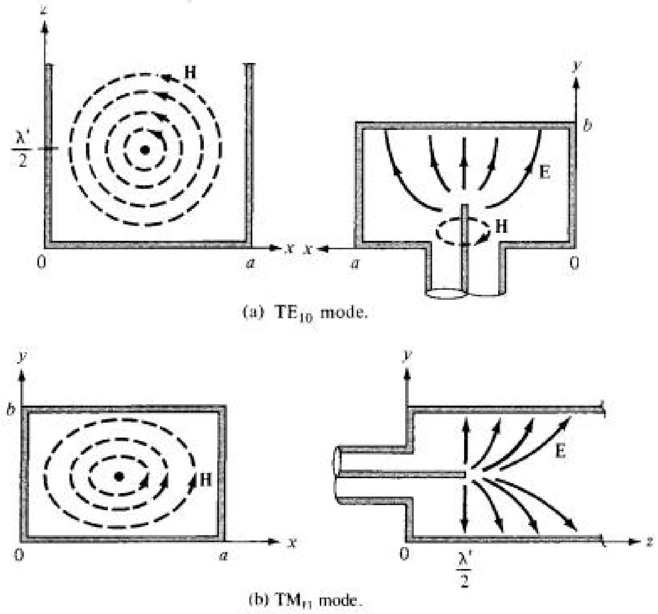
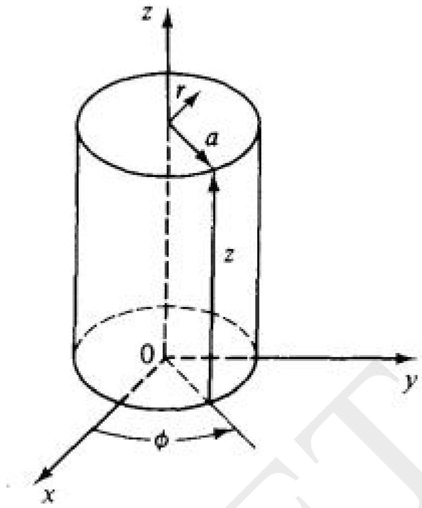


Figure 2: Excitation of modes in a rectangular waveguide

CIRCULAR WAVEGUIDE

A circular waveguide is a tubular, circular conductor. A plane wave propagating through a circular waveguide results in a transverse electric (TE) or transerse magnetic (TM) mode. Several other types of waveguides, such as elliptical and reentrant guides, also propagate electromagnetic waves.

As described in earlier section for rectangular waveguides, only a sinusoidal steadystate or frequency-domain solution will be attempted for circular waveguides. A cylindrical coordinate system is shown in Figure below.



The field expressions for TE_{np} mode of circular waveguide are given by

$$E_r = E_{0r} J_n \left(\frac{X'_{np} r}{a} \right) \sin (n\phi) e^{-j\beta_g z}$$

$$E_\phi = E_{0\phi} J'_n \left(\frac{X'_{np} r}{a} \right) \cos (n\phi) e^{-j\beta_g z}$$

$$E_z = 0$$

$$H_r = -\frac{E_{0\phi}}{Z_g} J_n \left(\frac{X'_{np} r}{a} \right) \cos (n\phi) e^{-j\beta_g z}$$

$$H_\phi = \frac{E_{0r}}{Z_g} J_n \left(\frac{X'_{np} r}{a} \right) \sin (n\phi) e^{-j\beta_g z}$$

$$H_z = H_{0z} J_n \left(\frac{X'_{np} r}{a} \right) \cos (n\phi) e^{-j\beta_g z}$$

where $Z_g = E_r / H_\phi = - E_\phi / H_r$ has been replaced for the wave impedance in the guide and where $n = 0, 1, 2, 3, ...$ and p = 1, 2, 3, 4, ...

The mode propagation constant is given by

$$\beta_g = \sqrt{\omega^2 \mu \varepsilon - \left(\frac{X'_{np}}{a} \right)^2}$$

The cutoff frequency for TE modes in a circular guide is then given by

$$f_c = \frac{X'_{np}}{2\pi a \sqrt{\mu\varepsilon}}$$

and the phase velocity for TE modes is

$$v_p = \frac{\omega}{\beta_g} = \frac{v}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

Where

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_0 \varepsilon_0}}$$

The wavelength and wave impedance for TE modes in a circular guide are given, respectively, by

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$Z_g = \frac{\omega\mu}{\beta_g} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

The field expressions for TM_{np} mode of circular waveguide are given by

$$E_r = E_{0r} J_n^{'}\left(\frac{X_{np} r}{a}\right) \cos{(n\phi)} e^{-j\beta_g z}$$

$$E_\phi = E_{0\phi} J_n \left(\frac{X_{np} r}{a}\right) \sin{(n\phi)} e^{-j\beta_g z}$$

$$E_z = E_{0z} J_n \left(\frac{X_{np} r}{a}\right) \cos{(n\phi)} e^{-j\beta_g z}$$

$$H_r = -\frac{E_{0\phi}}{Z_g} J_n \left(\frac{X_{np} r}{a}\right) \sin{(n\phi)} e^{-j\beta_g z}$$

$$H_\phi = \frac{E_{0r}}{Z_g} J_n^{'} \left(\frac{X_{np} r}{a}\right) \cos{(n\phi)} e^{-j\beta_g z}$$

$$H_z = 0$$

Some of the TM-mode characteristic equations in the circular guide are identical to those of the TE mode, but some are different. For convenience, all are shown here:

The mode propagation constant is given by

$$\beta_g = \sqrt{\omega^2 \mu\varepsilon - \left(\frac{X_{np}}{a}\right)^2}$$

The cutoff frequency for TE modes in a circular guide is then given by

$$f_c = \frac{X_{np}}{2\pi a \sqrt{\mu\varepsilon}}$$

and the phase velocity for TE modes is

$$v_p = \frac{\omega}{\beta_g} = \frac{v}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

Where

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_0 \varepsilon_0}}$$

The wavelength and wave impedance for TE modes in a circular guide are given, respectively, by

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$Z_g = \frac{\beta_g}{\omega \varepsilon} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

Impossibility of tem modes in waveguide

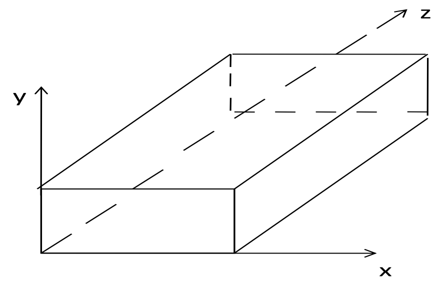


Fig: Wave guide

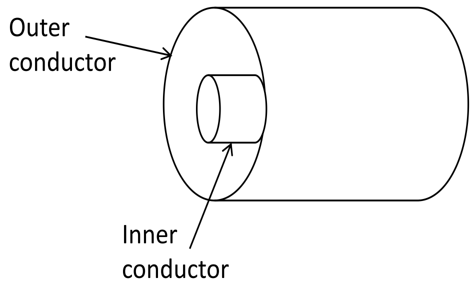


Fig: Coaxial Cable

The Transverse Electromagnetic (TEM) modes will not possible in waveguides, but they are possible in coaxial transmission lines because of the following reasons:

(i) For the existence of any mode such as TE, TM, TEM, etc., the requirement is a physical inner conductor or at least an existence of axial component of the field.

(ii) In case of coaxial transmission line, there is physical inner conductor and hence it is possible, where as in case of waveguide there is no physical inner conductor.

(iii) TE and TM modes are possible in waveguides because of the existence of axial components such as E_z in TM mode and H_z in TE mode.

Therefore due to the above reasons the TEM mode will not possible in waveguides. Note that, the magnetic field can exist around any physical conductor or at least around the axial component.

CAVITY RESONATORS

RECTANGULAR CAVITY RESONATOR:

In general, a cavity resonator is a metallic enclosure that confines the electromagnetic energy. The stored electric and magnetic energies inside the cavity will determine the equivalent inductance and capacitance. The energy dissipated by the finite conductivity of the cavity walls determines the equivalent resistance.

The rectangular cavity resonator is shown in the following figure.

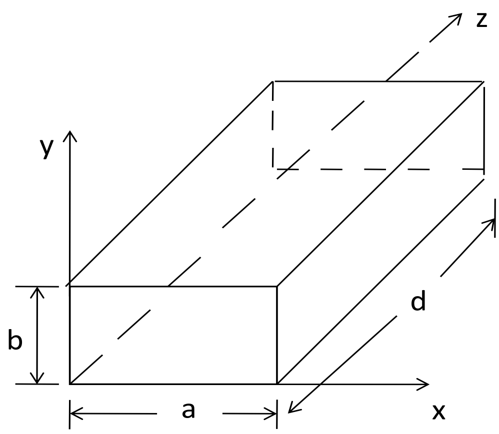


Fig: Rectangular cavity resonator

The field equation of rectangular cavity resonator for TE and TM modes are given by

$$H_z = H_{0z} cos\left(\frac{m\pi x}{a}\right) cos\left(\frac{n\pi y}{b}\right) sin\left(\frac{p\pi z}{d}\right) \quad for\ TE_{mnp}\ mode$$

Where

m = 0, 1, 2, 3, … represents the number of the half-wave periodicity in the x direction

n = 0, 1, 2, 3, … represents the number of the half-wave periodicity in the y direction

p = 1, 2, 3, … represents the number of the half-wave periodicity in the z direction

$$E_z = E_{0z} sin\left(\frac{m\pi x}{a}\right) sin\left(\frac{n\pi y}{b}\right) cos\left(\frac{p\pi z}{d}\right) \quad for\ TM_{mnp}\ mode$$

Where

m = 1, 2, 3, … represents the number of the half-wave periodicity in the x direction

n = 1, 2, 3, … represents the number of the half-wave periodicity in the y direction

p = 0, 1, 2, 3, … represents the number of the half-wave periodicity in the z direction

Dominant modes and resonant frequencies

A mode having the lowest resonant frequency is known as the dominant mode. The resonant frequency of the rectangular cavity resonator is given by

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

From the above equation we can find the resonant frequency of the rectangular cavity resonator. The lowest value of resonant frequency will occur for the mode TE_{101} when the condition a > b < d satisfied. Therefore the dominant mode of rectangular cavity resonator is given by

$$TE_{101} \quad when\ a > b < d$$

The frequency at which the response of resonator is maximum is called as resonant frequency. At resonant frequency the peak energies stored in the electric and magnetic fields will be equal.

The characteristic equation of rectangular cavity resonator is given by

$$k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

By solving the above equation we can obtain the resonant frequency as

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

CYLINDRICAL CAVITIES:

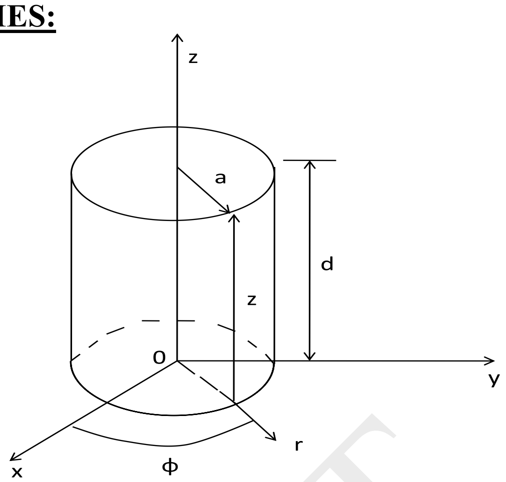


Fig: Cylindrical cavity resonator

A circular cylindrical cavity resonator is shown in the figure above.
The circular or cylindrical cavity resonator is made from the circular wave guide. The field equations of circular cavity resonator are given by

$$H_z = H_{0z} J_n\left(\frac{X'_{np} r}{a}\right)\cos(n\emptyset)\, sin\left(\frac{q\pi z}{d}\right) \qquad for\ TE_{npq}$$

Where

n = 0,1,2,3,… is the number of periodicity in the φ direction

p = 1,2,3,4,… is the number of zeros of the field in the radial direction

q = 1,2,3,4,… is the number of half-waves in the axial direction

J_n = Bessell's function of the first kind

H_{0z} = amplitude of the magnetic field

$$E_z = E_{0z} J_n\left(\frac{X_{np} r}{a}\right)\cos(n\emptyset)\, cos\left(\frac{q\pi z}{d}\right) \qquad for\ TM_{npq}$$

Where

n = 0,1,2,3,… is the number of periodicity in the φ direction

p = 1,2,3,4,… is the number of zeros of the field in the radial direction

q = 0,1,2,3,… is the number of half-waves in the axial direction

J_n = Bessell's function of the first kind

E_{0z} = amplitude of the electric field

Dominant modes and resonant frequencies

A mode having the lowest resonant frequency is known as the dominant mode. The resonant frequency of the cylindrical cavity resonator is given by

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{X'_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2} \qquad for\ \ TE\ mode$$

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{X_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2} \qquad for\ \ TM\ mode$$

From the above equation we can find the resonant frequency of the cylindrical cavity resonator. The lowest value of resonant frequency will occur for the mode TM_{110} when the 2a > d and TE_{111} when d ≥ 2a. Therefore the dominant modes of cylindrical cavity resonator are given by

$$TM_{110} \qquad when\ \ 2a > d$$
$$TE_{111} \qquad when\ \ d \geq 2a$$

The frequency at which the response of resonator is maximum is called as resonant frequency. At resonant frequency the peak energies stored in the electric and magnetic fields will be equal.

The characteristic equation of cylindrical cavity resonator is given by

$$k^2 = \left(\frac{X'_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2 \qquad for\ TE\ mode$$
$$k^2 = \left(\frac{X_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2 \qquad for\ TM\ mode$$

By solving the above equation we can obtain the resonant frequency as

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{X'_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2} \qquad for\ \ TE\ mode$$
$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{X_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2} \qquad for\ \ TM\ mode$$

SOLVED PROBLEMS

1. **Two identical point sources separated by a distance'd' each source having a field pattern given by $E_o = E_1 \sin\theta$. If $d = \lambda/2$ and phase angle $\alpha = 0$ derive the expression for field pattern. Plot the pattern.**

Ans:

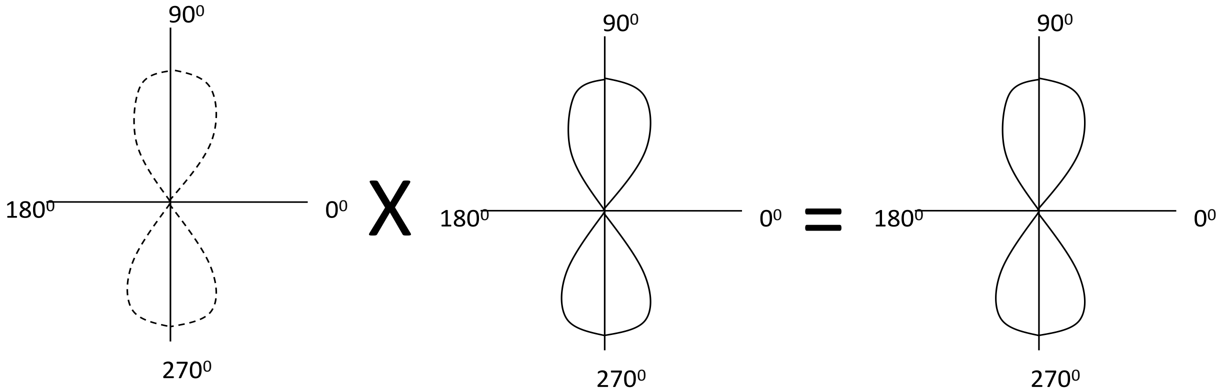
Given data:

$$E_0 = E_1 \sin\theta$$
$$d = \lambda/2$$

For broad side array, the electric field strength is given by

$$E = 2E_0 \cos\left(\frac{\psi}{2}\right)$$
$$\psi = \beta d \cos\theta = \frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\theta = \pi\cos\theta$$
$$E = 2(E_1 \sin\theta)\cos\left(\frac{\pi\cos\theta}{2}\right)$$

The pattern is shown in figure below.



2. **Calculate the directivity of a given linear broad side uniform array of 10 isotropic elements with separation of λ/4 between the elements.**

Ans:

Given data:

No.of elements(N) = 10

Spacing between the elements(d) = λ/4

$$Directivity(D) = 2N\left(\frac{d}{\lambda}\right) = 2 \times 10\left(\frac{\lambda/4}{\lambda}\right) = 5$$

Or

$$Directivity(D) = 10\log(6) = 6.99\ dB$$

3. **Calculate the directivity of a given linear end fire uniform array of 10 isotropic elements with separation of λ/4 between the elements.**

Ans:

Given data:

No.of elements(N) = 10

Spacing between the elements(d) = λ/4

$$Directivity(D) = 4N\left(\frac{d}{\lambda}\right) = 4 \times 10\left(\frac{\lambda/4}{\lambda}\right) = 10$$

Or

$$Directivity(D) = 10\log(10) = 10\ dB$$

4. **Calculate the directivity of a given linear end fire array with improved directivity , Hansen-Woodyard uniform array of 10 elements with separation of λ/4 between the elements.**

Ans:

Given data:

No.of elements(N) = 10

Spacing between the elements(d) = λ/4

$$Directivity(D) = 1.789\left[4N\left(\frac{d}{\lambda}\right)\right] = 1.789\left[4 \times 10\left(\frac{\lambda/4}{\lambda}\right)\right] = 17.89$$

Or

$$Directivity(D) = 10\log(17.89) = 12.551\ dB$$

5. **A uniform linear end fire array consists of 16 isotropic point sources with spacing of λ/4. If the phase difference α = -90^0 calculate**
   (a) HPBW (b) Beam solid angle (c) Directivity (d) Effective aperture

Ans:

Given data:

No.of elements (N) = 16
Spacing between the elements(d) = λ/4

(a)

$$HPBW = \sqrt{\frac{2\lambda}{\mathrm{Nd}}} \times 57.3 \ = 57.3\sqrt{\frac{2\lambda}{16 \times \frac{\lambda}{4}}} = 42^0$$

(c)

$$Directivity(D) = 4N\left(\frac{d}{\lambda}\right) = 4 \times 16\left(\frac{\lambda/4}{\lambda}\right) = 16$$

Or

$$Directivity(D) = 10\log(10) = 12.04\ dB$$

(b)

$$Beam\ solid\ angle = \frac{4\pi}{Directivity} = \frac{4\pi}{16} = 0.8373\ Steradians$$

(d)

$$G = D = \frac{4\pi A_e}{\lambda^2}$$

$$A_e = \frac{D\,\lambda^2}{4\pi} = \frac{0.8373 \times \lambda^2}{4\pi} = 1.194\,\lambda^2$$

6. **In order to scan the beam of linear array to 30^0 off broadside, calculate the inner element phase shift required if the elements are 3 cm spaced and the frequency is 64 KHz.**

Ans:

Given data:

$$\theta = 90^0 - 30^0 = 60^0$$
$$d = 3\ cm = 0.03\ m$$
$$f = 64\ KHz$$
$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{64 \times 10^3} = 4.68 \times 10^3$$

$$Inner\ element\ phase\ shift(\alpha) = -\beta d\cos\theta = -\frac{2\pi}{\lambda} \times 0.03 \times \cos(60)$$
$$= -\frac{2\pi}{4.68 \times 10^3} \times 0.03 \times 0.5$$
$$= 1.153 \times 10^{-3}\ degree$$

7. **Calculate**
   (i) HPBW (ii) Solid angle if a linear end fire array having 10 isotropic point source with λ/2 spacing and phase difference δ = 90^0

Ans:

Given data:

$$\delta = \alpha = 90^0$$
N = 10
d = λ/2

(i)

$$HPBW = \sqrt{\frac{2\lambda}{\mathrm{Nd}}} \times 57.3 \ = 57.3\sqrt{\frac{2\lambda}{10 \times \frac{\lambda}{2}}} = 36.24^0$$

$$Directivity(D) = 4N\left(\frac{d}{\lambda}\right) = 4 \times 10\left(\frac{\lambda/2}{\lambda}\right) = 20$$

$$Beam\ solid\ angle = \frac{4\pi}{Directivity} = \frac{4\pi}{20} = 0.628\ Steradians$$

8. Draw the radiation pattern of 8-element broad side array with spacing d = λ/2

Ans:

Given data:

No.of elements (N) = 8

Spacing between the elements (d) = λ/2

Pattern maxima:

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2Nd}\right) = \cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2(8)\lambda/2}\right)$$

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{(2n+1)}{8}\right)$$

When n = 1,

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{3}{8}\right) = \pm 67.98^0\ \ \&\ \ \pm 112.02^0$$

When n = 2,

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{5}{8}\right) = \pm 51.32^0\ \ \&\ \ \pm 128.68^0$$

When n = 3,

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{7}{8}\right) = \pm 28.95^0\ \ \&\ \ \pm 151.045^0$$

When n = 4,

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{9}{8}\right) = \ not\ satisfied$$

Pattern minima:

$$(\theta_{min})_{minor} = \cos^{-1}\left(\pm\frac{n\lambda}{Nd}\right) = \cos^{-1}\left(\pm\frac{n\lambda}{8\lambda/2}\right) = \cos^{-1}\left(\pm\frac{n}{4}\right)$$

When n = 1,

$$(\theta_{min})_{minor} = \cos^{-1}\left(\pm\frac{1}{4}\right) = \pm 75.5^0,\ \pm 104.47^0$$

When n = 2,

$$(\theta_{min})_{minor} = \cos^{-1}\left(\pm\frac{2}{4}\right) = \pm 60^0,\ \pm 120^0$$

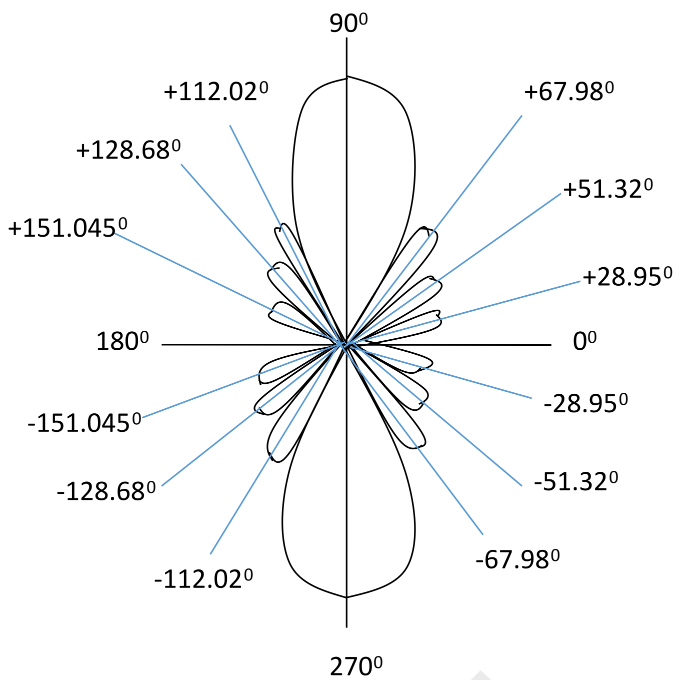
When n = 3,

$$(\theta_{min})_{minor} = \cos^{-1}\left(\pm\frac{3}{4}\right) = \pm 41.409^0,\ \pm 138.59^0$$

When n = 4,

$$(\theta_{min})_{minor} = \cos^{-1}(\pm 1) = 0^0,\ \ 180^0$$

The radiation pattern is shown in figure below.



9. Draw the radiation pattern of 4-element broad side and end fire array with d = λ/2

Ans:

Broad side array:

Given data:

No.of elements (N) = 4

Spacing between the elements (d) = λ/2

Pattern maxima:

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2Nd}\right) = \cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2(4)\lambda/2}\right)$$

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{(2n+1)}{4}\right)$$

When n = 1,

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{3}{4}\right) = \pm 41.405^0 \ \ \& \ \ \pm 138.59^0$$

When n = 2,

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{5}{4}\right) = \ not\ satified$$

Pattern minima:

$$(\theta_{min})_{minor} = \cos^{-1}\left(\pm\frac{n\lambda}{Nd}\right) = \cos^{-1}\left(\pm\frac{n\lambda}{4\lambda/2}\right) = \cos^{-1}\left(\pm\frac{n}{2}\right)$$

When n = 1,

$$(\theta_{min})_{minor} = \cos^{-1}\left(\pm\frac{1}{2}\right) = \pm 60^0,\ \pm 120^0$$

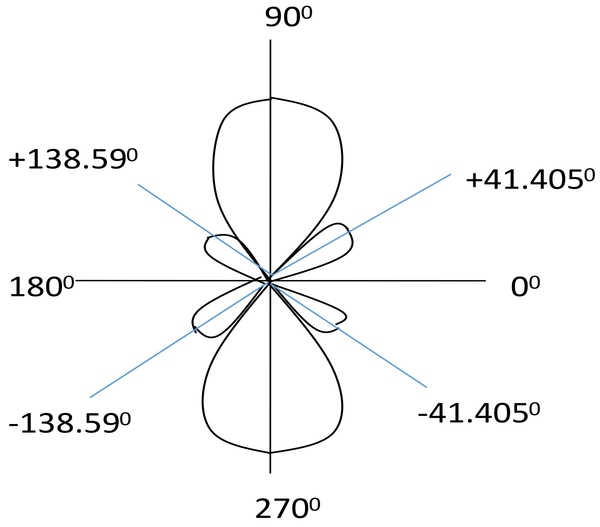
When n = 2,

$$(\theta_{min})_{minor} = \cos^{-1}(\pm 1) = 0^0, 180^0$$

When n = 3,

$$(\theta_{min})_{minor} = \cos^{-1}\left(\pm\frac{3}{2}\right) = not\ satisfied$$

The radiation pattern is shown in figure below.



End fire array:

Pattern maxima:

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2Nd}+1\right) = \cos^{-1}\left(\pm\frac{(2n+1)\lambda}{\frac{2(4)\lambda}{2}}+1\right)$$

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{(2n+1)}{4}+1\right)$$

When n = 1,

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{3}{4}+1\right) = \cos^{-1}\left(\frac{7}{4}\right)\ \&\ \cos^{-1}\left(\frac{1}{4}\right) = \pm\ 75.5^0$$

When n = 2,

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{5}{4}+1\right) = \cos^{-1}\left(\frac{9}{4}\right)\ \&\ \cos^{-1}\left(-\frac{1}{4}\right) = \pm\ 104.4^0$$

When n = 3,

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{7}{4}+1\right) = \cos^{-1}\left(\frac{11}{4}\right)\ \&\ \cos^{-1}\left(-\frac{3}{4}\right) = \pm\ 138.59^0$$

When n = 4,

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{9}{4}+1\right) = \cos^{-1}\left(\frac{13}{4}\right)\ \&\ \cos^{-1}\left(-\frac{5}{4}\right) = not\ satisfied$$

Pattern minima:

$$(\theta_{min})_{minor} = 2\sin^{-1}\left(\pm\sqrt{\frac{n\lambda}{2Nd}}\right) = 2\sin^{-1}\left(\pm\sqrt{\frac{n\lambda}{2(4)\lambda/2}}\right)$$

$$(\theta_{min})_{minor} = 2\sin^{-1}\left(\pm\sqrt{\frac{n}{4}}\right)$$

When n = 1,

$$(\theta_{min})_{minor} = 2\sin^{-1}\left(\pm\sqrt{\frac{1}{4}}\right) = \pm\ 60^0$$

When n = 2,

$$(\theta_{min})_{minor} = 2\sin^{-1}\left(\pm\sqrt{\frac{2}{4}}\right) = \pm 90^0$$

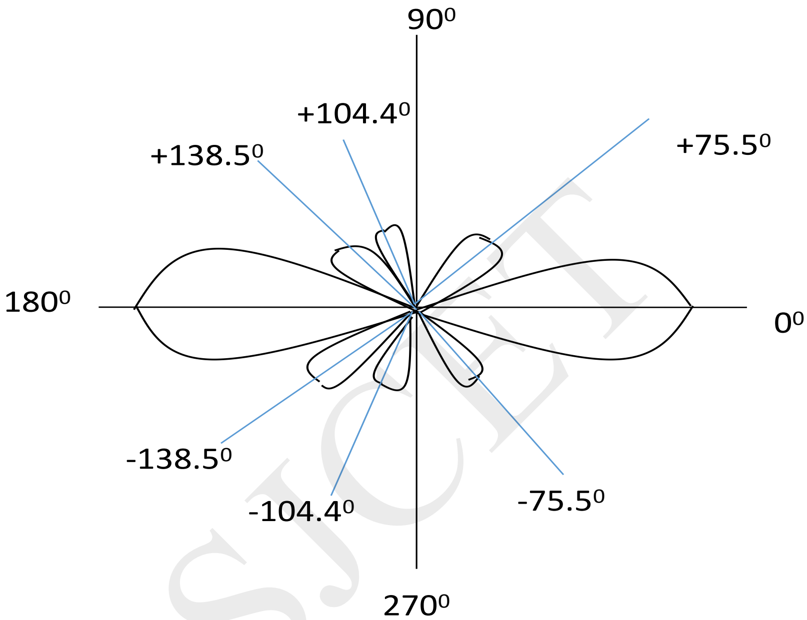
When n = 3,

$$(\theta_{min})_{minor} = 2\sin^{-1}\left(\pm\sqrt{\frac{3}{4}}\right) = \pm 120^0$$

When n = 4,

$$(\theta_{min})_{minor} = 2\sin^{-1}\left(\pm\sqrt{\frac{4}{4}}\right) = \pm 180^0$$

The resultant pattern is shown in figure below.



10. Calculate the distance beyond which the earth's curvature is to be accounted at frequency of (a) 100 KHz (b) 1 MHz (c) 10 MHZ.

Ans: f = 100 KHz = 0.1 MHz

$$d = \frac{50}{(f_{MHz})^{1/3}} \quad in\ miles = \frac{50}{0.1^{1/3}} = 107.75\ miles$$

(a) f = 1 MHz

$$d = \frac{50}{(f_{MHz})^{1/3}} \quad in\ miles = \frac{50}{1^{1/3}} = 50\ miles$$

(b) f = 10 MHz

$$d = \frac{50}{(f_{MHz})^{1/3}} \quad in\ miles = \frac{50}{10^{1/3}} = 23.21\ miles$$

11. Obtain the roughness factor at 3 MHz for an earth having σ = 0.5 with θ = 30⁰. Calculate the ratio of roughness factors for the same earth and same θ if the frequency is doubled.

Ans: Given data: Frequency (f) = 3 MHz

Wavelength (λ) = c/f = $3X10^8/3X10^6$ = 100 m

Standard deviation (σ) = 0.5

Angle of incidence (θ) = 30^0

$$Roughness\ factor\ (R) = \frac{4\pi\sigma sin\theta}{\lambda} = \frac{4\pi \times 0.5 \times \sin(30)}{100} = 0.031415927$$

When the frequency is doubled, Frequency (f) = 2X3 MHz = 6 MHz

Wavelength (λ) = c/f =$3X10^8/6X10^6$ = 50 m

Standard deviation (σ) = 0.5

Angle of incidence (θ) = 30^0

$$Roughness\ factor\ (R) = \frac{4\pi\sigma sin\theta}{\lambda} = \frac{4\pi \times 0.5 \times \sin(30)}{50} = 0.06282$$

12. Evaluate the roughness factors for the earth at 10 MHz if σ = 5 for θ equal to (a) 30^0 (b) 45^0 (c) 60^0

Ans:

(a) Given data: Frequency (f) = 10 MHz

Wavelength (λ) = c/f =$3X10^8/10X10^6$ = 10 m

Standard deviation (σ) = 5

Angle of incidence (θ) = 30^0

$$Roughness\ factor\ (R) = \frac{4\pi\sigma sin\theta}{\lambda} = \frac{4\pi \times 5 \times \sin(30)}{10} = 3.141$$

(b) Given data: Frequency (f) = 10 MHz

Wavelength (λ) = c/f =$3X10^8/10X10^6$ = 10 m

Standard deviation (σ) = 5

Angle of incidence (θ) = 45^0

$$Roughness\ factor\ (R) = \frac{4\pi\sigma sin\theta}{\lambda} = \frac{4\pi \times 5 \times \sin(45)}{10} = 4.442$$

(c) Given data: Frequency (f) = 10 MHz

Wavelength (λ) = c/f =$3X10^8/10X10^6$ = 10 m

Standard deviation (σ) = 5

Angle of incidence (θ) = 60^0

$$Roughness\ factor\ (R) = \frac{4\pi\sigma sin\theta}{\lambda} = \frac{4\pi \times 5 \times \sin(60)}{10} = 5.44$$

13. The transmitting and receiving antennas with respective heights of 49 m and 25 m are installed to establish communication at 100 MHz with transmitted power of 100 watts. Determine the LOS range and received signal Strength.

Ans: Given data: $\text{Height of the transmitting antenna}(\mathrm{h_t}) =$ 49 m

$$\text{Height of the receiving antenna}(\mathrm{h_r}) = 25\ \mathrm{m}$$

$$\text{Frequency}(\mathrm{f}) = 100\ \mathrm{MHz}$$

$$\text{Wavelength}(\lambda) = \frac{\mathrm{c}}{\mathrm{f}} = \frac{3 \times 10^8}{100 \times 10^6} = 3\ \mathrm{m}$$

$$\text{Power transmitted}(\mathrm{P}) = 100\ \text{watts}$$

LOS range is given by (including effective earth's radius)

$$d = 4.12\left[\sqrt{h_t} + \sqrt{h_r}\right]km$$

$$d = 4.12\left[\sqrt{49} + \sqrt{25}\right]km = 4.12[7+5]km = 49.44\ km$$

The received field strength is given by

$$|E_R| = \frac{88\sqrt{P}\ h_t h_r}{\lambda d^2} = \frac{88\sqrt{100} \times 49 \times 25}{3 \times (49.44 \times 1000)^2} = 1.47\ V/m$$

14. Calculate the maximum distance at which signal from transmitting antenna with 144 m height would be received by the receiving antenna of 25 m height.

Ans: Given data: Height of the transmitting antenna$(h_t) = 144$ m

Height of the receiving antenna$(h_r) = 25$ m

Maximum distance or LOS range is given by (including effective earth's radius)

$$d = 4.12\left[\sqrt{h_t} + \sqrt{h_r}\right]km = 4.12\left[\sqrt{144} + \sqrt{25}\right]km = 4.12[12 + 5]km$$

$$d = 70.04\ km$$

15. A transmitting antenna of 100 m height radiates 40 kW at 100 MHz uniformly in azimuth plane. Calculate the maximum LOS range and strength of the received signal at 16 m high receiving antenna at a distance of 10 km. At what distance would the signal strength reduces to 1 mV/m.

Ans: Given data: Height of the transmitting antenna$(h_t) = 100$ m

Height of the receiving antenna$(h_r) = 16$ m

Frequency$(f) = 100$ MHz

$$\text{Wavelength}(\lambda) = \frac{c}{f} = \frac{3 \times 10^8}{100 \times 10^6} = 3\ \text{m}$$

Power transmitted$(P) = 40$ kW

Distance$(d) = 10$ km

LOS range is given by (including effective earth's radius)

$$d = 4.12\left[\sqrt{h_t} + \sqrt{h_r}\right]km = 4.12\left[\sqrt{100} + \sqrt{16}\right]km = 4.12[10 + 4]km$$
$$= 57.68\ km$$

The received field strength is given by

$$|E_R| = \frac{88\sqrt{P}\,h_t h_r}{\lambda d^2} = \frac{88\sqrt{40 \times 10^3} \times 100 \times 16}{3 \times (10 \times 1000)^2} = 98.36\ mV/m$$

The distance at which the field strength reduces to 1 mV/m is

$$|E_R| = \frac{88\sqrt{P}\,h_t h_r}{\lambda d^2}$$

$$d = \sqrt{\frac{88\sqrt{P}\,h_t h_r}{|E_R|\lambda}} = \sqrt{\frac{88\sqrt{40 \times 10^3} \times 100 \times 16}{1 \times 10^{-6} \times 3}} = 96.88\ km$$

16. A directional antenna with 10 dB gain radiates 500 watts. The receiving antenna at 15 km distance receives 2 µW. Find the effective area of the receiving antenna. Assume negligible ground and ionospheric reflections.

Ans: Given data: Gain of transmitting antenna$(G_T) = 10$ dB

Gain without dB is given by

$$10 = 10\log(G_T)$$

$$G_T = 10$$

Power transmitted$(P_T) = 500$ W

Distance$(d) = 15$ km

Power received$(P_R) = 2\ \mu$W

We know that,

$$P_R = P_T G_T G_R\left(\frac{\lambda}{4\pi d}\right)^2$$

$$P_R = \frac{P_T G_T}{4\pi d^2} A_e$$

$$A_e = \frac{P_R\, 4\pi d^2}{P_T G_T} = \frac{2 \times 10^{-6} \times 4\pi \times (15 \times 1000)^2}{500 \times 10} = 1.13\ m^2$$

17. **Find the basic path loss for communication between two points 3000 km apart at a frequency of 3 GHz**.

Ans: Given data: Distance(d) = 3000 km

Frequency(f) = 3 GHz

$$Path\ loss = 32.45 + 20 \log_{10} f_{MHz} + 20\ \log_{10} d_{km}$$

$$Path\ loss = 32.45 + 20 \log_{10} (3 \times 1000) + 20\ \log_{10} (3000) = 171.534 \text{ dB}$$

18. Calculate the skip distance for flat earth with MUF of 10 MHz if the wave is reflected from a height of 300 km where the maximum value of n is 0.9

Ans: Given data: MUF(f_{MUF}) = 10 MHz

Height of the layer(h) = 300 km

Refractive index(μ or n) = 0.9

$$\mu = \sqrt{1 - \frac{81\, N_{max}}{f^2}}$$

$$N_{max} = \frac{(1-\mu^2)f^2}{81} = 23.45 \times 10^{10}$$

Critical frequency is $f_c = 9\sqrt{N_{max}} = 9\sqrt{23.45 \times 10^{10}} = 4.36\ MHz$

Skip distance is given by

$$D = 2h\sqrt{\frac{f_{MUF}^2}{f_c^2} - 1} = 2 \times 300 \times 10^3 \sqrt{\frac{(10 \times 10^6)^2}{(4.36 \times 10^6)^2} - 1} = 3916.2\ km$$

19. The critical frequencies at an instant observed for E, F_1 and F_2 layers were found to be 3, 5 and 9 MHz. Find the corresponding concentration of electrons in these layers.

Ans: The critical frequency is given by

$$f_c = 9\sqrt{N_{max}}$$

Then,

$$N_{max} = \frac{f_c^2}{81}$$

For E layer, f_c = 3 MHz,

$$N_{max} = \frac{(3 \times 10^3)^2}{81} = 0.111 \times 10^{12}$$

For F_1 layer, f_c = 5 MHz,

$$N_{max} = \frac{(5 \times 10^3)^2}{81} = 0.3086 \times 10^{12}$$

For F_2 layer, f_c = 9 MHz,

$$N_{max} = \frac{(9 \times 10^3)^2}{81} = 10^{12}$$

20. An air-filled rectangular waveguide of inside dimensions 7 x 3.5 cm operates in the dominant TE10 mode

a. Find the cutoff frequency.

b. Determine the phase velocity of the wave in the guide at a frequency of 3 .5 GHz.

c. Determine the guided wavelength at the same frequency.

Solution

Width of waveguide(a) = 7 cm = 0.07 m

Height of the waveguide(b) = 3.5 cm = 0.035 m

Mode = TE_{10}, i.e m = 1, n = 0

(a) The cutoff frequency is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.07}\right)^2 + \left(\frac{0}{0.035}\right)^2}$$

$$f_c = 2.14\times10^9 = 2.14GHz$$

(b) The signal frequency (f) = 3.5 GHz

The phase velocity is given by

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_p = \frac{3\times10^8}{\sqrt{1-\left(\frac{2.14\times10^9}{3.5\times10^9}\right)^2}} = 3.78\times10^8 m/s$$

(c)

The signal frequency (f) = 3.5 GHz
Signal Wavelength is

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3.5\times10^9} = 0.086m$$

The Guide wavelemgth is given by

$$\lambda_g = \frac{\lambda}{\sqrt{\left(1-\left(\frac{f_c}{f}\right)^2\right)}}$$

$$\lambda_g = \frac{0.086}{\sqrt{\left(1-\left(\frac{2.14\times10^9}{3.5\times10^9}\right)^2\right)}} = 0.108m = 10.8cm$$

21. An air-filled rectangular waveguide has dimensions of a = 6 cm and b = 4 cm. The signal frequency is 3 GHz. Compute the following for the TE_{10}, TE_{01}, TE_{11}, and TM_{11} modes
(a) Cutoff frequency
(b) Wavelength in the waveguide
(c) Phase constant and phase velocity in the waveguide
(d) Group velocity and wave impedance in the waveguide

Solution:

Width of waveguide(a) = 6 cm = 0.06 m
Height of the waveguide(b) = 4 cm = 0.04 m

The signal frequency (f) = 3 GHz
Signal Wavelength is

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^9} = 0.1m$$

For TE$_{10}$ mode:

(a) The cutoff frequency is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.06}\right)^2 + \left(\frac{0}{0.04}\right)^2}$$

$$f_c = 25\times10^8 = 2.5GHz$$

(b)

The Guide wavelemgth is given by

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\lambda_g = \frac{0.1}{\sqrt{1-\left(\frac{2.5\times10^9}{3\times10^9}\right)^2}} = 0.18m = 18cm$$

(c) The phase constant is given by

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{\left(1-\left(\frac{f_c}{f}\right)^2\right)} = \frac{\omega}{c}\sqrt{\left(1-\left(\frac{f_c}{f}\right)^2\right)}$$

$$\beta = \frac{2\pi f}{c}\sqrt{\left(1-\left(\frac{f_c}{f}\right)^2\right)} = \frac{2\pi}{\lambda}\sqrt{\left(1-\left(\frac{f_c}{f}\right)^2\right)}$$

$$\beta = \frac{2\pi}{0.1}\sqrt{\left(1-\left(\frac{2.5\times10^9}{3\times10^9}\right)^2\right)} = 34.73rad/m$$

The phase velocity is given by

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_p = \frac{3\times10^8}{\sqrt{1-\left(\frac{2.5\times10^9}{3\times10^9}\right)^2}} = 5.427\times10^8 m/s$$

(d) The group velocity is given by

$$v_g = c\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 3\times10^8\sqrt{1-\left(\frac{2.5\times10^9}{3\times10^9}\right)^2} = 1.6583\times10^8 m/s$$

For TE$_{01}$ mode:

(a) The cutoff frequency is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{0}{0.06}\right)^2 + \left(\frac{1}{0.04}\right)^2}$$

$$f_c = 37.5\times10^8 = 3.75GHz$$

f < f_c (3 GHz < 3.75 GHz), hence TE_{01} mode is not possible though the waveguide

For TE_{11} mode:

(a) The cutoff frequency is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.06}\right)^2 + \left(\frac{1}{0.04}\right)^2}$$

$$f_c = 45.06\times10^8 = 4.506GHz$$

f < f_c (3 GHz < 4.506 GHz), hence TE_{11} mode is not possible though the waveguide

For TM_{11} mode:

(b) The cutoff frequency is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.06}\right)^2 + \left(\frac{1}{0.04}\right)^2}$$

$$f_c = 45.06\times10^8 = 4.506GHz$$

f < f_c (3 GHz < 4.506 GHz), hence TM_{11} mode is not possible though the waveguide

22. **The dominant mode TE_{10} is propagated in a rectangular waveguide of dimension a = 6 cm and b = 4 cm. The distance between maximum and minimum is 4.47 cm. Determine the signal frequency of the dominant mode**

Solution:

Width of waveguide(a) = 6 cm = 0.06 m

Height of the waveguide(b) = 4 cm = 0.04 m

The distance between maximum and minimum (λg /4) = 4.47 cm = 0.0447 m

$$\frac{\lambda_g}{4} = 0.0447$$

$$\lambda_g = 4\times0.0447 = 0.1788m$$

For dominant mode (TE_{10}) the cutoff wavelength is given by

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{0}{b}\right)^2}}$$

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2}} = \frac{2}{\frac{1}{a}} = 2a$$

$$\lambda_c = 2a = 2\times0.06 = 0.12m$$

The signal wavelength is obtained as follows:

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$$

$$\frac{1}{\lambda^2} = \frac{1}{(0.1788)^2} + \frac{1}{(0.12)^2} = \frac{1}{0.0319} + \frac{1}{0.0144} = 31.347 + 69.44 = 100.8$$

$$\lambda^2 = \frac{1}{100.8} = 0.0099$$

$$\lambda = \sqrt{0.0099} = 0.099m$$

The signal frequency is given by

$$f = \frac{c}{\lambda} = \frac{3\times10^8}{0.099} = 3.03GHz$$

23. A rectangular waveguide is filled by dielectric material of $\varepsilon_r = 9$ and has inside dimensions of 7 X 3.5 cm. It operates in the dominant TE_{10} mode

(a) Determine the cutoff frequency

(b) Find the phase velocity in the guide at a frequency of 2 GHz.

(c) Find the guided wavelength at the same frequency

Solution:

Width of waveguide(a) = 7 cm = 0.07 m

Height of the waveguide(b) = 3.5 cm = 0.035 m

Dielectric constant or relative permitivity (ε_r) = 9

(a) The cutoff frequency of dominant mode is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{1}{2\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{1}{2\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{3\times10^8}{2\sqrt{\mu_r\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2\sqrt{1\times9}}\sqrt{\left(\frac{1}{0.07}\right)^2 + \left(\frac{0}{0.035}\right)^2}$$

$$f_c = 7.14\times10^8 = 0.714GHz$$

(b) The phase velocity is given by

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_p = \frac{3\times10^8}{\sqrt{\mu_r\varepsilon_r}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_p = \frac{3\times10^8}{\sqrt{1\times9}\sqrt{1-\left(\frac{0.714\times10^9}{2\times10^9}\right)^2}} = 1.07\times10^8 m/s$$

(c) The signal frequency (f) = 2 GHz

Signal Wavelength is

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{2\times10^9} = 0.15m$$

The guided wavelemgth is given by

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\lambda_g = \frac{0.15}{\sqrt{1-\left(\frac{0.714\times10^9}{2\times10^9}\right)^2}} = 0.16m = 16cm$$

24. A TE_{11} mode of 10 GHz is propagated in an air-filled rectangular waveguide. The magnetic field in the z-direction is given by

$$H = H_0 \cos\left(\frac{\pi x}{\sqrt{6}}\right)\cos\left(\frac{\pi y}{\sqrt{6}}\right) A/m$$

The phase constant is β = 1.0475 rad/cm. The quantities x and y are expressed in centimeters, and a = b = $\sqrt{6}$ are also in centimeters. Determine the cutoff frequency f_c, phase velocity v_p, guided wavelength λ_g and the magnetic field intensity in the y-direction.

Solution:

Mode = TE_{11}, i.e m = 1, n = 1

The signal frequency (f) = 10 GHz

Signal Wavelength is

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^9} = 0.03m$$

$$H = H_z = H_0 \cos\left(\frac{\pi x}{\sqrt{6}}\right)\cos\left(\frac{\pi y}{\sqrt{6}}\right) A/m$$

Phase constant(β) = 1.0475 rad/cm = 104.75 rad/m

Width of waveguide(a) = $\sqrt{6}$ cm = 2.449 cm = 0.0245 m

Height of the waveguide(b) = $\sqrt{6}$ cm = 2.449 cm = 0.0245 m

The cutoff frequency is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.0245}\right)^2 + \left(\frac{1}{0.0245}\right)^2}$$

$$f_c = 86.58\times10^8 = 8.658GHz$$

The phase velocity is given by

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_p = \frac{3\times10^8}{\sqrt{1-\left(\frac{8.658\times10^9}{10\times10^9}\right)^2}} = 6\times10^8 m/s$$

The guided wavelemgth is given by

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\lambda_g = \frac{0.03}{\sqrt{1-\left(\frac{8.658\times10^9}{10\times10^9}\right)^2}} = 0.06m = 6cm$$

The magnetic field intensity in y-direction is given by

$$H_y = \frac{j\beta A' n\pi}{k^2 b}\cos\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$H_y = \frac{j\beta A'\pi}{k^2}\cos\left(\frac{\pi x}{\sqrt{6}}\right)\cdot\sin\left(\frac{\pi y}{\sqrt{6}}\right)e^{-j\beta z}$$

25. Determine the cutoff wavelength for the dominant mode in a rectangualar waveguide of breadth 10 cm. For a 2.5 GHz signal propagated in this waveguide in the dominant mode, calculate the guide wavelength, the group and phase velocities

Solution:

Breadth of waveguide (a) = 10 cm = 0.1 m

Signal Frequency (f) = 2.5 GHz

Signal Wavelength is given by

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{2.5\times10^9} = 0.12m$$

For dominant mode (TE_{10}) the cutoff wavelength is given by

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}} = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{0}{b}\right)^2}}$$

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2}} = \frac{2}{\frac{1}{a}} = 2a$$

$$\lambda_c = 2a = 2\times0.1 = 0.2m = 20cm$$

The cutoff frequency is given by

$$f_c = \frac{c}{\lambda_c} = \frac{3\times10^8}{0.2} = 15\times10^8 = 1.5GHz$$

The guided wavelemgth is given by

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\lambda_g = \frac{0.12}{\sqrt{1-\left(\frac{1.5\times10^9}{2.5\times10^9}\right)^2}} = 0.15m = 15cm$$

The phase velocity is given by

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_p = \frac{3\times10^8}{\sqrt{1-\left(\frac{1.5\times10^9}{2.5\times10^9}\right)^2}} = 3.75\times10^8 m/s$$

The group velocity is given by

$$v_g = c\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 3\times10^8\sqrt{1-\left(\frac{1.5\times10^9}{2.5\times10^9}\right)^2} = 2.4\times10^8 m/s$$

26. The dimensions of guide are 2.5 X 1 cm. The frequency is 8.6 GHz. Find the following:

(a) Possible modes

(b) Cutoff frequencies

(c) Guide wavelengths

Solution:

Width of waveguide(a) = 2.5 cm = 0.025 m

Height of the waveguide(b) = 1 cm = 0.01 m

Signal Frequency (f) = 8.6 GHz

Signal Wavelength is given by

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{8.6\times10^9} = 0.034m = 3.4cm$$

The cutoff frequency for TE_{10} mode is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.025}\right)^2+\left(\frac{0}{0.01}\right)^2}$$

$$f_c = 60\times10^8 = 6GHz$$

$f > f_c$, the mode TE_{10} is possible.

The cutoff frequency for TE_{01} mode is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{0}{0.025}\right)^2+\left(\frac{1}{0.01}\right)^2}$$

$$f_c = 150\times10^8 = 15GHz$$

$f < f_c$, the mode TE_{01} not possible.

Simialry, the cutoff frequency for TE_{11} mode is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.025}\right)^2+\left(\frac{1}{0.01}\right)^2}$$

$$f_c = 161.55\times10^8 = 16.155GHz$$

$f < f_c$, the mode TE_{11} is not possible.

Therefore, only one mode is possible. That is TE_{10}

The guided wavelemgth for TE_{10} is given by

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\lambda_g = \frac{0.034}{\sqrt{1-\left(\frac{6\times10^9}{8.6\times10^9}\right)^2}} = 0.047m = 4.7cm$$

27. When the dominant mode is propagated in an air-filled rectangular waveguide, the guide wavelength for a frequency of 9000 MHz is 4 cm. Calculate the breadth of the guide.

Solution:

Signal Frequency (f) = 9000 MHz

Guide wavelength (λ_g) = 4 cm = 0.04 m

Signal Wavelength is given by

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{9000\times10^6} = 0.033m$$

The cutoff wavelength is obtained as follows:

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}$$

$$\frac{1}{\lambda_c^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_g^2}$$

$$\frac{1}{\lambda_c^2} = \frac{1}{(0.033)^2} - \frac{1}{(0.04)^2} = \frac{1}{0.001089} - \frac{1}{0.0016} = 918.273 - 625 = 293.273$$

$$\lambda_c^2 = \frac{1}{293.273} = 0.0034$$

$$\lambda_c = \sqrt{0.0034} = 0.0583m = 5.83cm$$

But for dominant mode

$$\lambda_c = 2a$$

$$a = \frac{\lambda_c}{2} = \frac{5.83}{2} = 2.91cm$$

28. A rectangular waveguide has a = 4 cm, b = 3 cm as its cross sectional dimensions. Find all the modes which will propagate at 5000 MHz

Solution:

Width of waveguide(a) = 4 cm = 0.04 m

Height of the waveguide(b) = 3 cm = 0.03 m

Signal Frequency (f) = 5000 MHz = 5 GHz

Signal Wavelength is given by

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{5000\times10^6} = 0.033m$$

The cutoff frequency for TE_{10} mode is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.04}\right)^2 + \left(\frac{0}{0.03}\right)^2}$$

$$f_c = 37.5\times10^8 = 3.75GHz$$

$f > f_c$, the mode TE_{10} is possible.

The cutoff frequency for TE_{01} mode is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{0}{0.04}\right)^2 + \left(\frac{1}{0.03}\right)^2}$$

$$f_c = 50\times10^8 = 5GHz$$

$f = f_c$, the mode TE_{01} not possible.

Simialry, the cutoff frequency for TE_{11} mode is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.04}\right)^2 + \left(\frac{1}{0.03}\right)^2}$$

$$f_c = 62.5\times10^8 = 6.25GHz$$

$f < f_c$, the mode TE_{11} is not possible.

Therefore, only one mode is possible. That is TE_{10}

29. A rectangular waveguide has dimension 2.5 X 5 cm. Determine the guide wavelength, phase constant and phase velocity at a wavelength of 4.5 cm for the dominant mode.

Solution:

Width of waveguide(a) = 5 cm = 0.05 m
Height of the waveguide(b) = 2.5 cm = 0.025 m
Signal wavelength (λ) = 4.5 cm = 0.045 m
Signal frequency is given by

$$f = \frac{c}{\lambda} = \frac{3\times10^8}{0.045} = 6.66GHz$$

The cutoff frequency for TE_{10} mode is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.05}\right)^2 + \left(\frac{0}{0.025}\right)^2}$$

$$f_c = 30\times10^8 = 3GHz$$

The guided wavelemgth for TE_{10} is given by

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\lambda_g = \frac{0.045}{\sqrt{1-\left(\frac{3\times10^9}{6.66\times10^9}\right)^2}} = 0.05 = 5cm$$

The phase constant is given by

$$\beta = \frac{2\pi}{\lambda}\sqrt{\left(1-\left(\frac{f_c}{f}\right)^2\right)}$$

$$\beta = \frac{2\pi}{0.045}\sqrt{\left(1-\left(\frac{3\times10^9}{6.66\times10^9}\right)^2\right)} = 124.65 rad/m$$

The phase velocity is given by

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{3\times10^8}{\sqrt{1-\left(\frac{3\times10^9}{6.66\times10^9}\right)^2}} = 3.36\times10^8 m/s$$

30. A rectangular waveguide with dimensions of 3 X 2 cm operates in the TM_{11} mode at 10 GHz. Determine the characterisitc wave impedance

Solution:

Width of waveguide(a) = 3 cm = 0.03 m
Height of the waveguide(b) = 2 cm = 0.02 m
Signal frequency (f) = 10 GHz
Signal wavelength is given by

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^9} = 0.03m$$

The cutoff frequency for TM_{11} mode is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.03}\right)^2+\left(\frac{1}{0.02}\right)^2}$$

$$f_c = 90.138\times10^8 = 9.013GHz$$

The characterisitc wave impedance is given by

$$\eta_{TM} = \eta_0\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\eta_{TM} = 120\pi\sqrt{1-\left(\frac{9.013\times10^9}{10\times10^9}\right)^2} = 163.3\Omega$$

31. An air-filled waveguide with a cross section 2 X 1 cm transports energy in the TE_{10} mode at the rate of 0.5 hp. The impressed frequency is 30 GHz. What is the peak value of electric field occuring in the guide.

Solution:

Width of waveguide(a) = 2 cm = 0.02 m
Height of the waveguide(b) = 1 cm = 0.01 m
Signal frequency (f) = 30 GHz
Energy (P) = 0.5 hp = 0.5 X 746 = 373 w

The field components of TE mode for dominant mode TE_{10} are given by

$$E_x = \frac{j\omega\mu}{k^2}A'\frac{n\pi}{b}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} = 0$$

$$E_y = -\frac{j\omega\mu}{k^2}A'\frac{m\pi}{a}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z} = -\frac{j\omega\mu}{k^2}A'\frac{\pi}{a}\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

$$H_x = \frac{j\beta A' m\pi}{k^2 a} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z} = \frac{j\beta A' \pi}{k^2 a} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$

$$H_y = \frac{j\beta A' n\pi}{k^2 b} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z} = 0$$

$$H_z = A' \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z} = 0$$

The equation E_y can be written as

$$E_y = E_{0y} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$

$$H_x = H_{0x} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$

$$-\frac{E_{0y}}{H_{0x}} = \eta_{TE}$$

$$H_{0x} = \frac{-E_{0y}}{\eta_{TE}}$$

$$H_x = \frac{-E_{0y}}{\eta_{TE}} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}$$

The average power from the poynting theorem is given by

$$P = \frac{1}{2}\int \left(E \times H^*\right) ds$$

$$P = \frac{1}{2}\int \left(E_{0y} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} a_y \times \left(\frac{-E_{0y}}{\eta_{TE}} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} \right)^* a_x \right) \cdot dxdya_z$$

$$P = \frac{1}{2}\int \left(E_{0y} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} a_y \times \frac{-E_{0y}}{\eta_{TE}} \sin\left(\frac{\pi x}{a}\right) e^{j\beta z} a_x \right) \cdot dxdya_z$$

$$P = \frac{1}{2}\int \left(E_{0y} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} \cdot \frac{-E_{0y}}{\eta_{TE}} \sin\left(\frac{\pi x}{a}\right) e^{j\beta z} \left(a_y \times a_x\right) \right) \cdot dxdya_z$$

$$P = \frac{1}{2}\int \left(E_{0y} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} \cdot \frac{-E_{0y}}{\eta_{TE}} \sin\left(\frac{\pi x}{a}\right) e^{j\beta z} \left(-a_z\right) \right) \cdot dxdya_z$$

$$P = \frac{1}{2}\int \left(E_{0y} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} \cdot \frac{E_{0y}}{\eta_{TE}} \sin\left(\frac{\pi x}{a}\right) e^{j\beta z} a_z \right) \cdot dxdya_z$$

$$P = \frac{1}{2}\int \left(\frac{E_{0y}^2}{\eta_{TE}} \sin^2\left(\frac{\pi x}{a}\right) a_z \right) \cdot dxdya_z$$

$$P = \frac{1}{2}\int \frac{E_{0y}^2}{\eta_{TE}} \sin^2\left(\frac{\pi x}{a}\right) dxdy$$

$$P = \frac{1}{2}\frac{E_{0y}^2}{\eta_{TE}} \int_0^a \sin^2\left(\frac{\pi x}{a}\right) dx \int_0^b dy$$

$$P = \frac{1}{4}\frac{E_{0y}^2}{\eta_{TE}} \left(a \times b\right)$$

The cutoff frequency for TE_{10} mode is given by

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.02}\right)^2 + \left(\frac{0}{0.01}\right)^2}$$

$$f_c = 75\times10^8 = 7.5GHz$$

The impedance of TE mode is given by

$$\eta_{TE} = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{7.5\times10^9}{30\times10^9}\right)^2}} = 389.46\Omega$$

$$P = \frac{1}{4}\frac{E_{0y}^2}{\eta_{TE}}(a\times b) \Rightarrow 0.5\times746 = \frac{1}{4}\frac{E_{0y}^2}{389.46}(0.02\times0.01)$$

$$E_{0y}^2 = \frac{0.5\times746\times4\times389.46}{0.02\times0.01} = 2905371600$$

$$E_{0y} = \sqrt{2905371600} = 53901.5 = 53.9kw$$

32. A certain microstrip line has the following parameters. ε_r = 5.23, h = 7 mils, t = 2.8 mils, w = 10 mils. Calculate the characteristic impedance Z_0 of the line

Solution:

Relative permitivity (ε_r) = 5.23
Height of the substrate (h) = 7 mils (1 mil = 1/1000 inch)
Thickness of the strip(t) = 2.8 mils
Width of the strip (w) = 10 mils
The characteristic impedance is given by

$$Z_0 = \frac{87}{\sqrt{\varepsilon_r + 1.41}}\ln\left[\frac{5.98h}{0.8w+t}\right]$$

$$Z_0 = \frac{87}{\sqrt{5.23+1.41}}\ln\left[\frac{5.98\times7}{0.8\times10+2.8}\right] = 45.78\Omega$$

UNIT-4 (PASSIVE MICROWAVE DEVICES, MICROWAVE AMPLIFIERS AND OSCILLATORS)

Syllabus: Passive Microwave Devices: Introduction to scattering parameters and their properties, Terminations, Variable short circuit, Attenuators, Phase shifters, Hybrid Tees (H-plane, E-plane, Magic Tees), Hybrid ring, Directional Couplers – Bethe hole and Two hole Couplers, Microwave propagation in Ferrites, Microwave devices employing Faraday rotation – Isolator, Circulator, Deriving Scattering matrix for Microwave passive devices.

Microwave Amplifiers and Oscillators: Microwave Tubes: Linear Beam Tubes – Two cavity Klystron amplifier -velocity modulation, bunching process, output power, Reflex Klystron oscillator, power output and efficiency, Travelling Wave Tube (TWT) – Bunching process and amplification JNTUA B.Tech. R20 Regulations process (**Qualitative treatment only**). Crossed Field Tubes – Magnetron oscillator, pi-mode operation, power output and efficiency, Hartree Condition.

INTRODUCTION TO MICROWAVES

Microwaves are the Electromagnetic waves whose wavelength is in the order of microns. The typical frequency range of microwaves is from 1 GHz to 300 GHz. As we know that the wavelength and frequecny are inversily proportional to each other, the wavelength will be very very small when the frequency is in the order of GHz. James Clerk Maxwell(1831-1879) predicted the exixtence of electromagnetic waves. Heinrich Rudolf Hertz(1857-1937) experimentally confirmed Maxwell's prediction. Guglielmo Marconi(1874-1937) transmitted information on an experimental basis at microwave frequencies.

Advantages of Microwaves:

1. Increased Bandwidth

For example,

in 3G, the frequency is 1.6 GHz-2.0 GHz, Bandwidth is 100 MHz

in 4G, the frequency is 2 GHz-8 GHz, Bandwidth is 100 MHz

in 5G, the frequency is 2 GHz-60 GHz, Bandwidth is order of GHz.

2. Improved directivity
3. Fading effect and reliability
4. Power requirements
5. Transparency property of microwaves.

MICROWAVE SPECTRUM AND BANDS

Electromagnetic spectrum is given by

S.No	Band designation	Frequency range	Wavelength
1	VLF	3-30 KHz	100-10 Km
2	LF	30-300 KHz	10-1 Km
3	MF	300-3000 KHz	1-0.1 Km
4	HF	3-30 MHz	100 -10 m
5	VHF	30-300 MHz	10-1 m
6	UHF	300-3000 MHz	100-10 cm
7	SHF	3-30 GHz	10-1 cm
8	EHF	30-300 GHz	10-1 mm
9	Millimeter	> 300 GHz	< 1 mm

The typical frequency range of microwaves is 1 GHz to 300 GHz. The Microwave frequency bands and their frequency range is shown in the following table:

S.No	Band designation	Frequency range	Wavelength
1	L-Band	1-2 GHZ	30-15 cm
2	S-Band	2-4 GHz	15-7.5 cm

3	C-Band	4-8 GHz	7.5-3.8 cm
4	X-Band	8-12 GHz	3.8-2.5 cm
5	Ku-Band	12-18 GHz	2.5-1.7 cm
6	k-Band	18-27 GHz	1.7-1.1 cm
7	Ka-Band	27-40 GHz	1.1-0.75 cm
8	Millimeter	40-300 GHZ	0.75-0.1 cm
9	Submillimeter	> 300 GHZ	< 0.1 cm

APPLICATIONS OF MICROWAVES

The following is the list of microwave applications:

a) Communication applications:

(i) Telecommunication: Intercontinental Telephone and TV, Space Communication, Telemetry communication link for railways.

(ii) Radar systems: Detect aircrafts, track/guide supersonic missiles, observe and track weather patterns, Air Traffic Control(ATC), Burglar alarms, Garage Door openers, police speed detectors

(iii) Satellite communications

(iv) Terrestrial communications

b) Industrial applications:

i) Microwave Oven
ii) Drying machines-Textile, food and paper industry for drying cloths, potato chips, printed maters.
iii) Food processing industry-Precooling/cooking, pasteurising/sterlity, roasting of food grains/beans.
iv) Plastic industry
v) Rubber industry
vi) Chemical industry
vii) Mining/public works, braking rock, tunnel boring, drying/breaking up concrete, curing of cements.

c) **Medical applications:** Diathermy for localized superficial heating, deep electromagnetic heating for tratment of cancer, electromagnetic waves through the human body will be used for monitoring heart beat, lung water detection.

d) **Agriculture applications:**
Microwaves will be used to change the taste of vegetables by reducing the acidity.

INTRODUCTION TO SCATTERING PARAMETERS AND THEIR PROPERTIES

Significance, Formulation and Properties of s-matrix:

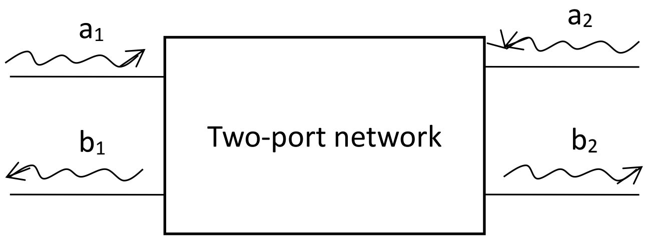
The ordinary parameters such as Z-parameters, Y-parameters, h-parameters, etc cannot be used at microwave frequencies because of the following reasons:

(i) The equipment is not readily available to measure the voltage and current at microwave frequencies.

(ii) Obtaining of open and short circuits at microwave frequencies are difficult.

(iii) The active devices such as tunnel diodes and power transistors will not have stability at open and short circuits.

Due to the above reasons, new parameters called S-parameters or simply S-matrix will be used to analyze the microwave components. S-parameters will be expressed in terms of waves instead of voltages and currents. Consider the simple two port network shown in the following figure.



In above figure a represents the input signals and b represents the reflected or output signals. The relation between these two parameters can be expressed in terms of s-parameters as

$$b_1 = S_{11}\, a_1 + S_{12}\, a_2$$
$$b_2 = S_{21}\, a_1 + S_{22}\, a_2$$

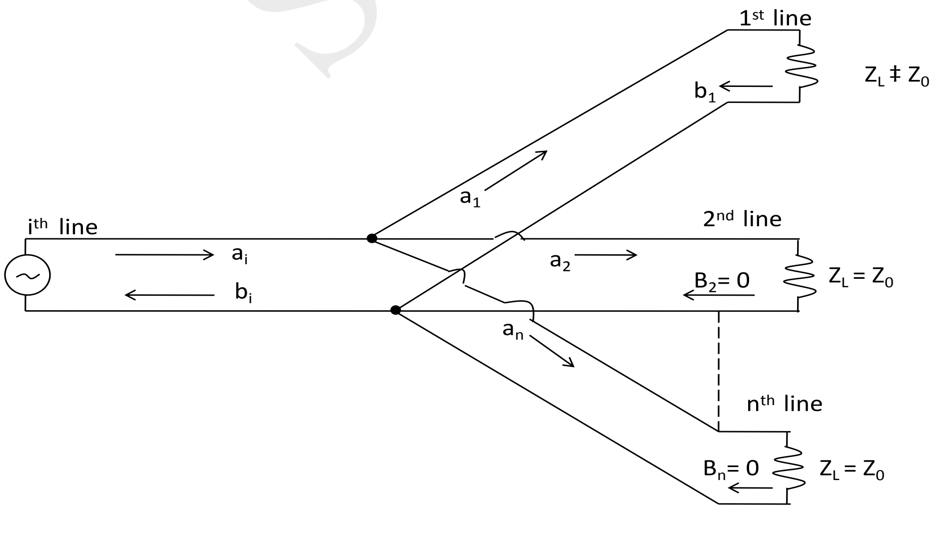
In above equation, S_{11}, S_{12}, S_{21}, and S_{22} represent the reflection coefficients.

In order to form the S-Matrix, consider the following figure. To obtain the relationship between the scattering matrix and the input/output powers at different ports, consider a junction of 'n' number of transmission lines wherein the i^{th} line (i can be any one line from 1 to n) is connected a source as shown in figure.

Let the first line be terminated in an impedance other than the characteristic impedance ($Z_L \neq Z_0$) and all the remaining lines in an impedance equal to Z_0. If 'a_i' be the incident wave at the junction due to source at the i^{th} line, then it divides itself among n-1 number of lines as a_1, a_2, a_3,…… a_n as shown in figure. There will be no reflections from 2^{nd} to n^{th} line and all incident waves absorbed due to impedance matching. But there is a reflected signal (b_1) from first line due to impedance mismatch and this reflected is going back into the junction. The reflected signal b_1 is related to a_1 by

$$b_1 = reflection\, coefficient\ \times a_1 = S_{i1} a_1$$

Where S_{i1} is the reflection coefficient of first line.



Hence the contribution to the outward traveling wave in the i^{th} line is given by

$$b_i = S_{i1} \cdot a_1 \quad [Because\ b_2 = b_3 = \dots .\, b_n = 0$$

Now let all the n-1 lines are terminated in an impedance not equal to Z_0. Then there will be reflections from all the n-1 lines into the junction and hence total contribution to the outward wave in the i^{th} line is given by

$$b_i = S_{i1} \cdot a_1 + S_{i2} \cdot a_2 + S_{i3} \cdot a_3 + \ldots + S_{in} \cdot a_n$$

The line 'i' can be any one from 1 to n, then

$$b_1 = S_{11} \cdot a_1 + S_{12} \cdot a_2 + S_{13} \cdot a_3 + \ldots + S_{1n} \cdot a_n$$
$$b_2 = S_{21} \cdot a_1 + S_{22} \cdot a_2 + S_{23} \cdot a_3 + \ldots + S_{2n} \cdot a_n$$
$$b_3 = S_{31} \cdot a_1 + S_{32} \cdot a_2 + S_{33} \cdot a_3 + \ldots + S_{3n} \cdot a_n$$

. . . .

. . . .

. .

$$b_n = S_{n1} \cdot a_1 + S_{n2} \cdot a_2 + S_{n3} \cdot a_3 + \ldots + S_{nn} \cdot a_n$$

In matrix form,

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \cdot \\ \cdot \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \cdot & \cdot & S_{1n} \\ S_{21} & S_{22} & S_{23} & \cdot & \cdot & S_{2n} \\ S_{31} & S_{32} & S_{33} & \cdot & \cdot & S_{3n} \\ \cdot & \cdot & \cdot & & \cdot \; \cdot & \cdot \\ \cdot & \cdot & \cdot & & & \\ S_{n1} & S_{n2} & S_{n3} & \cdot & \cdot & S_{nn} \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \cdot \\ \cdot \\ a_n \end{bmatrix}$$

Outputs = S-Matrix X Inputs

Properties of S-Matrix:

The properties of S-matrix are

(i) S-Matrix is a square matrix of order n x n

(ii) It is a symmetric matrix. i.e $S_{ij} = S_{ji}$

(iii) It is a unitary matrix. i.e $[S][S]^* = [I]$
Where $[S]^*$ is complex conjugate of [S] and [I] is a Identity matrix of same order of [S].

(iv) The sum of the products of each term of any row or column multiplied by the complex conjugate of the corresponding terms of any other row or column is zero. i.e.

$$\sum_{i=1}^{n} S_{ik} S_{ij}^* = 0 \qquad for\ k \neq j$$

(v) If any of the terminal or reference plane (say k^{th} port) are moved away the junction by an electric distance $\beta_k l_k$, each of the coefficients S_{ij} involving k will be multiplied by the factor $e^{-j\beta_k l_k}$.

TERMINATIONS (MATCHED LOADS)

Matched loads or matched terminations are used for impedance matching purpose. That is when the matched loads are connected in the waveguide, there will be no reflections. There are different types of matched loads are there such as using of graphite sand in the waveguide as the termination, using of resistive rod at the end of the waveguide, etc. These two matched loads are shown in the following figures.

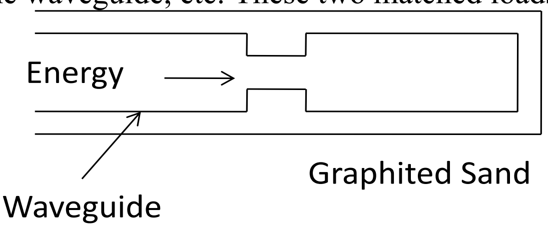


Fig: Matched load with graphited sand

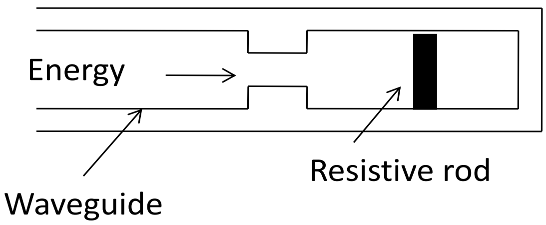
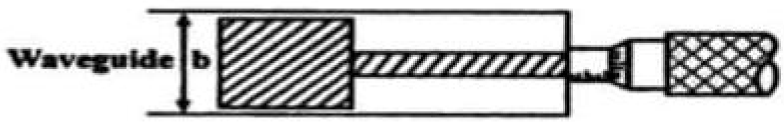


Fig: Matched load with resistive rod

VARIABLE SHORT CIRCUIT

Waveguide short-circuit terminations provide standard reflection at desired, precisely measurable position. The basic idea behind it is to provide short-circuit by changing the reactance of the termination. The simplest form of adjustable waveguide short-circuit is shown in the following figure.



It consists of a sliding block of a good conductor (such as copper) which makes as snug fit in the waveguide. The position of the block is varied by means of a micrometer drive.

ATTENUATORS

Resistive card attenuator:

Attenuator will be used to reduce the signal strength. The basic principle involved in attenuators is absorbing the signal by using the absorbing materials such as carbon film or aquadog. The structure of resistive card attenuator is shown in the following figure.

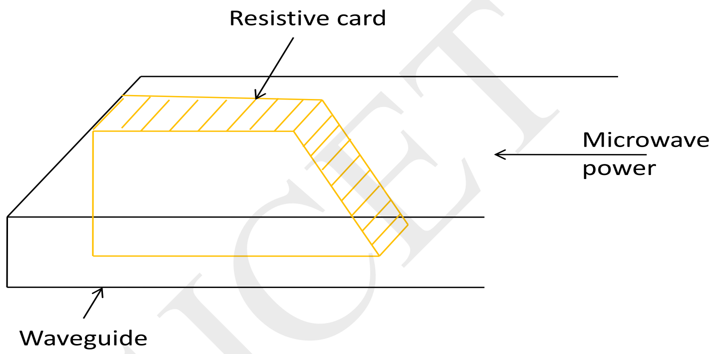


Fig: Fixed Resistive card attenuator

A resistive card coated with carbon film or aquadog is inserted into the waveguide. The card is inserted at the center of broader wall of the waveguide. When the signal is incident on the card, the signal strength will be absorbed. The variable resistive card attenuator is shown in the following figure.

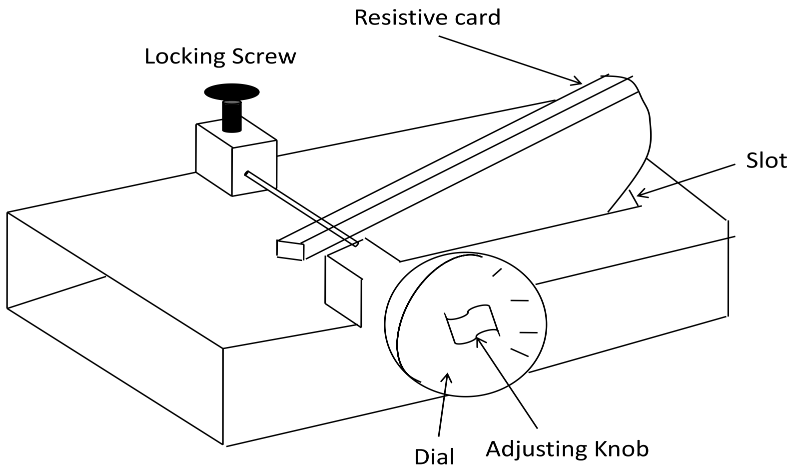


Fig: Variable Resistive card attenuator

The operation of variable resistive card attenuator can be explained as follows:
When the adjusting knob is rotated in anticlock wise direction, then the resistive card will be outoff the waveguide. At this position, the attenuation will be minimum. When the knob is go on rotating in clock wise direction, the resistive card will be go on inserting into the waveguide and hence the attenuation is go on increases. Once the knob is fully clock wise direction then the attenuation is maximum.

Rotary vane attenuators

The rotary vane attenuator will be used in all the practical applications. The rotary vane attenuator is a precession attenuator. The values measured by the rotary vane attenuator are accurate. The structure of rotary vane attenuator is shown in the following figure.

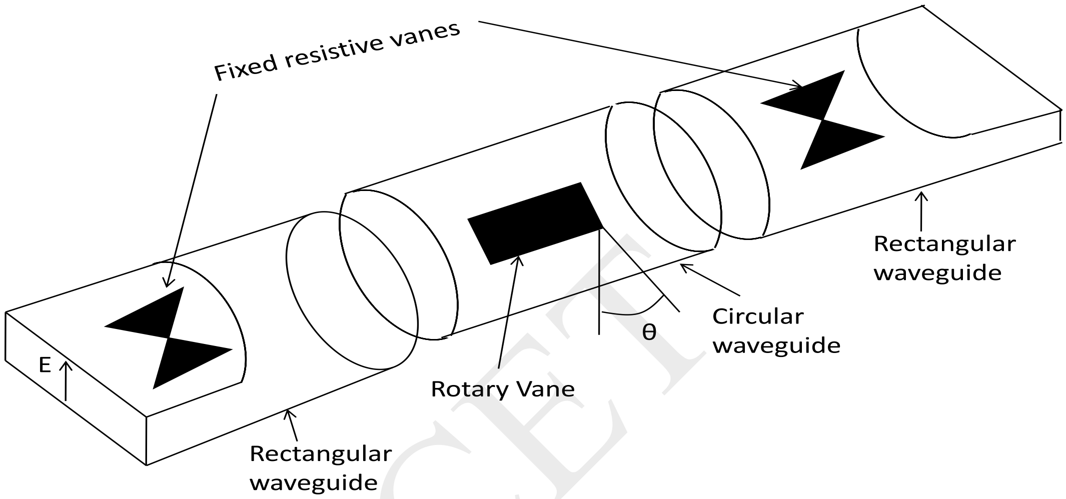


Fig: Rotary Vane Attenuator

It consists of two rectangular waveguide sections and one rotatable circular waveguide section. When the signal is applied at the port-1, it will pass through the first fixed resistive vane without any attenuation. When this signal is passed through the rotary vane, its E sinθ component will be attenuated and E cosθ component will be passed to the input of the second fixed resistive card. Again E cosθ sinθ component will be attenuated by the second fixed resistive card and E $\cos^2\theta$ component will be available as the output at the second port. Therefore the attenuation equal to 20 log $\cos^2\theta$ = 40 log cosθ.

PHASE SHIFTERS

Dielectric phase shifter:

$$\beta = \frac{2\pi}{\lambda}, \quad \lambda = \frac{v}{f}$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}}$$

A phase shifter is a microwave component which will introduce a certain amount of phase shift to the input signal. The basic principle involved in phase shifter is, changing the dielectric medium inside the guide by inserting some dielectric slab. When the slab is inserted into the waveguide the medium of wave propagation is varied and hence the phase constant of the signal changes. Due to the change of phase constant, the phase of the signal will change. The structure of dielectric phase shifter is shown in the following figure.

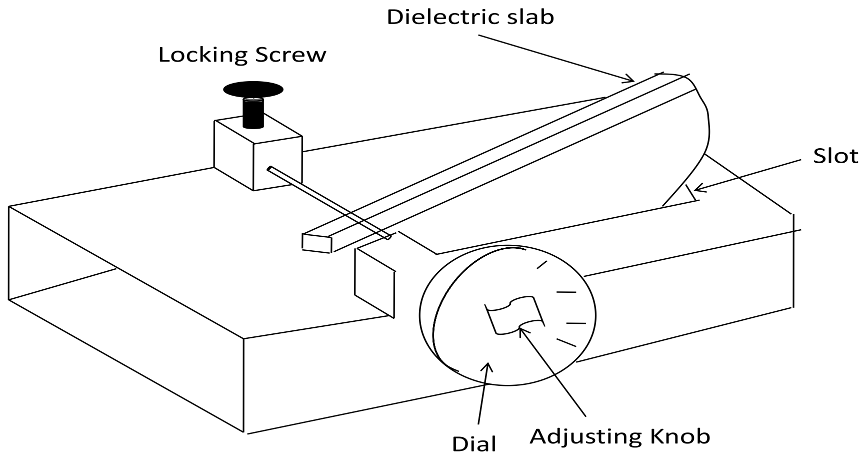


Fig: Dielectric phase shifter

The operation of dielectric phase shifter can be explained as follows:
When the adjusting knob is rotated in anticlock wise direction, then the dielectric slab will be outoff the waveguide. At this position, the phase shift will be minimum. When the knob is go on rotating in clock wise direction, the dielectric slab will be go on inserting into the waveguide and hence the phase shift is go on increases. Once the knob is fully clock wise direction then the phase shift is maximum.

<u>Rotary vane phase shifters:</u>

The rotary vane phase shifter is shown in the following figure. It consists of three dielectric vanes (two are the fixed vanes and one is the rotary vane). It also consists of two rectangular waveguide sections and one rotatable circular waveguide section. When the signal is applied to the port one of the device, it will not attenuated by the first fixed dielectric vane because the direction of E is perpendicular with respect to the vane. This signal will be appeared as the input at the rotary vane. Now the signal will be attenuated by the rotary vane because of the rotation. Finally some of the input signal will be available as the output at the second port.

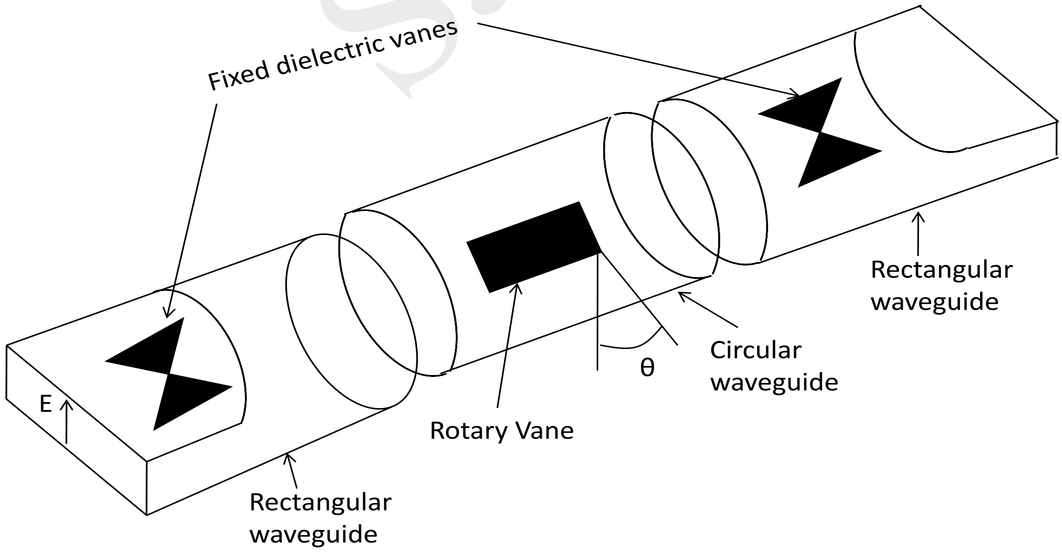


Fig: Rotary Vane phase shifter

HYBRID TEES

H-Plane Tee:

H-plane Tee is three port waveguide junction. The structure of H-plane Tee is shown in the following figure. H-plane Tee is also called as shunt Tee because; the axis of the side arm is in shunt or in perpendicular with the electric field in the main waveguide. The main properties of H-plane Tee are given by

(i) When the input is applied at the port-3, it divides between the ports 1 & 2 with equal amplitudes and same phase.
(ii) When the two collinear ports are supplied with inputs of equal amplitudes and same phase, the sum of these two inputs available as the output at the port-3.
(iii) When the two collinear ports are supplied with inputs of equal amplitudes and opposite phase, the difference of these two input amplitudes (i.e. zero) available as the output at the port-3.
(iv) In H-plane Tee H-arm is also called as the sum arm.

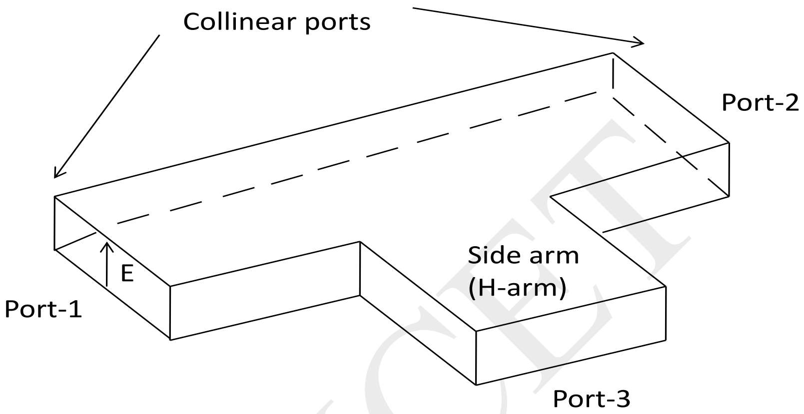


Fig: H-Plane Tee

Its S-Matrix will be derived as follows:
The general S-matrix for three port device can be written as

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \qquad --1$$

From the plane symmetry

$$S_{23} = S_{13} \qquad -(2)$$

When the port-3 is perfectly matched to the junction then

$$S_{33} = 0 \qquad -(3)$$

Form the symmetry property of s-matrix we can write

$$S_{12} = S_{21}, S_{13} = S_{31}, S_{23} = S_{32} \qquad -(4)$$

Substitute equations 2, 3, 4 in equation1

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \qquad -(5)$$

Form the unitary property of s-matrix we can write

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1C_1 \Rightarrow \qquad |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \qquad \text{- (6)}$$

$$R_2C_2 \Rightarrow \qquad |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \qquad \text{- (7)}$$

$$R_1C_1 \Rightarrow \qquad |S_{13}|^2 + |S_{13}|^2 = 1 \qquad \text{- (8)}$$

From the zero property we can write

$$R_3C_1 \Rightarrow \qquad S_{13}.S_{11}^* + S_{13}.S_{12}^* = 0 \qquad \text{- (9)}$$

From Equation 8, we can have

$$2|S_{13}|^2 = 1$$

$$|S_{13}|^2 = \frac{1}{2}$$

$$S_{13} = \frac{1}{\sqrt{2}} \qquad \text{- (10)}$$

By Comparing equations 6 and 7, we can write

$$S_{11} = S_{22} \qquad \text{- (11)}$$

From equation 9, we can have

$$S_{13}\left(S_{11}^* + S_{12}^*\right) = 0$$

To satisfy the above relation, we can have

$$S_{11}^* + S_{12}^* = 0$$

$$S_{11} = -S_{12} \qquad \text{- (12)}$$

Substitute equations 10 and 12 in equation 6

$$|S_{11}|^2 + |S_{11}|^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$2|S_{11}|^2 + \frac{1}{2} = 1$$

$$2|S_{11}|^2 = \frac{1}{2}$$

$$|S_{11}|^2 = \frac{1}{4}$$

$$S_{11} = \frac{1}{2} \qquad \text{- (13)}$$

Substitute equation 13 in equation 11

$$S_{22} = S_{11} = \frac{1}{2} \qquad \text{- (14)}$$

Substitute equation 13 in equation 12

$$S_{12} = -S_{11} = -\frac{1}{2} \qquad \text{- (15)}$$

Substitute equations 10, 14 and 15 in equation 5

$$[S] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$b_1 = \frac{1}{2}a_1 - \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3 \qquad \text{- (16)}$$

$$b_2 = -\frac{1}{2}a_1 + \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3 \qquad \text{- (17)}$$

$$b_3 = \frac{1}{\sqrt{2}}a_1 + \frac{1}{\sqrt{2}}a_2 \qquad \text{- (18)}$$

Case-1: Input applied at port-3($a_1 = 0, a_2 = 0, a_3 \neq 0$ **)**

$$b_1 = \frac{1}{2}(0) - \frac{1}{2}(0) + \frac{1}{\sqrt{2}}a_3 = \frac{1}{\sqrt{2}}a_3$$

$$b_2 = -\frac{1}{2}(0) + \frac{1}{2}(0) + \frac{1}{\sqrt{2}}a_3 = \frac{1}{\sqrt{2}}a_3$$

$$b_3 = \frac{1}{\sqrt{2}}(0) + \frac{1}{\sqrt{2}}(0) = 0$$

Case-2: Input applied at port-1 and port-2 with equal amplitude and same phase ($a_1 = a_2 = a, a_3 = 0$ **)**

$$b_1 = \frac{1}{2}a - \frac{1}{2}a + \frac{1}{\sqrt{2}}(0) = 0$$

$$b_2 = -\frac{1}{2}a + \frac{1}{2}a + \frac{1}{\sqrt{2}}(0) = 0$$

$$b_3 = \frac{1}{\sqrt{2}}a + \frac{1}{\sqrt{2}}a = \frac{2a}{\sqrt{2}}$$

Case-3: Input applied at port-1 and port-2 with equal amplitude and opposite phase ($a_1 = a, a_2 = -a, a_3 = 0$ **)**

$$b_1 = \frac{1}{2}a + \frac{1}{2}a + \frac{1}{\sqrt{2}}(0) = a$$

$$b_2 = -\frac{1}{2}a - \frac{1}{2}a + \frac{1}{\sqrt{2}}(0) = -a$$

$$b_3 = \frac{1}{\sqrt{2}}a - \frac{1}{\sqrt{2}}a = 0$$

E-Plane Tee:

E-plane Tee is three port waveguide junction. The structure of E-plane Tee is shown in the following figure. E-plane Tee is also called as series Tee because the axis of the side arm is in parallel or in series with the electric field in the main waveguide. The main properties of E-plane Tee are given by

(i) When the input is applied at the port-3, it divides between the ports 1 & 2 with equal amplitudes and opposite phase.

(ii) When the two collinear ports are supplied with inputs of equal amplitudes and same phase, the difference of these two inputs (i.e. zero) available as the output at the port-3.

(iii) When the two collinear ports are supplied with inputs of equal amplitudes and opposite phase, the sum of these two inputs available as the output at the port-3.

(iv) In E-plane Tee E-arm is also called as the subtractive arm.

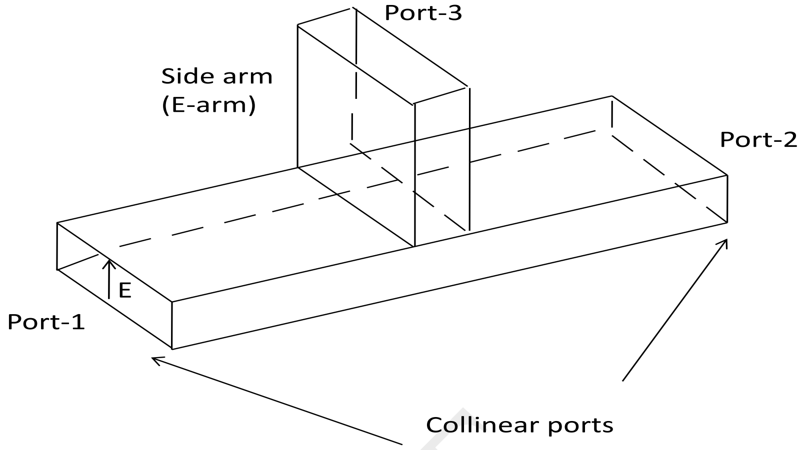
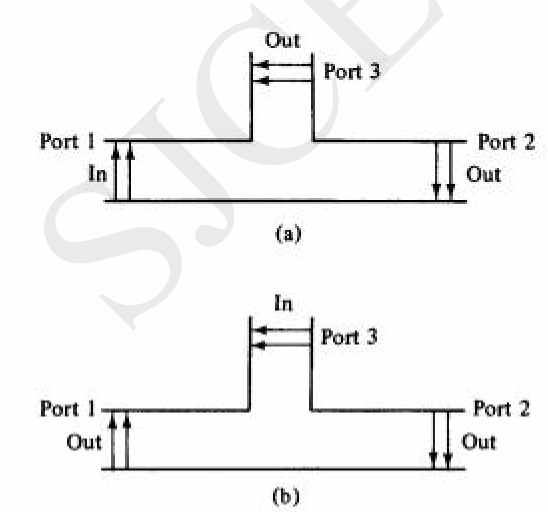


Fig: E-Plane Tee



The structure of E-Plane Tee is shown in the figure above. Its S-Matrix will be derived as follows:
The general S-matrix for three port device can be written as

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \qquad -- 1$$

From the plane symmetry

$$S_{23} = -S_{13} \qquad - (2)$$

When the port-3 is perfectly matched to the junction then

$$S_{33} = 0 \qquad - (3)$$

Form the symmetry property of s-matrix we can write

$$S_{12} = S_{21}, S_{13} = S_{31}, S_{23} = S_{32} \qquad - (4)$$

Substitute equations 2, 3, 4 in equation1

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \qquad - (5)$$

Form the unitary property of s-matrix we can write

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1C_1 \Rightarrow \quad |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \qquad - (6)$

$R_2C_2 \Rightarrow \quad |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \qquad - (7)$

$R_1C_1 \Rightarrow \quad |S_{13}|^2 + |S_{13}|^2 = 1 \qquad - (8)$

From the zero property we can write

$R_3C_1 \Rightarrow \quad S_{13}S_{11}^* - S_{13}S_{12}^* = 0 \qquad - (9)$

From Equation 8, we can have

$$2|S_{13}|^2 = 1$$

$$|S_{13}|^2 = \frac{1}{2}$$

$$S_{13} = \frac{1}{\sqrt{2}} \qquad - (10)$$

By Comparing equations 6 and 7, we can write

$$S_{11} = S_{22} \qquad - (11)$$

From equation 9, we can have

$$S_{13}\left(S_{11}^* - S_{12}^*\right) = 0$$

To satisfy the above relation, we can have

$$S_{11}^* - S_{12}^* = 0$$

$$S_{11} = S_{12} \qquad - (12)$$

Substitute equations 10 and 12 in equation 6

$$|S_{11}|^2 + |S_{11}|^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$2|S_{11}|^2 + \frac{1}{2} = 1$$

$$2|S_{11}|^2 = \frac{1}{2}$$

$$|S_{11}|^2 = \frac{1}{4}$$

$$S_{11} = \frac{1}{2} \qquad - (13)$$

Substitute equation 13 in equation 11

$$S_{22} = S_{11} = \frac{1}{2} \qquad - (14)$$

Substitute equation 13 in equation 12

$$S_{12} = S_{11} = \frac{1}{2} \qquad - (15)$$

Substitute equations 10, 14 and 15 in equation 5

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$b_1 = \frac{1}{2}a_1 + \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3 \qquad - (16)$$

$$b_2 = \frac{1}{2}a_1 + \frac{1}{2}a_2 - \frac{1}{\sqrt{2}}a_3 \qquad - (17)$$

$$b_3 = \frac{1}{\sqrt{2}}a_1 - \frac{1}{\sqrt{2}}a_2 \qquad - (18)$$

Case-1: Input applied at port-3($a_1 = 0, a_2 = 0, a_3 \neq 0$ **)**

$$b_1 = \frac{1}{2}(0) + \frac{1}{2}(0) + \frac{1}{\sqrt{2}}a_3 = \frac{1}{\sqrt{2}}a_3$$

$$b_2 = \frac{1}{2}(0) + \frac{1}{2}(0) - \frac{1}{\sqrt{2}}a_3 = -\frac{1}{\sqrt{2}}a_3$$

$$b_3 = \frac{1}{\sqrt{2}}(0) - \frac{1}{\sqrt{2}}(0) = 0$$

Case-2: Input applied at port-1 and port-2 with equal amplitude and same phase ($a_1 = a_2 = a, a_3 = 0$ **)**

$$b_1 = \frac{1}{2}a + \frac{1}{2}a + \frac{1}{\sqrt{2}}(0) = a$$

$$b_2 = \frac{1}{2}a + \frac{1}{2}a - \frac{1}{\sqrt{2}}(0) = a$$

$$b_3 = \frac{1}{\sqrt{2}}a - \frac{1}{\sqrt{2}}a = 0$$

Case-3: Input applied at port-1 and port-2 with equal amplitude and opposite phase ($a_1 = a, a_2 = -a, a_3 = 0$ **)**

$$b_1 = \frac{1}{2}a - \frac{1}{2}a + \frac{1}{\sqrt{2}}(0) = 0$$

$$b_2 = \frac{1}{2}a - \frac{1}{2}a - \frac{1}{\sqrt{2}}(0) = 0$$

$$b_3 = \frac{1}{\sqrt{2}}a + \frac{1}{\sqrt{2}}a = \frac{2a}{\sqrt{2}}$$

Magic Tee:

Magic Tee is four port waveguide junction. The structure of magic Tee is shown in the following figure.

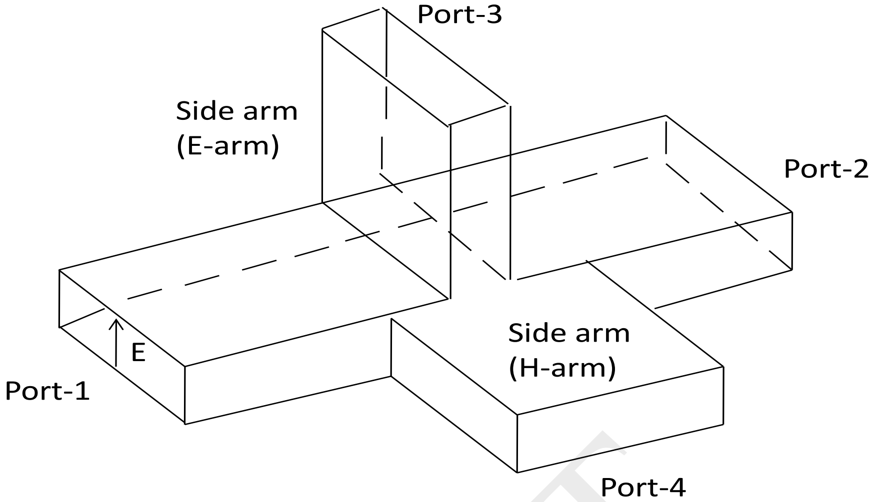


Fig: Magic-Tee

The magic Tee name arises because; when the input applied at any one of the collinear ports it will not available at the other collinear port even though they are the collinear. The magic is a combination of both E-plane Tee and H-plane Tee. The main properties of H-plane Tee are given by

(i) When the input is applied at the port-3, it divides between the ports 1 & 2 with equal amplitudes and opposite phase.
(ii) When the input is applied at the port-4, it divides between the ports 1 & 2 with equal amplitudes and same phase.
(iii) When the two collinear ports are supplied with inputs of equal amplitudes and same phase, the sum of these two inputs available as the output at the port-4 and zero output at port-3.
(iv) When the two collinear ports are supplied with inputs of equal amplitudes and opposite phase, the sum of these two input amplitudes available as the output at the port-3 and zero output at port-4.

The structure of Magic-Tee is shown in the figure above. Let port-4 is the E-arm and port-3 is the H-arm. From the properties of s-matrix and by utilizing the plane symmetry of magic tee we can derive the s-matrix. S-matrix of magic tee is derived as follows:

The general s-matrix of four port device is given by

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

From the plane symmetry

$$S_{23} = S_{13}$$
$$S_{24} = -S_{14}$$

Ports 3&4 are isolated and hence $S_{34} = S_{43} = 0$

From symmetry property of s-matrix we can write as

$$S_{12} = S_{21}, S_{13} = S_{31}, S_{23} = S_{32}, S_{34} = S_{43}, S_{24} = S_{42}, S_{41} = S_{14}$$

When the ports 3 and 4 are perfectly matched to the junction, then

$$S_{33} = S_{44} = 0$$

By applying all the above properties, we can write the s-matrix as

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \qquad \text{- (1)}$$

From the unitary property of s-matrix we can write as

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \qquad \text{-(2)}$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \qquad \text{- (3)}$$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \qquad \text{- (4)}$$

$$|S_{14}|^2 + |S_{14}|^2 = 1 \qquad \text{- (5)}$$

Also from zero property of s-matrix we can write

$$S_{14}.S_{11}^* - S_{14}S_{12}^* = 0 \qquad \text{- (6)}$$

From the equation 4, we can have

$$2|S_{13}|^2 = 1$$

$$S_{13} = \frac{1}{\sqrt{2}} \qquad \text{- (7)}$$

From the equation 5, we can have

$$2|S_{14}|^2 = 1$$

$$S_{14} = \frac{1}{\sqrt{2}} \qquad \text{- (8)}$$

By comparing equations 2 and 3, we can have

$$S_{11} = S_{22} \qquad \text{- (9)}$$

From equation 6,

$$S_{14}\left(S_{11}^* - S_{12}^*\right) = 0$$

$$S_{11} - S_{12} = 0$$

$$S_{11} = S_{12} \qquad \text{-(10)}$$

Substitute equations 7, 8 and 10 in equation 2

$$|S_{11}|^2 + |S_{11}|^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$2|S_{11}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$2|S_{11}|^2 + 1 = 1$$

$$2|S_{11}|^2 = 0$$

$$S_{11} = 0 \qquad \text{-(11)}$$

From equations 9, 10 and 11,

$$S_{11} = S_{12} = S_{22} = 0 \qquad - (12)$$

Substitute equations 7, 8 and 12 in equation 1

$$[S] = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$b_1 = \frac{1}{\sqrt{2}} a_3 + \frac{1}{\sqrt{2}} a_4$$

$$b_2 = \frac{1}{\sqrt{2}} a_3 - \frac{1}{\sqrt{2}} a_4$$

$$b_3 = \frac{1}{\sqrt{2}} a_1 + \frac{1}{\sqrt{2}} a_2$$

$$b_4 = \frac{1}{\sqrt{2}} a_1 - \frac{1}{\sqrt{2}} a_2$$

Case-1: When $a_3 \neq 0, a_1 = a_2 = a_4 = 0$ (Input applied at port 3)

$$b_1 = \frac{1}{\sqrt{2}} a_3 + \frac{1}{\sqrt{2}} a_4 = \frac{1}{\sqrt{2}} a_3$$

$$b_2 = \frac{1}{\sqrt{2}} a_3 - \frac{1}{\sqrt{2}} a_4 = \frac{1}{\sqrt{2}} a_3$$

$$b_3 = \frac{1}{\sqrt{2}} a_1 + \frac{1}{\sqrt{2}} a_2 = 0$$

$$b_4 = \frac{1}{\sqrt{2}} a_1 - \frac{1}{\sqrt{2}} a_2 = 0$$

Case-2: When $a_4 \neq 0, a_1 = a_2 = a_3 = 0$ (Input applied at port 4)

$$b_1 = \frac{1}{\sqrt{2}} a_3 + \frac{1}{\sqrt{2}} a_4 = \frac{1}{\sqrt{2}} a_4$$

$$b_2 = \frac{1}{\sqrt{2}} a_3 - \frac{1}{\sqrt{2}} a_4 = -\frac{1}{\sqrt{2}} a_4$$

$$b_3 = \frac{1}{\sqrt{2}} a_1 + \frac{1}{\sqrt{2}} a_2 = 0$$

$$b_4 = \frac{1}{\sqrt{2}}a_1 - \frac{1}{\sqrt{2}}a_2 = 0$$

Case-3: When $a_1 = a_2 = a, a_3 = a_4 = 0$ (Input applied at port 1 and port 2 with equal amplitudes and same phase)

$$b_1 = \frac{1}{\sqrt{2}}a_3 + \frac{1}{\sqrt{2}}a_4 = 0$$

$$b_2 = \frac{1}{\sqrt{2}}a_3 - \frac{1}{\sqrt{2}}a_4 = 0$$

$$b_3 = \frac{1}{\sqrt{2}}a_1 + \frac{1}{\sqrt{2}}a_2 = \frac{1}{\sqrt{2}}a + \frac{1}{\sqrt{2}}a = \frac{2a}{\sqrt{2}}$$

$$b_4 = \frac{1}{\sqrt{2}}a - \frac{1}{\sqrt{2}}a = 0$$

Case-4: When $a_1 = a, a_2 = -a, a_3 = a_4 = 0$ (Input applied at port 1 and port 2 with equal amplitudes and opposite phase)

$$b_1 = \frac{1}{\sqrt{2}}a_3 + \frac{1}{\sqrt{2}}a_4 = 0$$

$$b_2 = \frac{1}{\sqrt{2}}a_3 - \frac{1}{\sqrt{2}}a_4 = 0$$

$$b_3 = \frac{1}{\sqrt{2}}a_1 + \frac{1}{\sqrt{2}}a_2 = \frac{1}{\sqrt{2}}a - \frac{1}{\sqrt{2}}a = 0$$

$$b_4 = \frac{1}{\sqrt{2}}a - \frac{1}{\sqrt{2}}(-a) = \frac{1}{\sqrt{2}}a + \frac{1}{\sqrt{2}}a = \frac{2a}{\sqrt{2}}$$

Case-5: When $a_1 \neq 0, a_2 = a_3 = a_4 = 0$ (Input applied at port 1)

$$b_1 = \frac{1}{\sqrt{2}}a_3 + \frac{1}{\sqrt{2}}a_4 = 0$$

$$b_2 = \frac{1}{\sqrt{2}}a_3 - \frac{1}{\sqrt{2}}a_4 = 0$$

$$b_3 = \frac{1}{\sqrt{2}}a_1 + \frac{1}{\sqrt{2}}a_2 = \frac{1}{\sqrt{2}}a_1$$

$$b_4 = \frac{1}{\sqrt{2}}a_1 - \frac{1}{\sqrt{2}}a_2 = \frac{1}{\sqrt{2}}a_1$$

Case-6: When $a_2 \neq 0, a_1 = a_3 = a_4 = 0$ (Input applied at port 2)

$$b_1 = \frac{1}{\sqrt{2}}a_3 + \frac{1}{\sqrt{2}}a_4 = 0$$

$$b_2 = \frac{1}{\sqrt{2}}a_3 - \frac{1}{\sqrt{2}}a_4 = 0$$

$$b_3 = \frac{1}{\sqrt{2}}a_1 + \frac{1}{\sqrt{2}}a_2 = \frac{1}{\sqrt{2}}a_2$$

$$b_4 = \frac{1}{\sqrt{2}}a_1 - \frac{1}{\sqrt{2}}a_2 = -\frac{1}{\sqrt{2}}a_2$$

Applications of magic-Tee

Magic-T as duplexer: Magic-T can be used as a duplexer as shown in figure below.

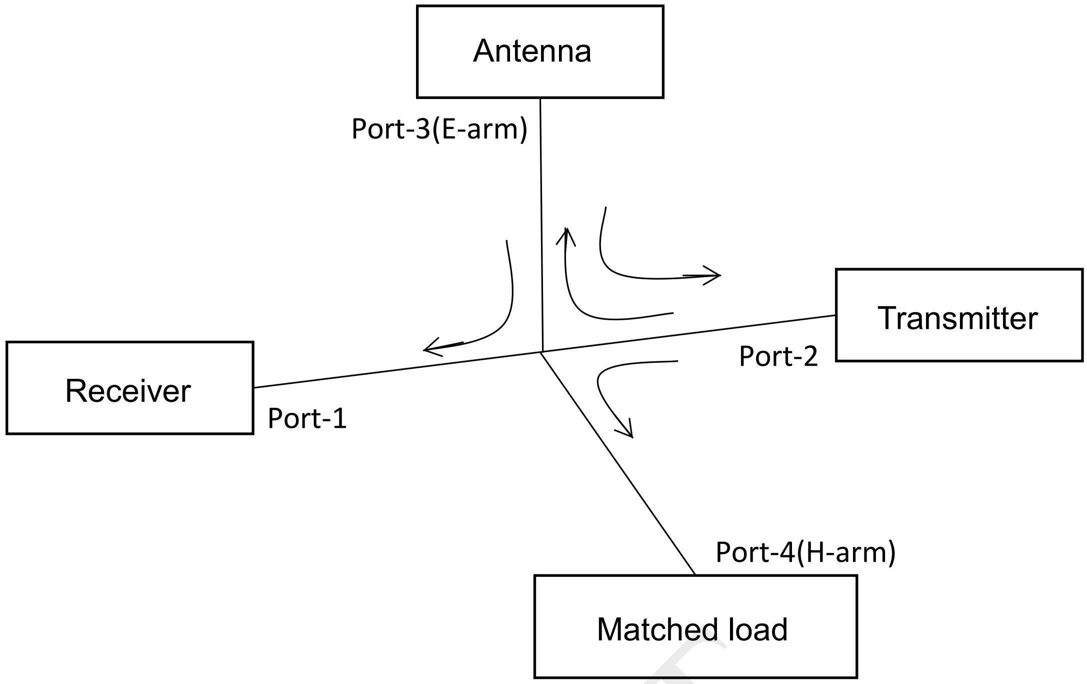


Fig: Magic-T as duplexer

The transmitter is connected at port-2, receiver is connected at port-1, antenna is connected at port-3(E-arm) and port-4(H-arm) is terminated with matched load. During the transmission, the signal generated by the transmitter will act as input at port of the magic-T and this transmitter power will be splitted and travels towards port-3 and port-4. The power traveling towards port-3 will be radiated with the antenna towards the target and power traveling towards port-4 will be absorbed by the matched load. Similarly, during the reception, the power received by th antenna will act as input at the port-3. This power will be splitted and travels towards port-1 and port-2. The power traveling towards port-1 will be received by the receiver of the radar system.

Measurement of impedance:

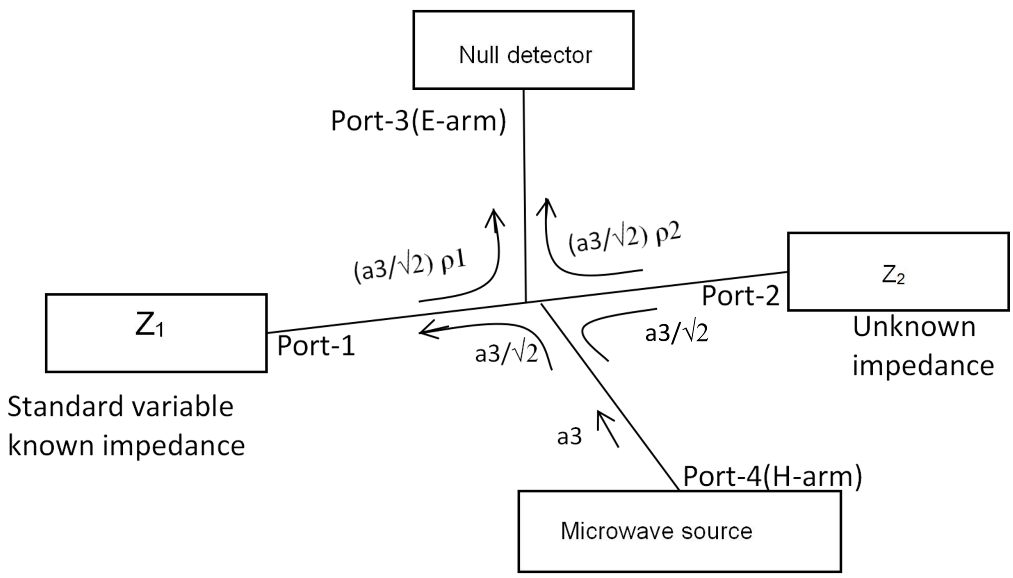


Fig: Magic-T for impedance measurement

The microwave signal generated by microwave source is devided between port 1 and 2 with equal amplitudes and same phase. When the signal $a_3/\sqrt{2}$ is incident at load Z_1, then some of the signal will be rflected back. This reflected power is ($a_3/\sqrt{2})\rho_1$. Similarly there is a reflected signal ($a_3/\sqrt{2})\rho_2$ from the load Z_2. These two reflected powers will act as inputs at the port 1 and port 2 of magic-T. Difference of these two signals will appear at the input of null detector. The null detector will show the value which will be equal to difference of two powers ($a_3/\sqrt{2})\rho_1$ and ($a_3/\sqrt{2})\rho_2$. We have to adjust the known impedance Z_2 such that the null detector shows zero valus. Zero value of the null detector indicates that two powers will be equal. That is

$$\frac{a_3}{\sqrt{2}}\rho_1 = \frac{a_3}{\sqrt{2}}\rho_2$$

$$\rho_1 = \rho_2$$

$$\frac{Z_1 - Z_g}{Z_1 + Z_g} = \frac{Z_2 - Z_g}{Z_2 + Z_g}$$

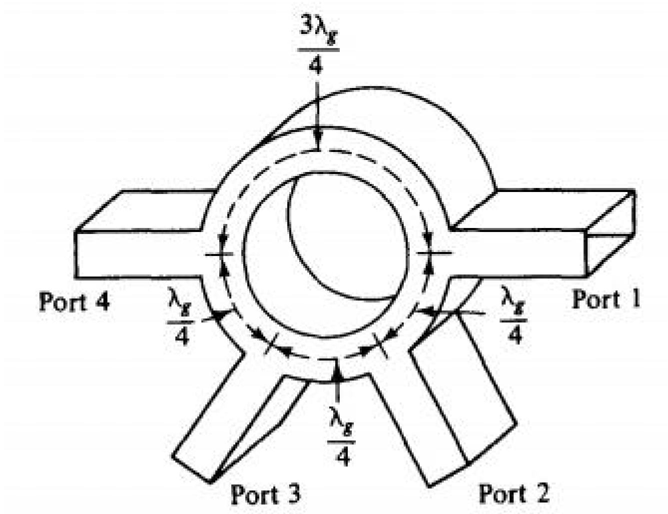
From the above equation the unknown impedance Z_2 can be obtained.

Magic-T as a mixer:

From the above figure, the received signal from the antenna and the signal from local oscillator will act as inputs at port 3 and port 4 respectively. As per the property of magic-T, some of these two signals will be traveled towards port-1 where the mixer is connected. The mixer mixup these two signals and produces corresponding IF signal.

HYBRID RING (RAT-RACE)

A hybrid ring consists of an annular line of proper electrical length to sustain standing waves, to which four arms are connected at proper intervals by means of series or parallel junctions.Following figure shows a hybrid ring with series junctions.



The hybrid ring has characteristics similar to those of the hybrid tee. When a wave is fed into port 1, it will not appear at port 3 because the difference of phase shifts for the waves traveling in the clockwise and counterclockwise directions is 180°. Thus the waves are canceled at port 3. For the same reason, the waves fed into port 2 will not emerge at port 4 and so on.

The S matrix for an ideal hybrid ring can be expressed as

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

DIRECTIONAL COUPLERS

A directional coupler is a four port junction made from two waveguides. The basic principle of directional coupler can be explained with the help of diagram shown in the following figure.

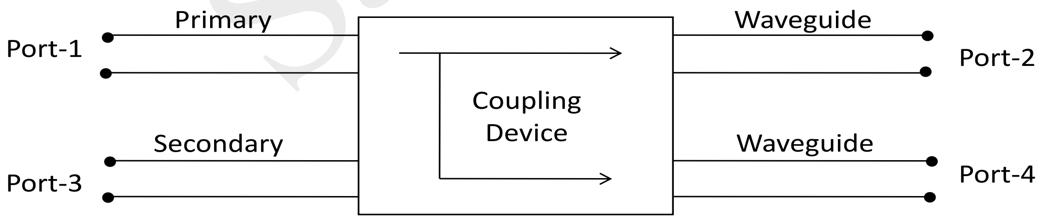


Fig: Directional Coupler

It consists of two waveguides called primary waveguide and secondary waveguide. The directional coupler is a reciprocal device. When the power is applied at the port-1, it divides between ports-2 & 4. But no power available at the port-3. Similarly when the power is applied at the port-2, it divides between ports-1 & 3. But, no power available at the port-4. The important characteristics of directional coupler are

$$\text{Coupling factor(C)} = 10 \log (P_i/P_f)$$

$$\text{Directivity (D)} = 10 \log (P_f/P_b)$$

Where P_i is the input power, P_f is the forward coupled power to the secondary waveguide and P_b is the back power.

There are different types of directional couplers such as four-hole directional coupler, two-hole directional coupler, reverse coupling directional coupler(schwinger coupler), Bethe-hole or single-hole directional coupler.

The general s-matrix for four port device is given by

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

From the symmetry property of s-matrix we can write

$$S_{12} = S_{21}, S_{13} = S_{31}, S_{23} = S_{32}, S_{34} = S_{43}, S_{24} = S_{42}, S_{41} = S_{14}$$

When all the four ports are perfectly matched to the junction then we can write

$$S_{11} = S_{22} = S_{33} = S_{44} = 0$$

Ports 1 & 3, ports 2 & 4 are isolated ports and hence we can write

$$S_{13} = S_{24} = 0$$

From the above all properties the s-matrix becomes

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \qquad \text{- (1)}$$

From the unitary property of s-matrix we can write

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1C_1 \Rightarrow \qquad |S_{12}|^2 + |S_{14}|^2 = 1 \qquad$ - (2)

$R_2C_2 \Rightarrow \qquad |S_{12}|^2 + |S_{23}|^2 = 1 \qquad$ - (3)

$R_3C_3 \Rightarrow \qquad |S_{23}|^2 + |S_{34}|^2 = 1 \qquad$ - (4)

$R_4C_4 \Rightarrow \qquad |S_{14}|^2 + |S_{34}|^2 = 1 \qquad$ - (5)

From zero property of s-matrix,

$R_1C_3 \Rightarrow \qquad S_{12} . S_{23}^* + S_{14} . S_{34}^* = 0 \qquad$ - (6)

Comparing equations 2 and 3, we can have

$$S_{14} = S_{23} \qquad \text{- (7)}$$

Comparing equations 3 and 4, we can have

$$S_{12} = S_{34} \qquad \text{- (8)}$$

Let us assume S_{12} is a real and positive (say P), then

$$S_{12} = S_{34} = P = S_{34}^*$$

From equation 6, we can have

$$PS_{23}^* + S_{14}P = 0$$

$$P\left[S_{23}^* + s_{14}\right] = 0 \qquad \text{- (9)}$$

Substitute equation 7 in equation 9

$$P\left[S_{23}^* + S_{23}\right] = 0$$

$$P \neq 0$$

Therefore,

$$S_{23} + S_{23}^* = 0$$

In order to satisfy the above equation, S_{23} must be imaginary term.
Let,

$$S_{23} = jq$$

Then

$$S_{23}^{*} = -jq$$

Therefore,

$$S_{12} = S_{34} = P \qquad \text{- (10)}$$

$$S_{23} = S_{14} = jq \qquad \text{- (11)}$$

Substitute equations 10 and 11 in equation 1

$$[S] = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix}$$

2-Hole type directional coupler

The principle of two-hole directional coupler can be explained with the help of diagram shown in the following figure.

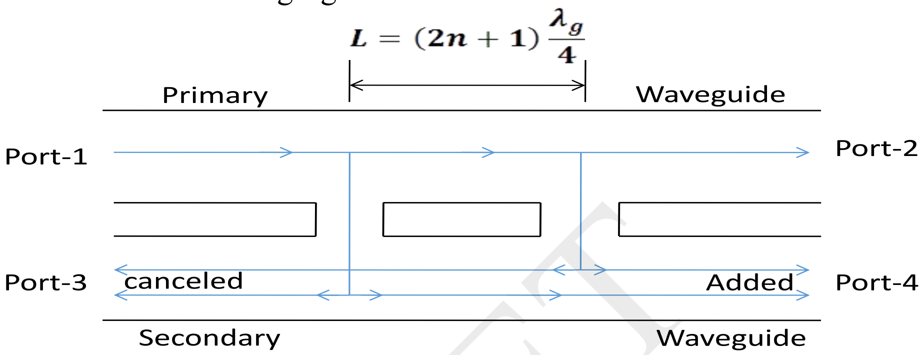


Fig: Two-hole directional coupler

Two-hole directional coupler contains two holes separated by the spacing given by the relation

$$L = (2n + 1)\frac{\lambda_g}{4}$$

Where n = 0,1,2,3,….is an integer and λ_g is the guide wavelength. When n = 0, then the spacing between the two holes is $\lambda_g/4$. The waves traveling towards port-4 will have the same phase and they will be added up. Whereas the waves traveling toward port-3 will have the opposite (180^0) phase and hence they will be canceled.

Bethe-hole type

A single-hole or Bethe-hole directional coupler is shown in the following figure.

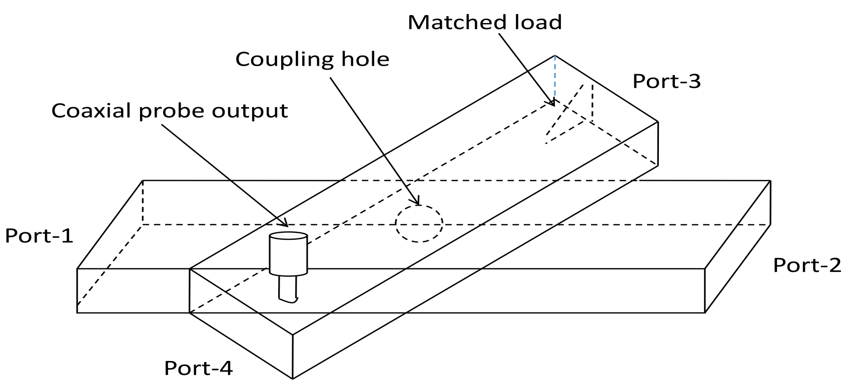


Fig: Bethe-hole or single hole directional coupler

It consists of two waveguides called as primary waveguide and secondary waveguide. The secondary waveguide is connected to the primary waveguide at some angle. Two waveguides are coupled through the single hole. The directivity of single-hole directional coupler is high as compared with the two-hole directional coupler because, coupling takes place through the single hole. The secondary waveguide is rotated such that maximum electric and magnetic coupling takes place between the two waveguides. When the signal is applied at the port-1, it available at the port-4 and port-2.

MICROWAVE PROPAGATION IN FERRITES

Ferrites-composition and characteristics:

Ferrite is an insulator but having magnetic properties. Examples of ferrites are manganese ferrite, zinc ferrite and associated ferromagnetic oxides such as Yttrium-Iron-Garnet or YIG in simple form. When electromagnetic waves propagate through the ferrite, they produce RF magnetic field inside and the direction of this RF field is at right angle to the direction of wave propagation. If an axial magnetic field from the permanent magnet is applied to the ferrite, an interaction will takes place within the ferrite. When only magnetic field is applied to the ferrite, electrons within the ferrite will align themselves along the lines of magnetic force, just as a magnetized needle aligns itself with the earth's magnetic field. The important characteristics of ferrites are **saturation magnetization, Line width and Curie temperature.** Saturation magnetization is defined as the minimum amount of d.c. magnetic field required to ensure that the axes of the spinning electrons are suitably aligned. Line width is defined as the range of magnetic field strengths over which absorption will takes place and is defined between the half power points for absorption. Curie temperature is defined as the temperature at which the ferrite loses its properties.

Faraday rotation:

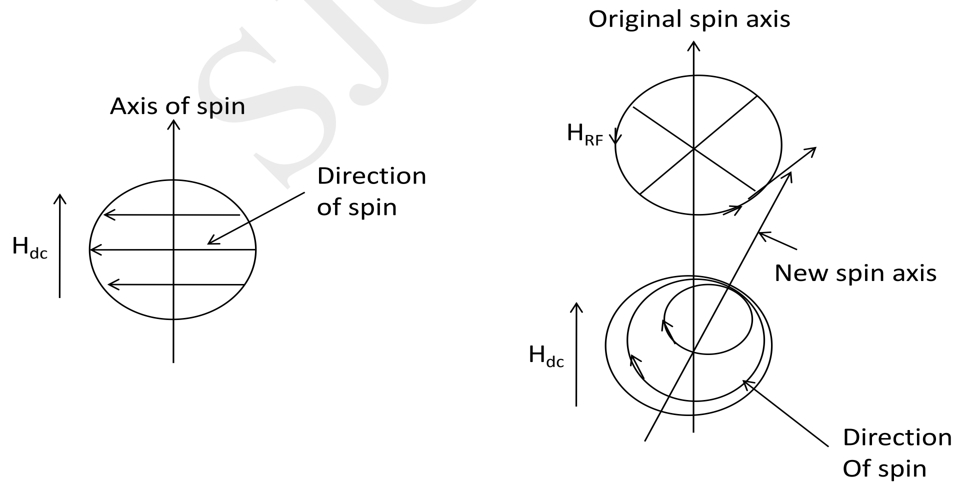


Fig: Effect of magnetic fields on spinning electrons.
(a) d.c.magnetic field only (b)d.c. and RF magnetic fields

Faraday rotation can be explained with the help of the above figure. Faraday rotation is defined as the rotation of plane of polarization of waves due to the interaction between the d.c magnetic field and RF magnetic field. When only d.c. magnetic field is applied, the electrons will align along the straight line called as spinning axis. But due to application of both d.c. magnetic field and RF magnetic field, the interaction will takes place and hence the plane of the wave will rotate. Therefore the phase of the waves which will propagate through the ferrite will shift by certain amount.

MICROWAVE DEVICES EMPLOYING FARADAY ROTATION

Gyrator:

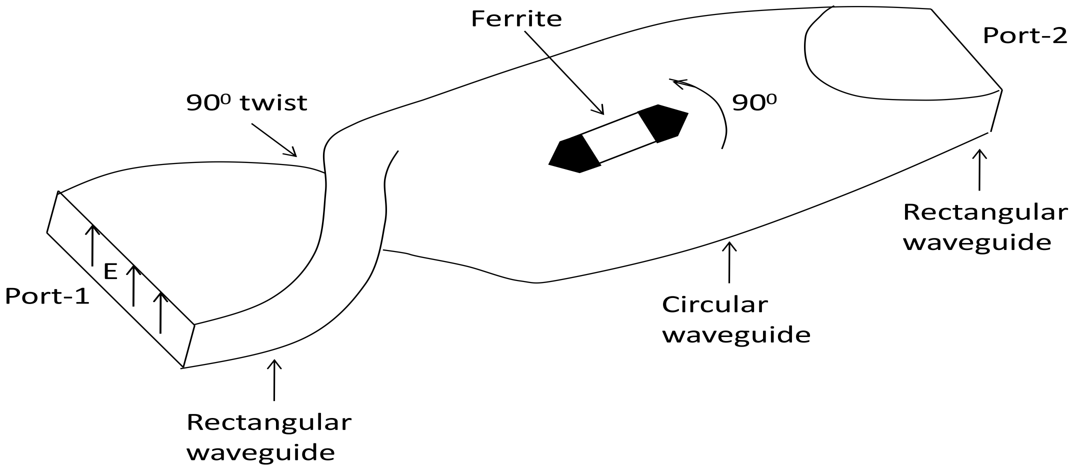


Fig: Structure of Gyrator

The structure of gyrator is shown in the figure above. It is a two port device. It introduces a phase shift of 180^0 when the signal travels from port-1 to port-2 and introduces zero phase shifts when the signal travels from the port-2 to port-1. It's simple operation can be explained as follows:

The signal traveling from port-1 will undergo a phase shift of 90^0 by the waveguide twist in anticlock wise direction and also undergo a phase shift of 90^0 by the ferrite in the same direction. Therefore the signal will undergo a total phase shift of 180^0 when it is travels from port-1 to port-2. Similarly the signal traveling from port-2 will undergo a phase shift of 90^0 by the ferrite in the anticlock wise direction and also undergo a phase shift of 90^0 by the waveguide twist in the clock wise direction. Therefore the signal will undergo a total phase shift of 0^0 when it is travels from port-2 to port-1.

Isolator:

The structure of Isolator is shown in the following figure.

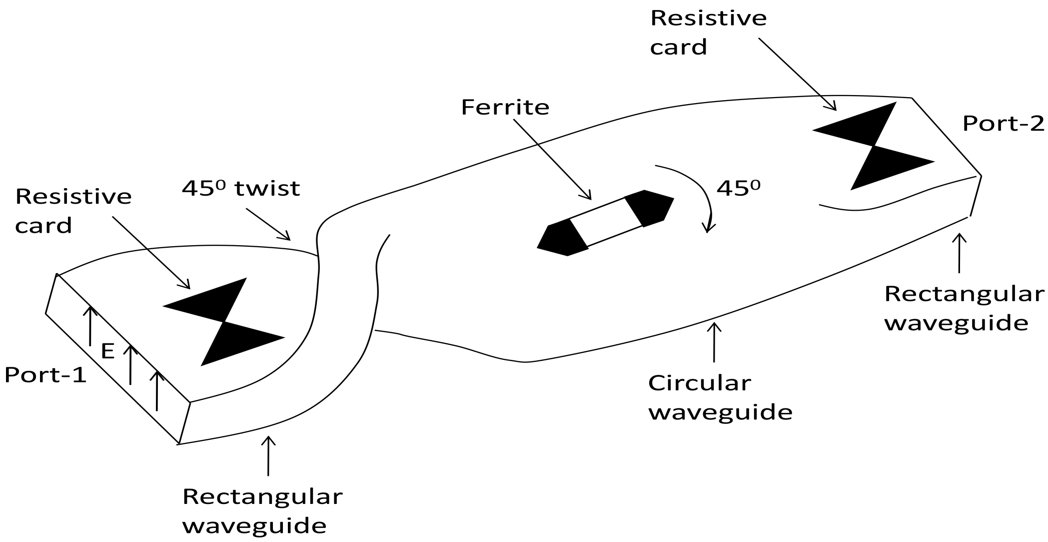


Fig: Structure of Isolator

It's simple operation can be explained as follows:
The signal traveling from port-1 will undergo a phase shift of 45^0 by the waveguide twist in anticlock wise direction and also undergo a phase shift of 45^0 by the ferrite in the opposite direction. Therefore the signal will undergo a total phase shift of 0^0 when it is travels from port-1 to port-2. During this traveling, the signal will be perpendicular to both the resistive cards and hence there will be no absorption of signal by the resistive cards. Similarly the signal traveling from port-2 will undergo a phase shift of 45^0 by the ferrite in the clock wise direction and also undergo a phase shift of 45^0 by the waveguide twist in the same direction. Therefore the signal will undergo a total phase shift of 90^0 when it is travels from port-2 to port-1. Due to this phase shift, the signal will be parallel to the resistive card in the port-1, and hence the signal will be absorbed by this resistive card. Therefore no signal will come out from port-1.
Isolator is a two port device and hence general s-matrix will be written as

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

When the two ports of isolator are perfectly matched, then we can write

$$S_{11} = S_{22} = 0$$

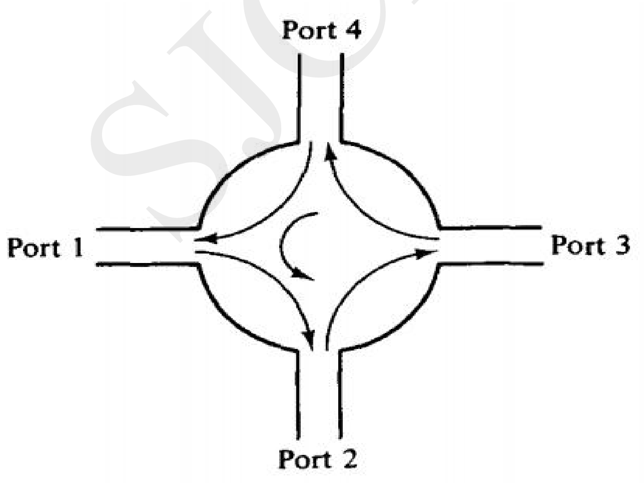
From the property of isolator we can write

$$S_{12} = \frac{b_1}{a_2} = 0 \, , S_{21} = \frac{b_2}{a_1} = 1$$

Therefore the s-matrix of isolator can be written as

$$[S] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Circulator:



Circulator is a 4 port component in which the power flow takes place from port-1 to port-2, from port-2 to port-3, port-3 to port-4 and port-4 to port-1 as shown in the figure above.

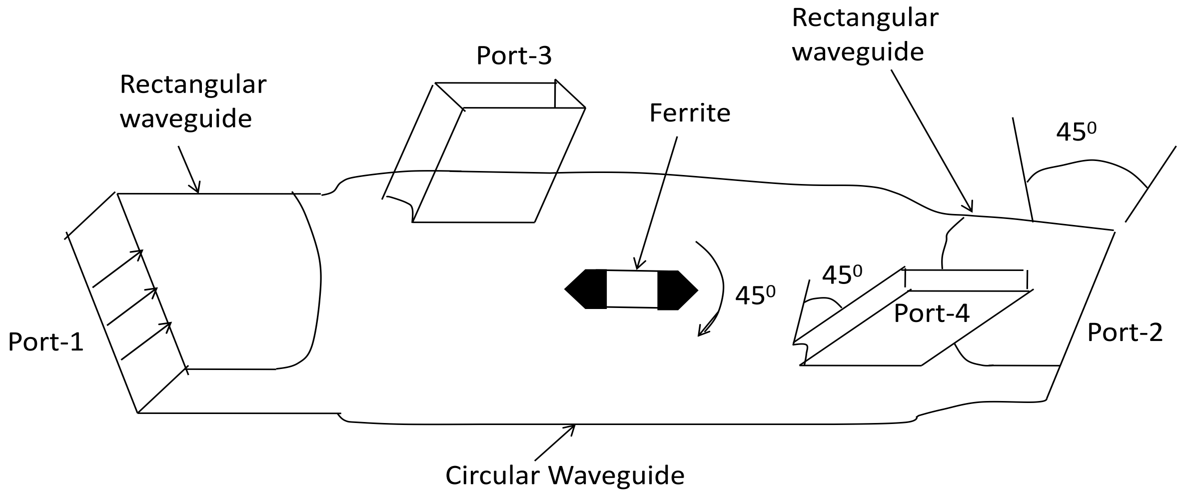


Fig: Four port ferrite circulator

The structure of four port ferrite circulator is shown in the above figure. It consists of ferrite material and waveguide twists. The ferrite will rotate the signal in clock wise direction by the amount of 45^0. When the signal is applied at port-1, it available only at port-2 as the output. Similarly when the signal is applied at port-2, it will be available at port-3. When the signal is applied to port-3, it will be available at port-4. Finally when the signal is applied at port-4, it will be available at port-1.

The general s-matrix for four port device can be written as

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

When all the ports are perfectly matched to the junction, then we can write

$$S_{11} = S_{22} = S_{33} = S_{44} = 0$$

From the property of circulator we can say

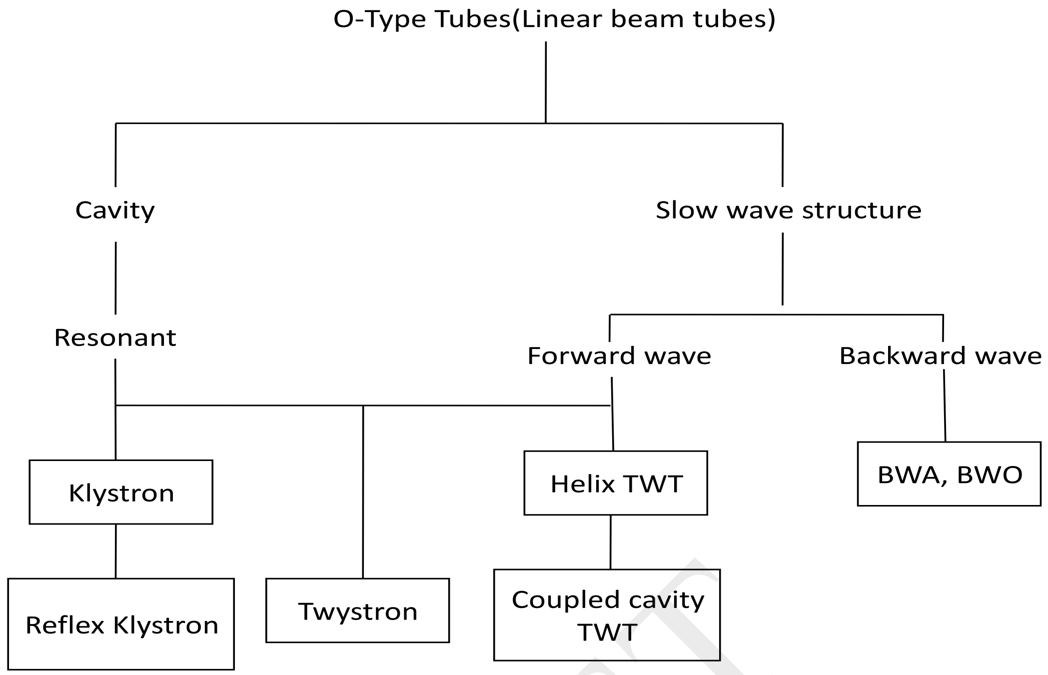
$$S_{12} = \frac{b_1}{a_2} = 0\,, S_{13} = \frac{b_1}{a_3} = 0\,, S_{14} = \frac{b_1}{a_4} = 1,$$
$$S_{21} = \frac{b_2}{a_1} = 1\,, S_{23} = \frac{b_2}{a_3} = 0\,, S_{24} = \frac{b_2}{a_4} = 0,$$
$$S_{31} = \frac{b_3}{a_1} = 0\,, S_{32} = \frac{b_3}{a_2} = 1\,, S_{34} = \frac{b_3}{a_4} = 0,$$
$$S_{41} = \frac{b_4}{a_1} = 0\,, S_{42} = \frac{b_4}{a_2} = 0\,, S_{43} = \frac{b_4}{a_3} = 1,$$

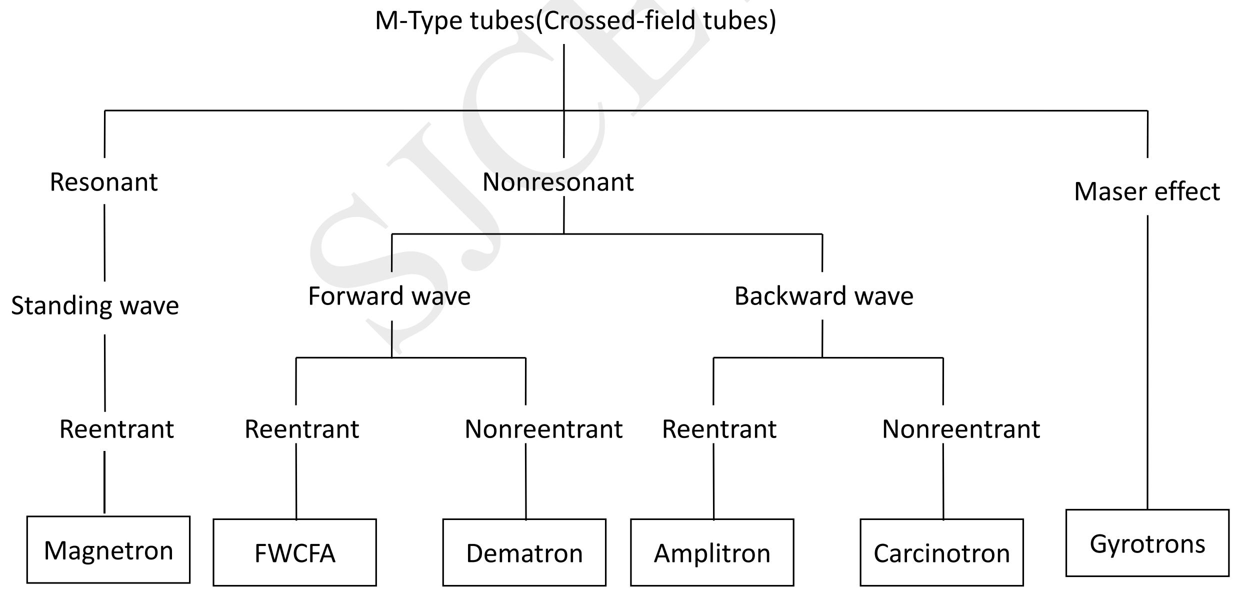
Therefore the s-matrix of four port circulator becomes

$$[S] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

MICROWAVE TUBES

Microwave tubes-O-type and M-type classifications:

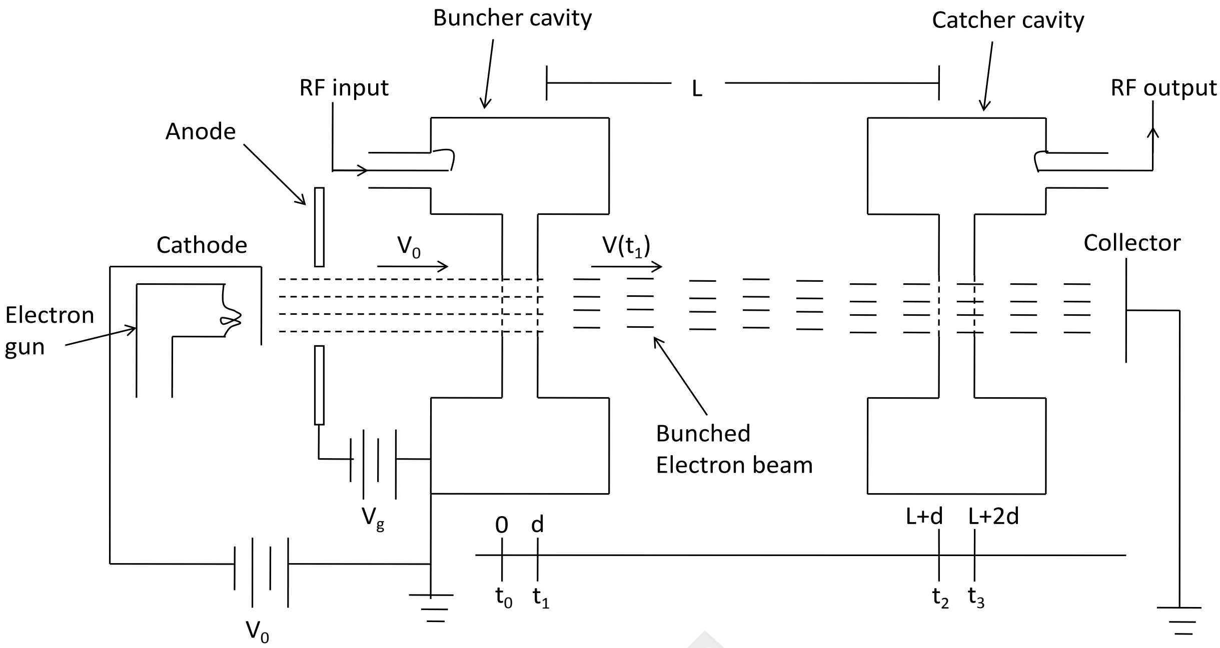




TWO CAVITY KLYSTRON AMPLIFIER

Structure:

The structure of two cavity klystron is shown in figure below. The two cavity klystron amplifier consists of two cavity resonators, Collector, electron gun, anode. The cavities are used for tuning purpose such that the gain of the amplifier is improved. The first cavity is known as buncher cavity which assists in bunching of electron beam. The second cavity is known as catcher cavity which will catch the bunched electron beam. The purpose of using anode is to avoid the spreading of electron beam or it helps in providing the linear electron beam. The electron gun is used to emit the beam of electrons.



The performance characteristics of two cavity klystron are given by

(i) Efficiency = 40%
(ii) Power output = 500 KW average power and 30 MW pulsed power
(iii) Power gain = 30 dB
(iv) Frequency = 250 MHz to 100 GHz
(v) Noise figure = 15 to 20 dB

Velocity modulation process and Applegate diagram:

The process of varying the velocity of the electrons with RF signal is known as velocity modulation. The beam of electrons will be emitted from the cathode. A high d.c. voltage is applied to the cathode of the electron gun such that the electrons will be accelerated towards the collector. The anode electrodes are provided such that the electrons will be further accelerated. The –ve d.c. supply is connected to the electron gun and anode such that the electrons travel with high velocity. The first cavity(Buncher cavity) having cavity gap or buncher grids. The buncher cavity is tuned such that it will be operated at the input RF signal frequency. The input signal to be amplified should be applied to the buncher cavity. The RF signal is existing across the buncher cavity gap.

Let the velocity of electrons is V_0 before entering the buncher cavity gap. The electrons will have potential energy and kinetic energy. Therefore,

$$eV_0 = \frac{1}{2} m\, v_0^2 \qquad -1$$

Where 'e' is the charge of electron, 'V_0' is the applied d.c.voltage, 'm' is the mass of electron, 'v_0' is the velocity of electrons.

$$2eV_0 = m\, v_0^2$$

$$v_0 = \sqrt{\frac{2eV_0}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$$v_0 = 0.593 \times 10^6 \sqrt{V_0} \qquad -2$$

The above equation give the velocity of electrons due to applied d.c. voltage V_0. The velocity of the electrons which will enter into the buncher cavity gap will be changes according to the amplitude of the RF signal. Since the electrons contains a.c. components also, so the current modulation also takes place in the buncher cavity gap. Let the input signal is $V_s = V_1 \sin\omega t$. To find out the velocity modulation equation either in terms of t_0 or t_1 we need to find the average microwave voltage in the buncher cavity gap which will be calculated as follows:

The cavity gap transit time is given by

$$\tau = \frac{d}{v_0} = t_1 - t_0$$

The transit angle is given by

$$\theta_g = \omega\tau = \frac{\omega d}{v_0} = \omega(t_1 - t_0) \qquad -3$$

The average microwave signal at the buncher cavity gap is given by

$$V_s = \frac{1}{\tau}\int_{t_0}^{t_1} V_1 \sin\omega t \, dt = \frac{V_1}{\tau}\int_{t_0}^{t_1} \sin\omega t \, dt = \frac{V_1}{\tau}\left[\frac{-\cos\omega t}{\omega}\right]_{t_0}^{t_1}$$

$$V_s = \frac{V_1}{\tau\omega}[-\cos\omega t_1 - -\cos\omega t_0 = \frac{V_1}{\tau\omega}[\cos\omega t_0 - \cos\omega t_1] \qquad -4$$

From equation 3,

$$\frac{\omega d}{v_0} = \omega t_1 - \omega t_0$$

$$\omega t_1 = \omega t_0 + \frac{\omega d}{v_0} \qquad -5$$

Substitute equation 5 in equation 4

$$V_s = \frac{V_1}{\tau\omega}\left[\cos\omega t_0 - \cos\left(\omega t_0 + \frac{\omega d}{v_0}\right)\right] \qquad -6$$

Let $\quad \omega t_0 + \frac{\omega d}{2v_0} = \omega t_0 + \frac{\theta_g}{2} = A \;\; and \;\; \frac{\omega d}{2v_0} = \frac{\theta_g}{2} = B$

Then equation 6 can be written as

$$V_s = \frac{V_1}{\tau\omega}[\cos(A - B) - \cos(A + B)]$$

But $\quad \cos(A - B) - \cos(A + B) = 2\sin A \, \sin B$

$$V_s = \frac{V_1}{\tau\omega}[2\sin A \, \sin B] = \frac{V_1}{\tau\omega}\left[2\sin\left(\omega t_0 + \frac{\theta_g}{2}\right) \, \sin\left(\frac{\theta_g}{2}\right)\right]$$

$$V_s = \frac{2V_1}{\tau\omega}\left[\sin\left(\frac{\theta_g}{2}\right)\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] = \frac{V_1}{\omega\tau/2}\left[\sin\left(\frac{\theta_g}{2}\right)\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

$$V_s = \frac{V_1}{\theta_g/2}\left[\sin\left(\frac{\theta_g}{2}\right)\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] = V_1\left[\frac{\sin\left(\frac{\theta_g}{2}\right)}{\theta_g/2}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

$$V_s = V_1\left[\beta_i \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \qquad -7$$

Where $\quad \beta_i$ is known as coupling coefficient

$$\beta_i = \frac{\sin\left(\frac{\theta_g}{2}\right)}{\theta_g/2} \qquad -8$$

From above equation it can be seen that, if transit angle θ_g decreases, the coupling between the electron beam and buncher cavity increases. Where as if θ_g increases, the coupling between the cavity and electron beam decreases.

At $\quad t = t_1, \quad v_0 = v(t_1)$

$$v(t_1) = \sqrt{\frac{2eV_0'}{m}} \qquad -9$$

Where $\quad V_0' = V_0 + \textit{Average value of microwave signal at cavity gap} = V_0 + V_s$

$$V_0' = V_0 + V_1\left[\beta_i \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \qquad -10$$

Substitute equation 10 in equation 9

$$v(t_1) = \sqrt{\frac{2e}{m}\left(V_0 + V_1\left[\beta_i \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]\right)}$$

$$v(t_1) = \sqrt{\frac{2eV_0}{m}\left(1 + \frac{V_1}{V_0}\left[\beta_i \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]\right)}$$

$$= \sqrt{\frac{2eV_0}{m}\left(1 + \frac{V_1\beta_i}{V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right)}$$

Where $\frac{V_1\beta_i}{V_0}$ is called depth of velocity modulation or modulation index.

$$v(t_1) = \sqrt{\frac{2eV_0}{m}}\sqrt{1 + \frac{V_1\beta_i}{V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)} = v_0\sqrt{1 + \frac{V_1\beta_i}{V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)}$$

$$v(t_1) = v_0\left[1 + \frac{V_1\beta_i}{V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]^{1/2}$$

Apply binomial expansion and neglect higher order terms

$$v(t_1) = v_0\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right) + \dots\right] \qquad -11$$

From equation 3,

$$\omega t_0 = \omega t_1 - \frac{\omega d}{v_0} = \omega t_1 - \theta_g \qquad -12$$

Substitute equation 12 in equation 11

$$v(t_1) = v_0\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_1 - \theta_g + \frac{\theta_g}{2}\right)\right]$$

$$v(t_1) = v_0\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \qquad -13$$

Equations 11 and 13 are called velocity modulation equations in terms of t_0 and t_1 respectively.

The Applegate diagram of two cavity klystron amplifier is shown in figure below.

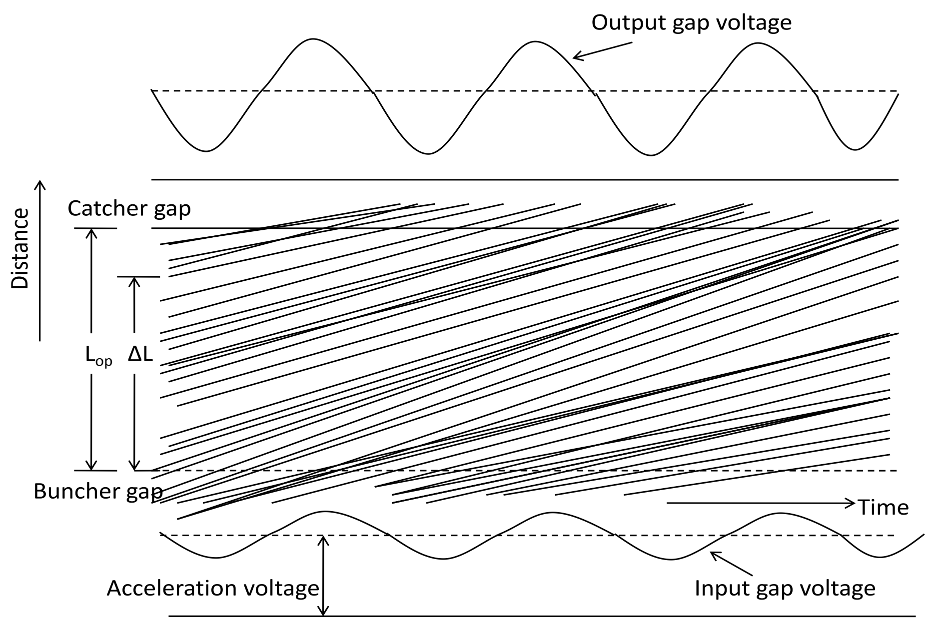


Fig: Applegate diagram of two cavity klystron

Bunching process:

The bunching process in two cavity klystron can be explained with the help of the following figure.

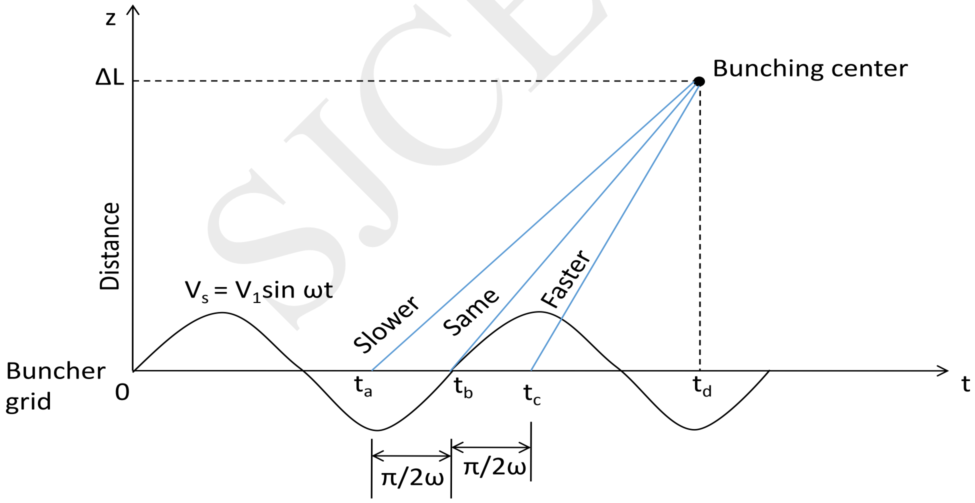


Fig: Bunching process

Once the electrons leave the buncher cavity, they drift with a velocity $v(t_1)$ along the field free space between the two cavities. The effect of velocity modulation produces bunching of the electron beam. The signal which existing across the buncher cavity gap is shown in figure. The electrons that passes the buncher cavity at $V_s = 0$, travel through unchanged velocity 'v_0' and become the bunching center. Those electrons that passes the buncher cavity during the positive half cycle of the microwave input voltage (V_s) travel faster than the electrons that passed the gap when $V_s = 0$. Those electrons that passes the buncher cavity during the negative half cycle of the microwave input voltage travel slower than the electrons that passed the gap when $V_s = 0$. At distance of ΔL along the beam from the buncher cavity, all the three electrons form as a bunch as shown in above figure.

The distance from the buncher grid to the location of dense electron bunching for the electron 'b' at 't_b' is given by

$$\Delta L_b = v_0(t_d - t_b) \qquad -1$$

Similarly the distance for the electron 'a' at 't_a' and electron 'c' at 't_c' are given by

$$\Delta L_a = v_{min}(t_d - t_a) = v_{min}\left(t_d - t_b + \frac{\pi}{2\omega}\right) \qquad -2$$

$$\Delta L_c = v_{max}(t_d - t_c) = v_{max}\left(t_d - t_b - \frac{\pi}{2\omega}\right) \qquad -3$$

The velocity modulation equation is given by

$$v(t_1) = v_0\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \qquad -4$$

v_{min} occurs when $\omega t_1 - \frac{\theta_g}{2} = -\frac{\pi}{2}$

Then equation 4 becomes

$$v(t_1) = v_{min} = v_0\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\frac{-\pi}{2}\right)\right] = v_0\left[1 - \frac{\beta_i V_1}{2V_0}\right] \qquad -5$$

v_{max} occurs when $\omega t_1 - \frac{\theta_g}{2} = \frac{\pi}{2}$

Then equation 4 becomes

$$v(t_1) = v_{max} = v_0\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\frac{\pi}{2}\right)\right] = v_0\left[1 + \frac{\beta_i V_1}{2V_0}\right] \qquad -6$$

Substitute equation 5 in equation 2

$$\Delta L_a = v_0\left[1 - \frac{\beta_i V_1}{2V_0}\right]\left(t_d - t_b + \frac{\pi}{2\omega}\right) = \left(v_0 - \frac{v_0\beta_i V_1}{2V_0}\right)\left(t_d - t_b + \frac{\pi}{2\omega}\right)$$

$$\Delta L_a = v_0(t_d - t_b) + \frac{v_0\pi}{2\omega} - \frac{v_0\beta_i V_1}{2V_0}\left(t_d - t_b + \frac{\pi}{2\omega}\right)$$

$$\Delta L_a = v_0(t_d - t_b) + \frac{v_0\pi}{2\omega} - \frac{v_0\beta_i V_1}{2V_0}(t_d - t_b) - \frac{v_0\beta_i V_1}{2V_0}\frac{\pi}{2\omega} \qquad -7$$

Similarly substitute equation 6 in equation 3

$$\Delta L_c = v_0\left[1 + \frac{\beta_i V_1}{2V_0}\right]\left(t_d - t_b - \frac{\pi}{2\omega}\right)$$

$$\Delta L_c = v_0(t_d - t_b) - \frac{v_0\pi}{2\omega} + \frac{v_0\beta_i V_1}{2V_0}(t_d - t_b) - \frac{v_0\beta_i V_1}{2V_0}\frac{\pi}{2\omega} \qquad -8$$

To form a bunch all the three electrons at t_a, t_b and t_c should travel the same distance ΔL. Therefore equate equations 1 and 7

$$v_0(t_d - t_b) = v_0(t_d - t_b) + \frac{v_0\pi}{2\omega} - \frac{v_0\beta_i V_1}{2V_0}(t_d - t_b) - \frac{v_0\beta_i V_1}{2V_0}\frac{\pi}{2\omega}$$

$$0 = \frac{v_0\pi}{2\omega} - \frac{v_0\beta_i V_1}{2V_0}(t_d - t_b) - \frac{v_0\beta_i V_1}{2V_0}\frac{\pi}{2\omega}$$

$$-\frac{v_0\beta_i V_1}{2V_0}(t_d - t_b) = \frac{v_0\beta_i V_1}{2V_0}\frac{\pi}{2\omega} - \frac{v_0\pi}{2\omega}$$

$$-\frac{v_0\beta_i V_1}{2V_0}(t_d - t_b) = \frac{v_0\pi}{2\omega}\left(\frac{\beta_i V_1}{2V_0} - 1\right)$$

$$t_d - t_b = -\frac{\pi}{2\omega}\left(\frac{\beta_i V_1}{2V_0} - 1\right)\frac{2V_0}{\beta_i V_1} = -\frac{\pi}{2\omega}\left(\frac{\beta_i V_1}{2V_0} \times \frac{2V_0}{\beta_i V_1} - \frac{2V_0}{\beta_i V_1}\right) = -\frac{\pi}{2\omega}\left(1 - \frac{2V_0}{\beta_i V_1}\right)$$

$$t_d - t_b = -\frac{\pi}{2\omega} + \frac{\pi}{2\omega} \times \frac{2V_0}{\beta_i V_1} = -\frac{\pi}{2\omega} + \frac{\pi V_0}{\beta_i V_1 \omega}$$

When $V_0 >> V_1$, then $\pi/2\omega$ can be neglected.

$$t_d - t_b = \frac{\pi V_0}{\beta_i V_1 \omega} \qquad -7$$

Substitute equation 7 in equation 1

$$\Delta L_b = \Delta L = v_0 \frac{\pi V_0}{\beta_i V_1 \omega} \qquad -8$$

The time taken for the electron bunches to travel the distance L (Drift space) is known as transit time (T) and is given by

$$T = t_2 - t_1 = \frac{L}{v(t_1)} \qquad -9$$

Substitute equation 4 in equation 9

$$T = \frac{L}{v_0\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]} = T_0\left[\frac{1}{1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)}\right]$$

Where $T_0 = \frac{L}{v_0}$ is called d.c. transit time.

$$T = T_0\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]^{-1}$$

Expand by using binomial expansion and neglect higher order terms

$$T = T_0\left[1 - \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \qquad -10$$

$$\omega T = \omega T_0\left[1 - \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] = \theta_0 - X\sin\left(\omega t_1 - \frac{\theta_g}{2}\right) \qquad -11$$

Where $\theta_0 = \omega T_0 = \frac{\omega L}{v_0}$ is known as d.c transit angle

$$X = \frac{V_1\beta_i}{2V_0}\theta_0 \quad \textit{is called bunching parameter of a klystron}$$

<u>Small signal theory expressions for o/p power and efficiency:</u>

To derive the equation for the power output and efficiency, let us assume the charge 'dQ_0' is passing through the buncher cavity gap at a time interval of dt_0. Then

$$i = \frac{dQ}{dt}$$

$$dQ = i\,dt$$

Or $\qquad dQ_0 = I_0\,dt_0$

Where I_0 is the d.c. current. From the conservation of charge, that is charge neither be created nor destroyed. The same amount of charge dQ_0 will pass through the catcher cavity gap at a time interval of dt_2. Then

$$dQ_0 = i_2\,dt_2$$

Where 'i_2' is the current in the catcher cavity.

$$dQ_0 = I_0\,dt_0 = i_2\,dt_2 \qquad -1$$

We have

$$T = t_2 - t_1 = \frac{L}{v(t_1)} \qquad -2$$

But

$$v(t_1) = v_0\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \qquad -3$$

Substitute equation 3 in equation 2

$$T = t_2 - t_1 = \frac{L}{v_0\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]}$$

$$t_2 - t_1 = T_0\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]^{-1}$$

Expand by using binomial expansion

$$t_2 - t_1 = T_0\left[1 - \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \qquad -4$$

$$t_2 = t_1 + T_0\left[1 - \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \qquad -5$$

We know that $\tau = t_1 - t_0$

$$t_1 = \tau + t_0 \qquad -6$$

Substitute equation 6 in equation 5

$$t_2 = \tau + t_0 + T_0\left[1 - \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \qquad -7$$

Differentiate above equation w.r.t t_0

$$\frac{dt_2}{dt_0} = \frac{dt_0}{dt_0} + T_0\frac{d}{dt_0}\left[1 - \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

$$\frac{dt_2}{dt_0} = 1 + T_0\left[0 - \frac{V_1\beta_i}{2V_0}\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)\cdot\omega\right]$$

$$\frac{dt_2}{dt_0} = 1 - \frac{\omega T_0 V_1\beta_i}{2V_0}\cos\left(\omega t_0 + \frac{\theta_g}{2}\right) = 1 - \frac{\omega L}{2v_0}\frac{\beta_i V_1}{V_0}\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

$$\frac{dt_2}{dt_0} = 1 - \theta_0\frac{V_1\beta_i}{2V_0}\cos\left(\omega t_0 + \frac{\theta_g}{2}\right) = 1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

$$dt_2 = dt_0\left(1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)\right) \qquad -8$$

From equation 1

$$i_2 = \frac{I_0\, dt_0}{dt_2} \qquad -9$$

Substitute equation 8 in equation 9

$$i_2 = \frac{I_0\, dt_0}{dt_0\left(1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)\right)}$$

$$= \frac{I_0}{\left(1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)\right)} \qquad -10$$

The above equation gives the current arriving the catcher cavity in terms of t_0. The equation 10 in terms of t_2 can be written as

From equation 7

$$t_2 = \tau + t_0 + T_0 \qquad if \qquad \omega t_0 = -\frac{\theta_g}{2}$$

$$\omega t_2 = \omega\tau + \omega t_0 + \omega T_0 = \theta_g + \omega t_0 + \theta_0$$

$$\omega t_0 = \omega t_2 - \theta_g - \theta_0 \qquad -11$$

Substitute equation 11 in equation 10

$$i_2 = \frac{I_0}{\left(1 - X\cos\left(\omega t_2 - \theta_g - \theta_0 + \frac{\theta_g}{2}\right)\right)}$$

$$i_2 = \frac{I_0}{\left(1 - X\cos\left(\omega t_2 - \theta_0 - \frac{\theta_g}{2}\right)\right)} \qquad -12$$

The beam current at the catcher cavity is a periodic signal having the period 'T'. Therefore, the current i_2 can be expanded by using Fourier series

$$i_2 = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos\left(n\omega t_2\right) + b_n \sin n\omega t_2\right] \qquad -13$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} i_2\, d(\omega t_2) = \frac{1}{2\pi}\int_{-\pi}^{\pi} I_0\, d(\omega t_0) = \frac{I_0}{2\pi}\left[\omega t_0\right]_{-\pi}^{\pi}$$

$$= I_0 \qquad -14$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} i_2\ \cos\left(n\omega t_2\right) d(\omega t_2)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} i_2\ \sin\left(n\omega t_2\right) d(\omega t_2)$$

The above two equations can be solved by suing Bessel functions and are given by

$$a_n = 2I_0\, J_n(n\mathrm{X})\cos\left(n\theta_g + n\theta_0\right) \qquad -15$$

$$b_n = 2I_0\, J_n(n\mathrm{X})\sin\left(n\theta_g + n\theta_0\right) \qquad -16$$

Where $J_n(nX)$ is the n^{th} order Bessel function.

Substitute equations 14, 15 and 16 in equation 13

$$i_2 = I_0 + \sum_{n=1}^{\infty}\left[2I_0\, J_n(n\mathrm{X})\cos\left(n\theta_g + n\theta_0\right)\cos\left(n\omega t_2\right) + 2I_0\, J_n(n\mathrm{X})\sin\left(n\theta_g + n\theta_0\right)\sin\left(n\omega t_2\right)\right]$$

$$i_2 = I_0 + \sum_{n=1}^{\infty} 2I_0\, J_n(n\mathrm{X})\left[\cos(n\omega t_2)\cos\left(n\theta_g + n\theta_0\right) + \sin\left(n\omega t_2\right)\sin\left(n\theta_g + n\theta_0\right)\right]$$

$$i_2 = I_0 + \sum_{n=1}^{\infty} 2I_0\, J_n(n\mathrm{X})\left[\cos\left(n\omega t_2 - n\theta_g - n\theta_0\right)\right]$$

$$i_2 = I_0 + \sum_{n=1}^{\infty} 2I_0\, J_n(n\mathrm{X})\left[\cos\left(n\omega t_2 - n\omega\tau - n\omega T_0\right)\right]$$

$$i_2 = I_0 + \sum_{n=1}^{\infty} 2I_0\, J_n(n\mathrm{X})\left[\cos n\omega(t_2 - \tau - T_0)\right] \qquad -17$$

In above equation the first term 'I_0' is the d.c. component and the second part is related to a.c. components. Let us consider only a.c. components.

$$i_2 = \sum_{n=1}^{\infty} 2I_0\, J_n(n\mathrm{X})\left[\cos n\omega(t_2 - \tau - T_0)\right]$$

The magnitude of fundamental component (when n = 1) of the above signal is given by

$$I_f = I_2 = 2I_0\, J_1(\mathrm{X}) \qquad -18$$

This fundamental component has the maximum magnitude at X = 1.841

Output Power: The induced current at the catcher cavity is given by

$$i_{2nd} = \beta_0 I_2 \qquad -19$$

Substitute equation 18 in equation 19

$$i_{2nd} = \beta_0 2 I_0 J_1(\mathrm{X}) = 2\beta_0 I_0 J_1(\mathrm{X}) \qquad -20$$

The average output power is given by

$$P_{out} = \frac{1}{2} {i_{2nd}}^2 R_{sh} = \frac{1}{2} V_2 i_{2nd} \qquad -21$$

Where R_{sh} is the effective shunt resistance which includes wall resistance of the catcher cavity, beam loading resistance and external load resistance.

Substitute equation 19 in equation 21

$$P_{out} = \frac{(\beta_0 I_2)^2}{2} R_{sh} = \frac{\beta_0 I_2 V_2}{2}$$

Efficiency: The efficiency of two cavity klystron amplifier is given by

$$\eta = \frac{P_{out}}{P_{in}} = \frac{\frac{\beta_0 I_2 V_2}{2}}{V_0 I_0} = \frac{\beta_0 I_2 V_2}{2 V_0 I_0}$$

REFLEX KLYSTRON OSCILLATOR

Structure:

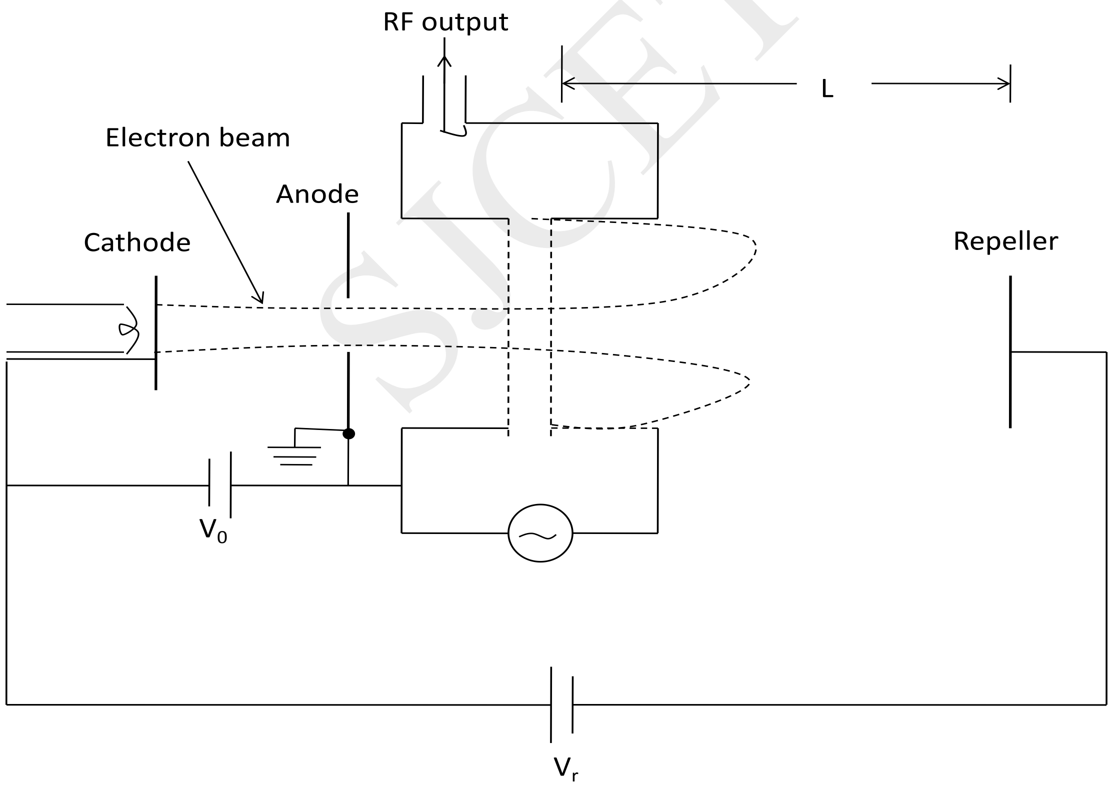
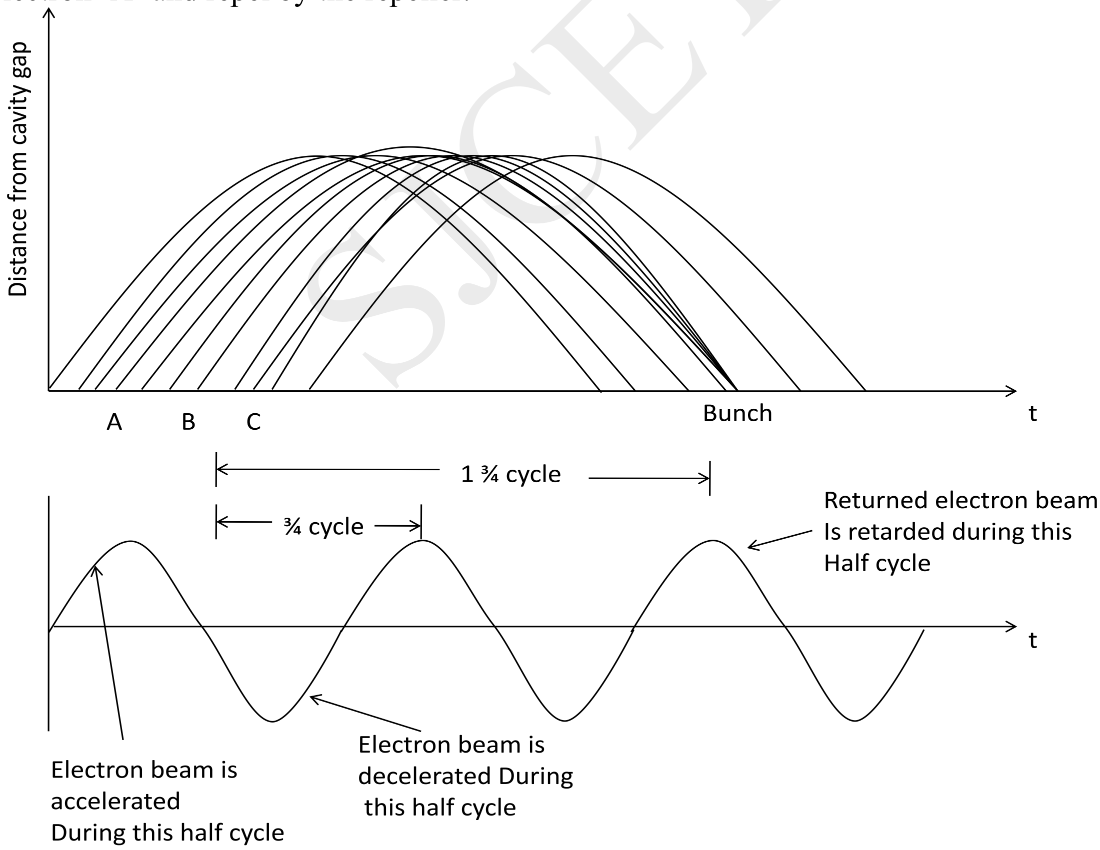


Fig: Structure of reflex klystron oscillator

The structure of reflex klystron oscillator is shown in above figure. It consists of cathode, Reentrant cavity, Repeller. The purpose of cathode is to emit the beam of electrons. A high d.c.voltage (V_0) is applied to the cathode. The cavity is used to velocity modulate the electrons. The repeller is used to repel the electrons. The repeller is supplied with high d.c. voltage known as repeller voltage(V_r). The two cavity klystron amplifier can be used as a oscillator by giving feedback such that bharkhauson criteria is satisfied. But the problem arises with two cavity klystron is when we want to operate the oscillator at some other frequency, we need to adjust t(tune) both the cavities. In order to avoid this problem a single cavity klystron known as reflex klystron is used.

Velocity modulation and Applegate diagram:

The basic principle in reflex klystron is by giving less energy to the electrons, more energy will be gained with the help of velocity modulation. Initially due to noise or transients, small oscillations will be exist within the cavity and this oscillations will be sustained by the energy of electron bunches. The function of repeller is it reflects or repels the electrons which are coming towards it. The velocity modulation process of reflex klystron oscillator can be explained with the help of Applegate diagram shown in figure below. In figure, the oscillations due to noise or transient also had shown which exist across the cavity gap. The electron beam is emitted from the electron gun and is accelerated with the help of high d.c. voltage (V_0). The electron beam is velocity modulated when it enters the cavity gap. As shown in figure, let the electron 'A' entering the cavity gap when the signal is at +ve maximum. The velocity of electron 'A' is increased since it would be accelerated by the +ve half cycle. This electron travels more distance in the repeller space and repelled by the repeller and finally travels towards the cavity. Similarly let us assume the electron 'B' enters the cavity gap when the gap voltage is zero. Then the velocity of the electron 'B' is not changed and travels the lesser distance in the repeller space as compared with the electron 'A' and repel by the repeller.



Third electron 'C' is enter the cavity gap later than the electron'B' when the gap voltage is at –ve maximum and its velocity will be decelerated. The electron 'C' will travels less distance in the repeller space as compared with electron 'B' since it is having less velocity compared to 'B' and will be repelled by the repeller. The time taken by the electrons to travel the distance towards repeller and returned to the cavity gap is called round trip transit time. Since the three electrons A,B & C enters the cavity gap with difference in time and the electrons are travelling different distances, as a result, all the three electrons forms as a bunch while traveling to the cavity.

Normally the electrons gains energy when those are accelerated by the voltage and loose energy when they are retarded or slowed down by the voltage(-ve). Therefore to gain more energy from the electron bunches, the electron bunch should returned to the cavity when the gap voltage is at retarding phase(+ve maximum). Therefore, the electron bunches will be retarded and give up their kinetic energy to the –ve half cycle(Since electrons are retarded by the +ve half cycle). Therefore, as shown in Applegate diagram the first +ve peak occurs (after reference electron 'B' since around it electron bunch takes place) after ¾ cycle. So we have to adjust the repeller voltage such that the electron bunches returned to the cavity when the cavity gap voltage is at first ¾ cycle. Therefore first mode occurs at first ¾ cycle. By varying the repeller voltage, the electron bunches will return to the cavity gap after 1 ¾ cycle, 2 ¾ cycle and so on. Therefore, different modes of operation of a reflex klystron is possible which is given as

$$N = (n + 3/4)$$

Where 'N' is the mode of operation and 'n' is the integer varies from 0 to infinity. First mode occurs when n = 0, 2nd mode occurs when n = 1, and so on. The velocity modulation equation of reflex klystron is similar to two cavity klystron and is given by

$$v(t_1) = v_0 \left[1 + \frac{V_1 \beta_i}{2V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2} \right) \right]$$

Mathematical theory of bunching:

The analysis of reflex klystron is similar to two cavity klystron. The electron enters the cavity gap with a velocity of $v_0 = 0.593 \times 10^6 \sqrt{V_0}$ at z = t_0 and the electron leave the cavity gap at z = t_1 with a velocity of

$$v(t_1) = v_0 \left[1 + \frac{V_1 \beta_i}{2V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2} \right) \right] \qquad -1$$

The same electron will be returned to the cavity gap at z = t_2 by the retarding electric field which is given by

$$E = \frac{V}{d} = \frac{V_r + V_0 + V_1 \sin \omega t}{L} \qquad -2$$

The force due to this electric field on the electron is given by

$$F = -eE \qquad -3$$

Substitute equation 2 in equation 3

$$F = -e \frac{V_r + V_0 + V_1 \sin \omega t}{L} \cong -e\left(\frac{V_r + V_0}{L} \right) \qquad -4$$

Since $V_1 \sin \omega t \ll V_r + V_0$

We know that,

$$F = ma = m\frac{d^2 z}{dt^2} \qquad -5$$

Equate equations 4 and 5

$$m\frac{d^2 z}{dt^2} = -e\left(\frac{V_r + V_0}{L} \right)$$

$$\frac{d^2 z}{dt^2} = -\frac{e}{m}\left(\frac{V_r + V_0}{L} \right)$$

Integrate above equation with respect to t

$$\frac{dz}{dt} = -\frac{e(V_r + V_0)}{mL}\int_{t_1}^{t} dt = -\frac{e(V_r + V_0)}{mL}(t - t_1) + k_1 \qquad -6$$

At $\quad t = t_1, \quad \frac{dz}{dt} = v(t_1)$

Apply above condition to the equation 6

$$v(t_1) = -\frac{e(V_r + V_0)}{mL}(t_1 - t_1) + k_1$$

$$v(t_1) = 0 + k_1$$

$$k_1 = v(t_1) \qquad -7$$

Substitute equation 7 in equation 6

$$\frac{dz}{dt} = -\frac{e(V_r + V_0)}{mL}(t - t_1) + v(t_1) \qquad -8$$

Integrate above equation with respect to t

$$z = -\frac{e(V_r + V_0)}{mL}\int_{t_1}^{t}(t - t_1)dt + \int_{t_1}^{t} v(t_1)dt$$

$$z = -\frac{e(V_r + V_0)}{mL}\int_{t_1}^{t}(t - t_1)dt + v(t_1)\int_{t_1}^{t} dt$$

$$z = -\frac{e(V_r + V_0)}{2mL}(t - t_1)^2 + v(t_1)(t - t_1) + k_2 \qquad -9$$

At $\quad t = t_1, \quad z = d$

Apply above condition to the equation 9

$$d = -\frac{e(V_r + V_0)}{2mL}(t_1 - t_1)^2 + v(t_1)(t_1 - t_1) + k_2 = 0 + 0 + k_2$$

$$k_2 = d \qquad -10$$

Substitute equation 10 in equation 9

$$z = -\frac{e(V_r + V_0)}{2mL}(t - t_1)^2 + v(t_1)(t - t_1) + d \qquad -11$$

The electrons are leaving the cavity (z = d) with time t_1 and returning to the cavity (z = d) at t = t_2. Therefore, $\quad at \;\; t = t_2, \quad z = d$

Apply above condition to equation 11

$$d = -\frac{e(V_r + V_0)}{2mL}(t_2 - t_1)^2 + v(t_1)(t_2 - t_1) + d$$

$$d - d = -\frac{e(V_r + V_0)}{mL}(t_2 - t_1)^2 + v(t_1)(t_2 - t_1)$$

$$0 = -\frac{e(V_r + V_0)}{2mL}(t_2 - t_1)^2 + v(t_1)(t_2 - t_1)$$

$$\frac{e(V_r + V_0)}{2mL}(t_2 - t_1)^2 = v(t_1)(t_2 - t_1)$$

$$\frac{e(V_r + V_0)}{2mL}(t_2 - t_1) = v(t_1)$$

$$T' = t_2 - t_1 = \frac{v(t_1)2mL}{e(V_r + V_0)} \qquad -12$$

Where T' is known as round trip transit time

Substitute equation 1 in equation 12

$$T' = t_2 - t_1 = \frac{v_0\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]2mL}{e(V_r + V_0)}$$

$$= \frac{2mLv_0}{e(V_r + V_0)}\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

$$t_2 - t_1 = T_0'\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \qquad -13$$

Where $T_0' = \frac{2mLv_0}{e(V_r+V_0)}$ is known as round trip d.c. transit time. Multiply equation 13 with ω on both sides

$$\omega(t_2 - t_1) = \omega T_0'\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

$$\omega(t_2 - t_1) = \theta_0'\left[1 + \frac{V_1\beta_i}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

Where $\theta_0' = \omega T_0'$ is known as round trip d.c. transit angle.

$$\omega(t_2 - t_1) = \theta_0' + \theta_0'\frac{V_1\beta_i}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)$$

$$= \theta_0' + X'\sin\left(\omega t_1 - \frac{\theta_g}{2}\right) \qquad -14$$

Where $X' = \frac{V_1\beta_i}{2V_0}\theta_0'$ is known as bunching parameter of reflex klystron oscillator.

Power output and efficiency:

For transferring of maximum energy the round trip transit angle, referring to the center of the bunch must be given by

$$\omega(t_2 - t_1) = \omega T_0' = (n - 1/4) = 2N\pi = 2\pi n - \pi/2$$

Where $N = n - 1/4$, 'n' is any positive integer for cycle number. 'N' is the mode numbers (no.of modes).

The induced current in the cavity of reflex klystron can be derived similar to the two cavity klystron and is given by

$$i_2 = -I_0 - \sum_{n=1}^{\infty} 2I_0 J_n(nX')[\cos n(\omega t_2 - \omega\tau - \omega T_0')]$$

$$i_2 = -I_0 - \sum_{n=1}^{\infty} 2I_0 J_n(nX')[\cos n(\omega t_2 - \theta_g - \theta_0')]$$

In above equation –ve sign indicates that the current induced after reflection by the repeller. The fundamental component of a.c. current is given by

$$i_2 = 2I_0 J_1(X')[\cos(\omega t_2 - \theta_g - \theta_0')]$$

The magnitude of induced current in the cavity is given by

$$I_2 = \beta_i i_2 = 2I_0\beta_i J_1(X')$$

The average value of the power delivered to the load is given by

$$P_{ac} = \frac{V_1 I_2}{2} = \frac{2I_0\beta_i J_1(X')V_1}{2} = V_1 I_0\beta_i J_1(X') \qquad -1$$

We know that,

$$X' = \frac{V_1\beta_i}{2V_0}\theta_0' \qquad -2$$

$$\theta_0' = \omega T_0' = 2n\pi - \frac{\pi}{2} \qquad -3$$

Substitute equation 3 in equation 2

$$X' = \frac{V_1\beta_i}{2V_0}\left(2n\pi - \frac{\pi}{2}\right)$$

$$V_1 = \frac{2X'V_0}{\beta_i\left(2n\pi - \frac{\pi}{2}\right)} \qquad -4$$

Substitute equation 4 in equation 1

$$P_{ac} = \frac{2X'V_0}{\beta_i\left(2n\pi - \frac{\pi}{2}\right)} I_0\beta_i J_1(X') = \frac{2V_0I_0X' J_1(X')}{2n\pi - \frac{\pi}{2}} \qquad -5$$

The above equation is known as output power of a reflex klystron oscillator.
The efficiency of the reflex klystron oscillator is given by

$$\eta = \frac{P_{ac}}{P_{dc}} = \frac{\frac{2V_0I_0X' J_1(X')}{2n\pi - \frac{\pi}{2}}}{V_0I_0} = \frac{2X' J_1(X')}{2n\pi - \frac{\pi}{2}} \qquad -6$$

In practice the mode of n = 2 has the most power output. The factor $X' J_1(X')$ reaches a maximum when X = 2.408. Then the efficiency for mode 2 is given by

$$\eta = \frac{2X' J_1(X')}{2n\pi - \frac{\pi}{2}} = \frac{2(2.408)\, J_1(2.408)}{2(2)\pi - \pi/2} = \frac{2(2.408)(0.52)}{2(2)\pi - \pi/2} \times 100 = 22.7\%$$

Oscillating modes and o/p characteristics:

The following graph represents the mode characteristics of reflex klystron oscillator. From the figure we can observe that, the output power is maximum for 1 ¾ mode. The power output is approximately 400 mW for 1 ¾ mode. The output power is less for 2 ¾ mode and 3 ¾ mode as compared with 1 ¾ mode. Therefore the reflex klystron oscillator should be operated at 1 ¾ mode to get high power output.

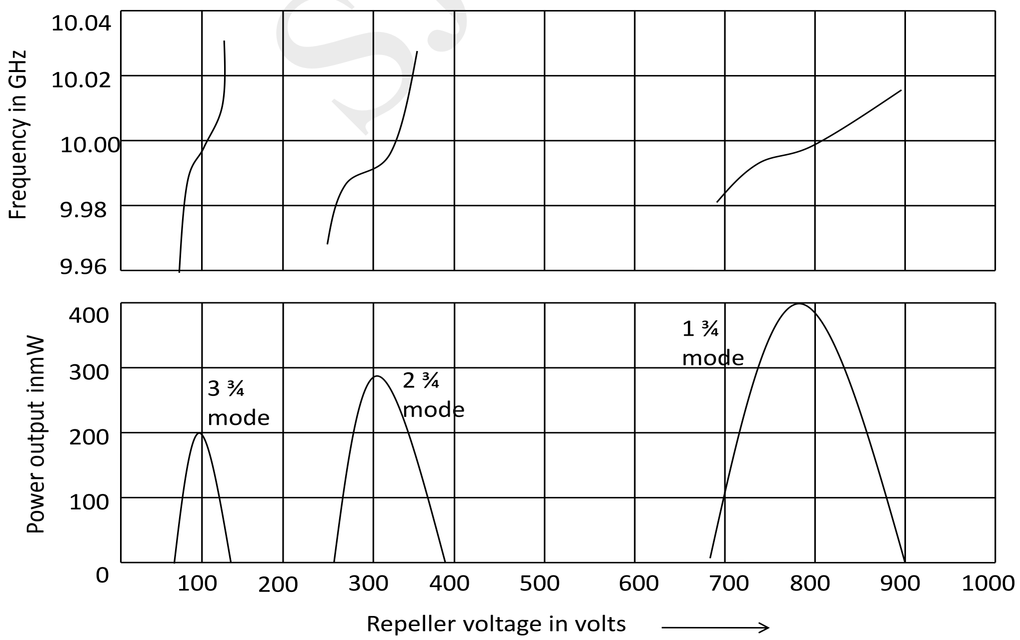


Fig: Power output and frequency characteristics of a reflex klystron

TRAVELLING WAVE TUBE (TWT)

Significance, types and characteristics of slow wave structure:

As the operating frequency is increased, both the inductance and capacitance of the resonant circuit must be decreased in order to maintain resonance at the operating frequency. Because the gain bandwidth product is limited by the resonant circuit, the resonator cannot generate a large output.

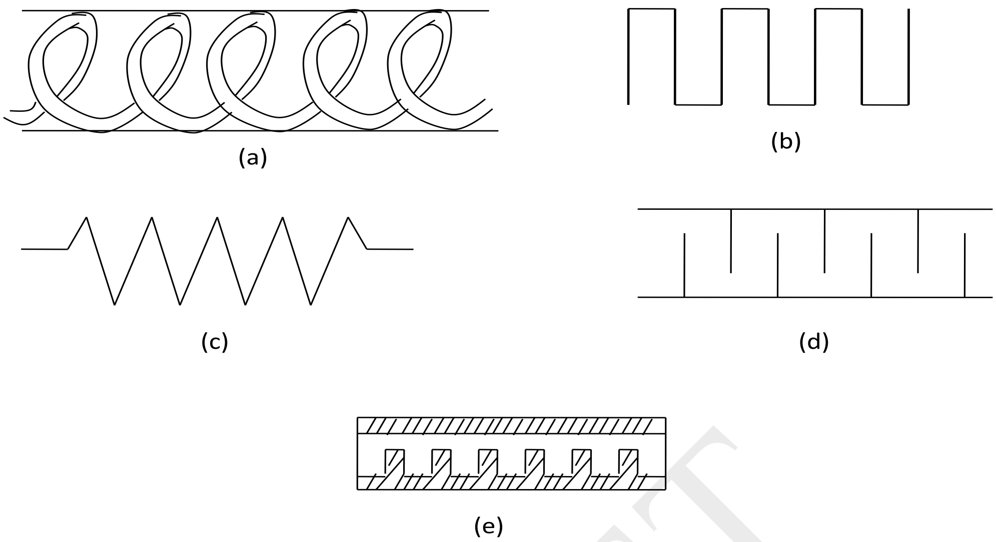
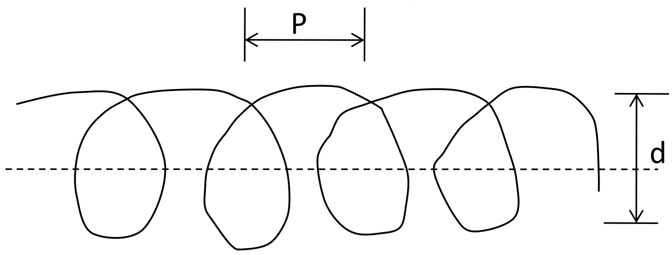
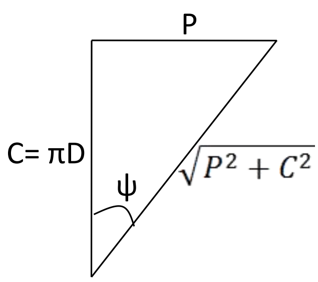


Fig: Slow-wave structures. (a) Helical line (b) Folded-back line (c) Zigzag line (d) Interdigital line (e) Corrugated waveguide

Several nonresonant periodic circuits or slow-wave structures are designed for producing large gain over a wide bandwidth. Various types of slow-wave structures are shown in figure above. Slow-wave structures are special circuits that are used in microwave tubes to reduce the wave velocity in a certain direction so that the electron beam and the signal wave can interact. The commonly used slow-wave structure is a helical line. So let us discuss about the helical line. In the helical line structure RF field or EM waves travels along the turns of the helix with a speed equal to velocity of light(c).





Let ‘τ’ be the time taken by the RF field to travel along one turn of the helix and is given by

$$\tau = \frac{\sqrt{P^2 + C^2}}{c} = \frac{\sqrt{P^2 + (\pi d)^2}}{c} \qquad -1$$

Where ‘d’ is the diameter of the helix, ‘C’ is the circumference of helix, ‘c’ is the velocity of light and ‘P’ is the helix pitch which is defined as the distance traveled by the wave along the helix axis. Let τ' be the time required by the axial electric field to travel the distance of ‘P’ and is given as

$$\tau' = \frac{P}{v_p} \qquad -2$$

Where v_p is the phase velocity of the axial electric field. The time taken by the RF field and axial electric field must be same, why because, these two waves traveling different distances with different velocities.
Therefore equate equations 1 and 2

$$\frac{\sqrt{P^2 + (\pi d)^2}}{c} = \frac{P}{v_p}$$

$$v_p = \frac{P \cdot c}{\sqrt{P^2 + (\pi d)^2}}$$

When $P \ll (\pi d)$, then P can be neglected

$$v_p = \frac{P \cdot c}{\sqrt{(\pi d)^2}} = \frac{P \cdot c}{\pi d}$$

Therefore the velocity of axial electric field is the product of velocity of light(c) and the ratio of pitch to circumference (P/πd).

$$v_p = \frac{\omega}{\beta} = \frac{P \cdot c}{\pi d} \qquad -3$$

The following figure shows the ω-β diagram or Brillouin diagram for helical slow-wave structure. The helix ω-β diagram is very useful in designing a helical slow-wave structure. Once β(phase constant) is known then the phase velocity (v_p) can be computed from the equation for a given dimension of the helix.

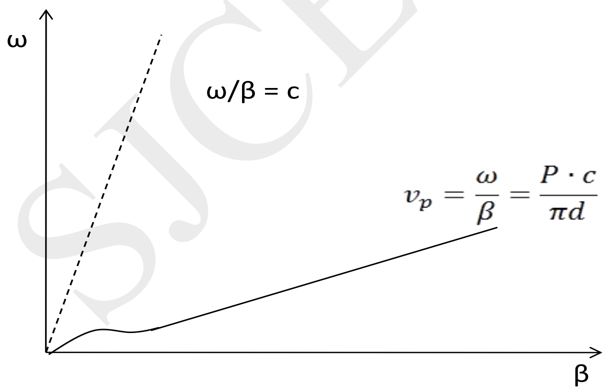


Fig: ω-β diagram for helical structure

Structure of TWT, Bunching process and amplification process:

The schematic structure of Traveling Wave Tube is shown in figure below. The simplified structure or circuit of helix TWT is shown in figure below. Kompfner invented the helix TWT in 1944. Before starting to describe the TWT, it seems appropriate to compare the basic operating principle of both TWT and klystron amplifier. In case of TWT the microwave circuit is a nonresonant and the wave travels with the same speed as the electrons in the beam. The initial effect on the beam is small amount of velocity modulation caused by the weak electric fields associated with the traveling wave and this velocity modulation later translates to current modulation, which then induces RF current in the circuit, causing amplification. However, there are some major differences between the TWT and klystron which are

(i) The interaction of electron beam and RF field in the TWT is continuous over the entire length of the circuit, but the interaction in the klystron occurs only at the gaps of a resonant cavities.

(ii) The wave in the TWT is a propagating wave, where as in klystron it not propagating wave.

In the coupled cavity TWT there is a coupling effect between cavities, where as in klystron each cavity operates independently.

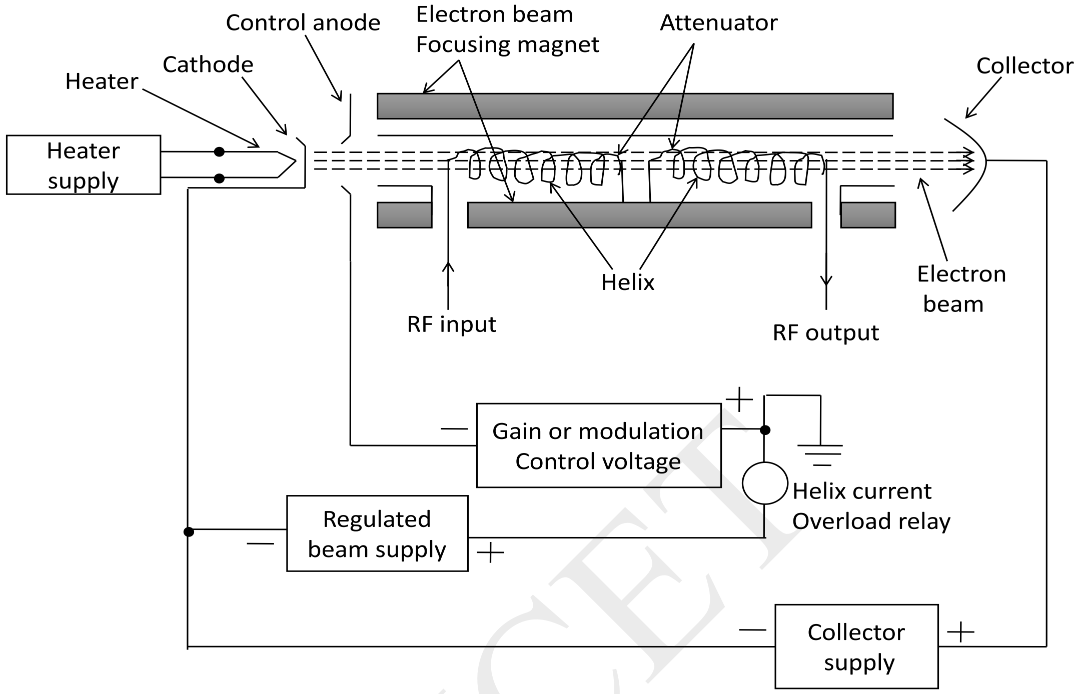


Fig: Schematic diagram of helix traveling wave tube

A helix TWT consists of electron beam and slow-wave structure. The electron beam is focused by a constant magnetic field along the electron beam and slow-wave structure. The slow-wave structure is either the helical type or folded-back line. The TWT has an electron gun to produce a narrow electron beam which in turn passed through the centre of the long axial helix. A magnetic focusing is provided to prevent the beam from spreading and to guide it through the centre of the helix. A signal to be amplified is applied to the one end of the helix adjacent to the electron gun. The amplified signal appears at the output or other end of the helix.

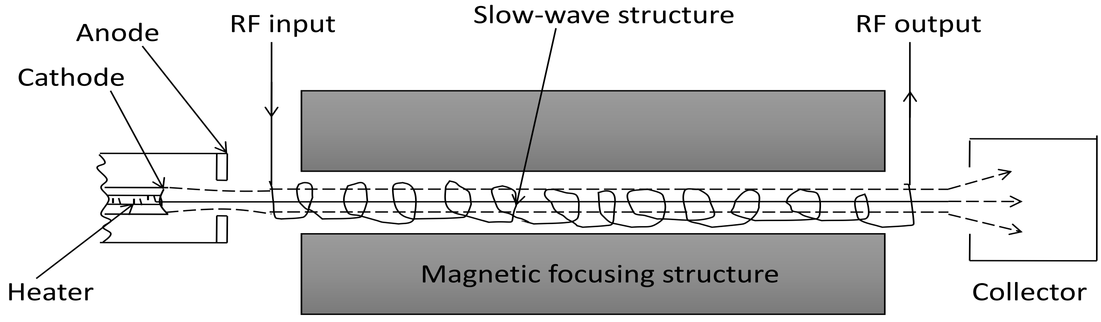


Fig: Simplified circuit of helix TWT

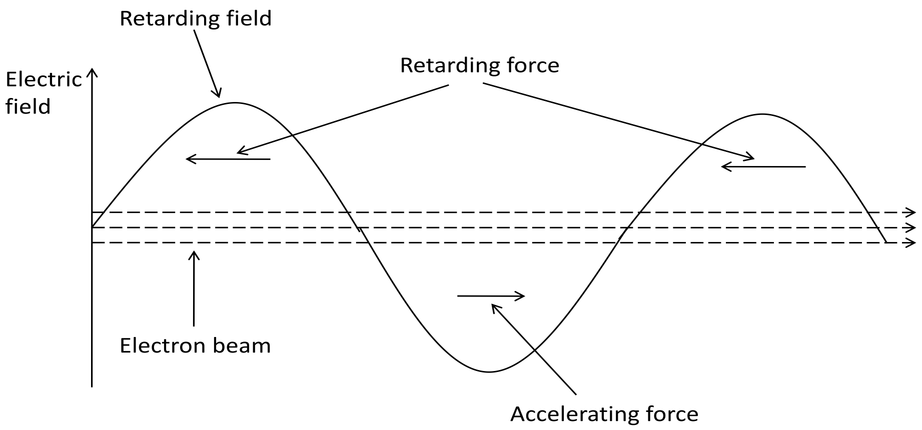


Fig: interaction between electron beam and electric filed

The basic principle can be explained as follows: The RF signal (input) propagating around the turns of the helix produces an electric field at the centre of the helix, which is called axial electric field. The RF signal is propagating along the helix with a velocity equal to velocity of light, where as the axial electric field travels with velocity which equal to the velocity of light multiplied with the ratio of helix pitch to helix circumference. The purpose of helix is to reduce the velocity of axial electric field such that the interaction will takes place between electron beam and axial electric field. The interaction is such that on an average the electron delivers energy to the wave on the helix. As a result, the signal wave grows and amplified output is obtained. The mechanism by which the electrons transfer energy to the RF input can be explained with the help of following figure.

As shown in above figure, the electrons which are entering the retarding field(+ve half cycle of input or RF signal) are decelerated because the force applied by that field is opposite to the motion of electrons(F = -QE). Similarly the electrons which are entering the RF field during its accelerating field(-ve half cycle) are accelerated by the accelerating force. The velocity of the electrons which are entering during the zero point of the input signal is not changed. Therefore due to above mechanism velocity modulation takes place and electrons form as a bunch. This electron bunch delivers energy to the input signal. Since the velocity of electrons is slightly greater than the axial wave velocity, more electrons are in the retarding field than in accelerating field and a great amount of energy is transferred to the electromagnetic field from the electron bunches because when electrons are slowed down they deliver the energy. Whereas accelerating electrons extracts the energy from the RF field. The bunch continuous to become more compact and large amplification of the signal voltage occurs at the end of helix. The attenuator is placed at the center of the helix to reduce all the waves which are traveling along the helix from load, so that the reflected waves from the mismatched load can be prevented from reaching the input and causing oscillations.

CROSSED FIELD TUBES

Cross field effects:

Crossed-field tubes derive their name from the fact that the dc electric field and the dc magnetic field are perpendicular to each other. They are also called M-type tubes. In crossed-filed tubes, the electrons emitted by the cathode are accelerated by the electric field and gain velocity, but the greater the velocity, the more their path is bent by the magnetic field. If an RF field is applied to the anode circuit, those electrons entering the circuit during the retarding filed are decelerated and give up some of their energy to the RF field. Consequently, their velocity is decreased, and these slower electrons will then travel the dc electric field far enough to regain essentially the same velocity as before. Because of the crossed-field interactions, only those electrons that have given up sufficient energy to the RF field can travel all the way to the anode.

MAGNETRON OSCILLATOR

Different types

Basically there are three types of magnetrons such as

(i) Split anode magnetron
(ii) Cyclotron frequency magnetron
(iii) Traveling wave magnetron

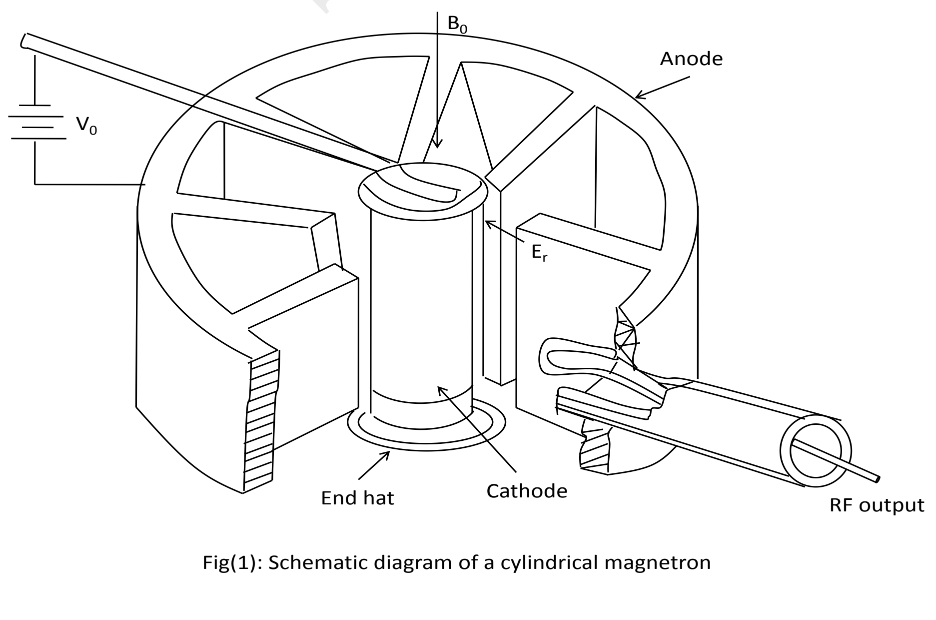
Commonly, traveling wave magnetron is preferable as compared with other two. In traveling wave magnetron there are different types and are given by

(i) Cylindrical magnetron
(ii) Linear or planar magnetron
(iii) Coaxial magnetron
(iv) Voltage tunable magnetron
(v) Inverted coaxial magnetron
(vi) Frequency-agile magnetron

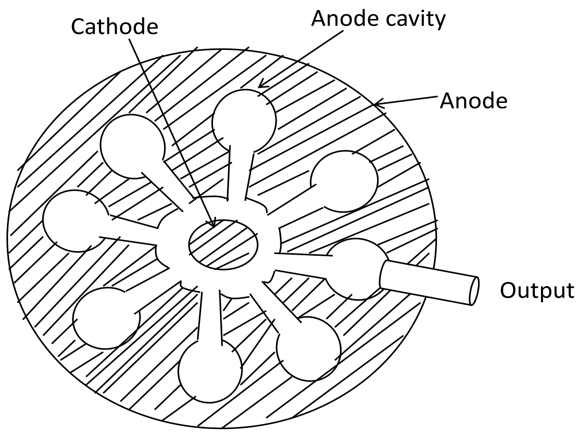
Cylindrical traveling wave magnetron

The schematic diagram of a cylindrical magnetron is shown in figure 1 below. This type of magnetron is also called as conventional magnetron.

Constructional features: The cavity magnetron or traveling wave magnetron has cylindrical construction employing a radial electric field (E_r) and axial magnetic field (B_0) and anode structure with permanent cavities. Figure 2, shows the cross sectional view of the magnetron. Here cylindrical cathode is surrounded by the anode with cavities and thus radial d.c. electric field will exist. With the help of some permanent magnet, a magnetic field will be applied to the magnetron such that magnetic lines of force are at right angles to the plane of the radial electric field. Since the magnetic field is perpendicular to the radial electric field, the magnetron also called as crossed-field device.



Fig(1): Schematic diagram of a cylindrical magnetron

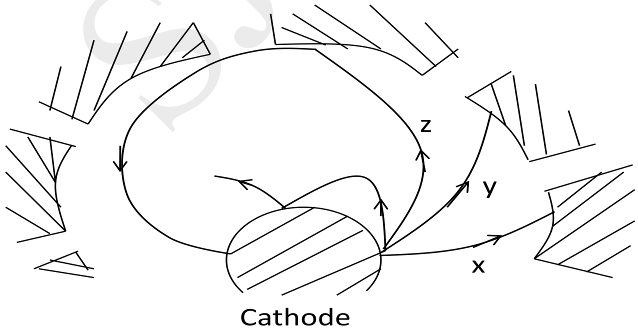


Fig(2): Simplified diagram of magnetron

The output of the magnetron oscillator is taken from one of the cavities by means of a coaxial line as shown in figure 2. The material used for cathode is copper. There are number of resonant cavities in magnetron and each cavity having its resonant frequency. Therefore magnetron
having many number of modes of operation which are equal to number of cavities. For example if a magnetron having 8 cavities, then it has 8 modes of operation. The phase difference of the waves which are existing between the adjacent cavities must be chosen such that total phase difference of all the anode cavities must be 360^0. For example if magnetron having 8 cavities, the phase difference between the adjacent cavities must be 45^0.

pi-mode operation:

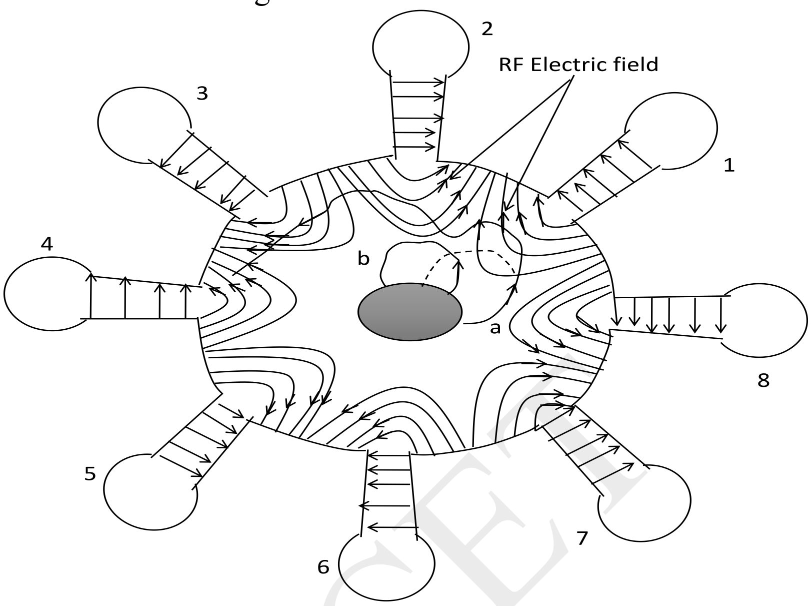
When magnetic and electric fields acts simultaneously upon the electron, its path can have any number of shapes depends upon their relative strength of the mutually perpendicular electric and magnetic fields. Some of these electron paths are shown in figure 3.



Fig(3): Electron paths in magnetron

The d.c voltage (V_0) is applied to the cathode of the magnetron with respect to the anode. Due to this d.c voltage electric field will be existing between the anode and cathode. The direction of electric field is from anode to cathode. This electric field is in the direction of radius of cylindrical magnetron and hence the electric field is known as radial eclectic field. The electrons will be emitted or generated at the cathode and travels towards the anode under the influence of force exerted by the electric field. When the magnetic field is zero, the electron goes straight away from the cathode to anode. This is indicated as path 'x' as shown in figure 3. When the magnetic field is small, but definite strength, it will exert as a lateral force on the electron, as a result the electrons takes a bending path which is shown as path 'y' in figure 3. As the electron approaches the anode, its velocity continuous to increase radially as it is accelerating. Therefore, the effect of magnetic field upon electron increases so that the path curvature becomes sharper as the electron approaches the anode.

It is possible to make the magnetic field so strong the electrons will not reach the anode at all. The magnetic field required to return the electrons to the cathode after they have just grazed the anode is called cutoff field. The resulting path ‘z’ is shown in figure 3. The electric field (RF field) distribution in π-mode (PI-mode) cavity magnetron is shown in figure 4 below.



Fig(4): Field distribution in PI-mode magnetron

The phase difference between the adjacent cavities is 180^0. Since the cavities are resonant in nature, RF oscillations are present due to noise or transient. These small RF oscillations will be sustained by the taking the energy from the electrons. We have to choose the axial magnetic field and radial electric field such that the more number of electrons should reach the cavity gap voltage(RF voltage) at a proper time interval. Then all the electrons give up their energy to the RF electric field. As shown in figure 4, the electron ‘a’ is enters the RF electric field in the cavity. The phase of the signal (RF Signal) in the cavity is such that the electron may accelerated or decelerated (retarded) depending upon the accelerating force or retarding force by the RF oscillations. If the electron ‘a’ is slowed down by the cavity gap voltage, then that electron gives its energy to the RF oscillations. Once electron slowed down, it will take different path as shown by the electron ‘a’ in figure 4.

In a magnetron a self consistent oscillations can exist only if the phase difference (φ) between the adjoining anode poles is nπ/4, where ‘n’ is integer. For best results n = 4 is used in practice. The resulting π-mode (if n = 4, φ = π) oscillations are shown in figure 4. In the absence of RF electric field electrons ‘a’ and ‘b’ would have followed the paths shown by the dotted line a & b respectively, but RF field naturally modifies these paths. The RF oscillations also exist inside the cavity resonators. Due to the application of d.c voltage to the cathode of the magnetron, the electrons tries to travel straight away toward the anode but due to the application of axial magnetic field the electrons will take a bending paths as shown in figure 4. The electron ‘a’ enter in the cavity gap RF oscillations. The direction of the motion of the electron ‘a’ is in the same direction of the RF electric field of 1st cavity and hence the electron ‘a’ will be slowed down by the electric field because the electric field produces opposing force on electrons. Therefore the electron ‘a’ give up energy to the RF oscillations. But the electron ‘b’ is entering the RF electric field of cavity 2 as shown in figure 4. The electron ‘b’ will be accelerated by the RF electric field and gains energy from the RF electric field instead of giving the energy. So we must choose the magnetic field and radial electric field such that large number of electrons should give their energy to the RF oscillations. Due to effect of RF electric field on the electron, bunching will takes place because of each cavity having opposite phase with respect to adjacent cavity. Instead of individual electrons, bunch of electrons traveling towards the anode cavities and give their large amount of energy to the RF oscillations.

Power output and efficiency:

The output characteristics of magnetron will be studied by means of Rieke diagram shown in figure below.

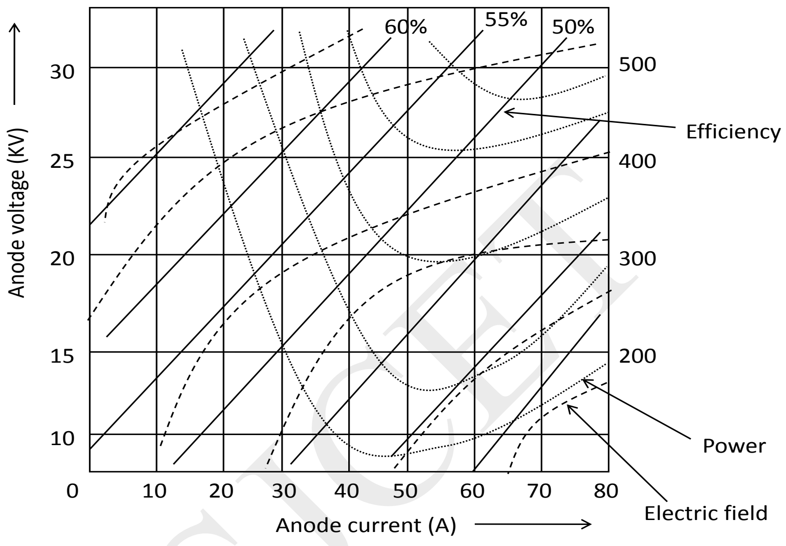
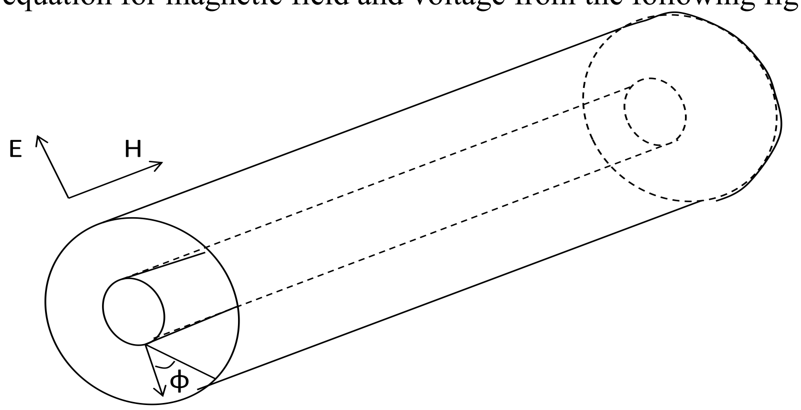


Fig: Rieke diagram

The Rieke diagram is nothing but a chart or graph which is used to design or study the performance characteristics of a magnetron. The Rieke diagram represented in terms of anode voltage and anode current. With the help of Rieke diagram we can find the power, electric field and efficiency by knowing the anode voltage and anode current.

Hartree Condition:

In order to understand the Hull cutoff condition and Hartree resonance condition, let us derive the equation for magnetic field and voltage from the following figure.



Let ‘a’ be the radius of cathode, ‘b’ be the radius of anode and ‘ϕ’ be the angular displacement of the electron bends. Force acting on the electron due to magnetic field is given by

$$F = Bev \qquad -1$$

Where ‘B’ is the magnetic flux density, ‘e’ is the charge of electron and ‘v’ is the velocity of electron. In the direction of ϕ the force is given by

$$F_{\phi} = Bev_{\rho} \qquad -2$$

Where ‘ρ’ is the radial distance from the cathode, ‘v_ρ’ is the velocity of electrons in the direction of radial distance (ρ). The torque in ϕ direction is given by

$$T_{\phi} = \rho F_{\phi} \qquad -3$$

Substitute equation 2 in equation 3

$$T_{\phi} = \rho Bev_{\rho} \qquad -4$$

Angular momentum is equal to the multiplication of angular velocity and moment of inertia. i.e.

$$Angular\ momentum = \frac{d\phi}{dt} \times m\rho^2$$

Since ϕ is the angular displacement, the rate of change of angular displacement is nothing but angular velocity. In equation 4 ‘m’ is the mass of electron. The time rate of change of angular momentum is nothing but a torque. i.e

$$T = \frac{d}{dt}\left(\frac{d\phi}{dt} \times m\rho^2\right) \qquad -5$$

Equate equations 4 and 5

$$\rho Bev_{\rho} = \frac{d}{dt}\left(\frac{d\phi}{dt} \times m\rho^2\right) = 2m\rho\frac{d\phi}{dt} + m\rho^2\frac{d^2\phi}{dt^2}$$

Take integration on both sides with respect to ‘t’,

Then $\int \rho Bev_{\rho} = \int 2m\rho\frac{d\phi}{dt} + \int m\rho^2\frac{d^2\phi}{dt^2}$

$$Be\int \rho v_{\rho}.dt = 2m\rho\int\frac{d\phi}{dt}.dt + m\rho^2\int\frac{d^2\phi}{dt^2}.dt$$

But $v_{\rho} = \frac{d\rho}{dt}$

$$Be\int \rho\frac{d\rho}{dt}.dt = 2m\rho\int\frac{d\phi}{dt}.dt + m\rho^2\int\frac{d^2\phi}{dt^2}.dt$$

$$Be\int \rho\, d\rho = 2m\rho\int d\phi + m\rho^2\int\frac{d^2\phi}{dt}$$

$$eB\frac{\rho^2}{2} = 2m\rho\phi + m\rho^2\frac{d\phi}{dt} \qquad -6$$

For particular direction the parameters ϕ, mρϕ, can be thought of constant and let it be ‘c’, then equation 6 becomes

$$eB\frac{\rho^2}{2} = m\rho^2\frac{d\phi}{dt} + c \qquad -7$$

From figure, at $\rho = a,\ \frac{d\phi}{dt} = 0$, then apply this condition to equation 7

$$eB\frac{a^2}{2} = 0 + c$$

$$c = eB\frac{a^2}{2} \qquad -8$$

Substitute equation 8 in equation 7

$$eB\frac{\rho^2}{2} = m\rho^2\frac{d\phi}{dt} + eB\frac{a^2}{2}$$

$$m\rho^2\frac{d\phi}{dt} = eB\frac{\rho^2}{2} - eB\frac{a^2}{2} = \frac{eB}{2}(\rho^2 - a^2)$$

$$\frac{d\phi}{dt} = \frac{eB}{2m\rho^2}(\rho^2 - a^2) = \frac{eB}{2m}\left(1 - \frac{a^2}{\rho^2}\right) \qquad -9$$

From above equation, when ρ = a, then $\frac{d\phi}{dt} = 0$ that is angular velocity is equal to zero and when $\rho \gg a$, then $\frac{d\phi}{dt}$ will be maximum. That is

$$\left(\frac{d\phi}{dt}\right)_{max} = \omega_{max} = \frac{eB}{2m} \qquad -10$$

In above equation $\frac{a^2}{\rho^2}$ is neglected, because ρ^2 is large as compared with a^2.

We know that,

$$eV_0 = \frac{1}{2}mv^2 = \frac{1}{2}m\left({v_\rho}^2 + {v_\phi}^2\right) \qquad -11$$

Where v_ρ $and\ v_\phi$ are the components of 'v' in 'ρ' and' ϕ' directions respectively. V_0 is the applied d.c voltage to the cathode. But

$$v_\rho = \frac{d\rho}{dt} \quad and\ v_\phi = \rho\frac{d\phi}{dt} \qquad -12$$

Substitute equation 12 in equation 11

$$eV_0 = \frac{1}{2}m\left(\left(\frac{d\rho}{dt}\right)^2 + \left(\rho\frac{d\phi}{dt}\right)^2\right) \qquad -13$$

From equations 9 and 10 we have

$$\frac{d\phi}{dt} = \omega_{max}\left(1 - \frac{a^2}{\rho^2}\right) \qquad -14$$

Substitute equation 14 in equation 13

$$eV_0 = \frac{1}{2}m\left(\left(\frac{d\rho}{dt}\right)^2 + \left(\rho\omega_{max}\left(1 - \frac{a^2}{\rho^2}\right)\right)^2\right) \qquad -15$$

At $\rho = b,\ \frac{d\rho}{dt} = 0,\ \ then$ equation 15 becomes

$$eV_0 = \frac{1}{2}m\left((0)^2 + \left(b\omega_{max}\left(1 - \frac{a^2}{b^2}\right)\right)^2\right)$$

$$= \frac{1}{2}mb^2{\omega_{max}}^2\left(1 - \frac{a^2}{b^2}\right)^2 \qquad -16$$

Substitute equation 10 in equation 16

$$eV_0 = \frac{1}{2}mb^2\left(\frac{eB}{2m}\right)^2\left(1 - \frac{a^2}{b^2}\right)^2 = \frac{b^2e^2B^2}{8m}\left(1 - \frac{a^2}{b^2}\right)^2 \qquad -17$$

$$Since\ \ b^2 \gg a^2,\ then\ \ \frac{a^2}{b^2}\ \ can\ be\ neglected$$

$$eV_0 = \frac{b^2e^2B^2}{8m} \qquad -18$$

$$B = B_c = \sqrt{\frac{8V_0m}{eb^2}} = \frac{1}{b}\sqrt{\frac{8V_0m}{e}} \qquad -19$$

The above equation is known as hull cutoff magnetic equation. That is hull cutoff magnetic field is defined as the magnetic field above which the electrons will not reach the anode. Therefore when the magnetic field is greater than B_c, then the electrons will return to the cathode and this returned electrons will produce a back heating. From equation 17, the equation for B_c can also expressed as

$$B_c = \frac{\sqrt{8V_0 m/e}}{b\left(1 - \frac{a^2}{b^2}\right)} \qquad -20$$

Again from equation 17, we can have

$$V_0 = V_{0c} = \frac{b^2e^2B^2}{8me}\left(1 - \frac{a^2}{b^2}\right)^2 = \frac{b^2eB^2}{8m}\left(1 - \frac{a^2}{b^2}\right)^2 \qquad -21$$

The equation 21 is known as hull cutoff voltage equation. That is the hull cutoff voltage is defined as the voltage below which the electrons will not reach the anode. The Hull cutoff condition determines the anode voltage or magnetic field necessary to obtain the non-zero anode current as a function of the magnetic field or anode voltage in the absence of the Electromagnetic field. The hartree anode voltage is given by

$$V_{oh} = \frac{\omega B_0 d}{\beta} - \frac{\omega^2}{\beta^2}\frac{m}{2e}$$

Where 'ω' is the angular frequency, B_0 is the magnetic field, 'd' is the spacing between the cathode and anode, 'β' is the phase constant, 'm' is the mass of the electron and 'e' is the charge of electron.

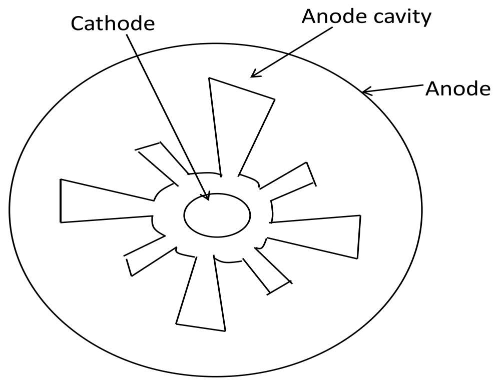
Mode jumping in Magnetron:

The number of resonant frequencies of a magnetron oscillator depends upon the number of cavities in the magnetron. For example, if magnetron having 8 cavities, then there are 8 possible modes of operation. In case of normal magnetron the wavelength of different modes differ very slightly from adjacent modes. Due to this small difference in frequency of one mode with adjacent mode, there is a possibility of mode jumping. That means when the magnetron is operating in one mode, it may enter into the other node which is adjacent to the operating mode. For example, let N = 8 cavities, then the phase difference between the adjacent cavities is given by

$$\Phi_n = \frac{2\pi n}{N} = \frac{2\pi n}{8} = \frac{n\pi}{4}$$

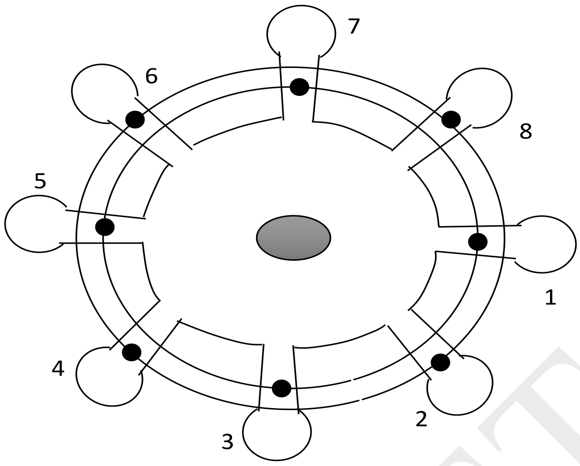
Where 'n' is integer

To get satisfactorily results, we need to avoid this mode jumping. To avoid the mode jumping one method is, changing the cavity structure of the anode which is shown in figure below.



Fig(2): Rising sun magnetron

The structure shown in above figure is known as rising sun magnetron. From it can be observed that, the adjacent cavities having different shape such that there will be large difference in the resonant frequency of each cavity from the adjacent cavity. One more method to avoid the mode jumping is strapping which is shown in figure below. In the strapping method two rings will be wounded around the anode structure such that the rings should the cavities. One ring is connected to cavity numbers 1, 3, 5 & 7 such that these four cavities operated together and more difference in the frequency between the cavities. Similarly second ring is connected to the cavity numbers 2, 4, 6 & 8.



Fig(4): Strapping scheme for PI-mode

SOLVED PROBLEMS

1. **A two cavity klystron amplifier has the following parameters:**

$$V_0 = 1000\,V, \quad R_0 = 40\,K\Omega, \quad I_0 = 25\,mA, \quad f = 3\,GHz$$

Gap spacing in either cavity (d) = 1 mm
Spacing between the two cavities (L) = 4 cm
Effective shunt impedance excluding beam loading (R_{sh}) = 30 KΩ

(a) Find the input gap voltage to give maximum voltage V_2
(b) Find the voltage gain, neglecting the beam loading in the output cavity
(c) Find the efficiency of the amplifier, neglecting beam loading
(d) Calculate the beam loading conductance and show that neglecting it was justified in the preceding calculations.

Answer: Given data:

$$V_0 = 1000\,V, \quad R_0 = 40\,K\Omega, \quad I_0 = 25\,mA, \quad f = 3\,GHz$$

Gap spacing in either cavity (d) = 1 mm
Spacing between the two cavities (L) = 4 cm
Effective shunt impedance excluding beam loading (R_{sh}) = 30 KΩ

(a) For maximum V_2, $J_1(X)$ must be maximum. That is $J_1(X) = 0.582$ at $X = 1.841$(from Bessel function table)

$$v_0 = 0.593 \times 10^6\sqrt{V_0} = 0.593 \times 10^6\sqrt{1000} = 1.88 \times 10^7\,m/s$$

$$\theta_g = \frac{\omega d}{v_0} = \frac{2\pi f d}{v_0} = \frac{2 \times \pi \times 3 \times 10^9 \times 1 \times 10^{-3}}{1.88 \times 10^7} = 1\,rad$$

$$\beta_i = \beta_o = \frac{sin(\theta_g/2)}{\theta_g/2} = \frac{sin(1X57.3/2)}{1/2} = 0.952$$

$$\theta_o = \omega T_o = \frac{\omega L}{v_o} = \frac{2\pi f L}{v_o} = \frac{2\pi \times 3 \times 10^9 \times 0.04}{1.88 \times 10^7} = 40\ rad$$

$$\mathrm{X} = \frac{\beta_i V_1 \theta_o}{2V_o}$$

$$V_1 = \frac{2V_o X}{\beta_i \theta_o} = \frac{2 \times 1000 \times 1.841}{0.952 \times 40} = 96.7\ V$$

(b)

$$A_v = \frac{|V_2|}{|V_1|}$$

$$I_2 = 2I_o J_1(X) = 2 \times 25 \times 10^{-3} \times 0.582 = 29.1\ mA$$

$$V_2 = \beta_o I_2 R_{sh} = 0.952 \times 29.1 \times 10^{-3} \times 30 \times 10^3 = 831\ V$$

$$A_v = \frac{|V_2|}{|V_1|} = \frac{831}{96.7} = 8.61$$

$$A_v = 20\ log(8.61) = 18.7\ dB$$

(c)

$$\%\ \mathrm{Efficiency} = \frac{\beta_o I_2 V_2}{2 I_o V_o} = \frac{0.952 \times 29.1 \times 10^{-3} \times 831}{2 \times 25 \times 10^{-3} \times 1000} \times 100 = 46.2\%$$

(d)

The beam loading conductance is given by

$$G_b = \frac{G_o}{2}\left(\beta_o^2 - \beta_o cos(\frac{\theta_g}{2})\right) = \frac{I_o/V_o}{2}\left(\beta_o^2 - \beta_o cos(\frac{\theta_g}{2})\right)$$

$$\mathrm{G_b} = \frac{25 \times 10^{-3}/1000}{2}\left(0.952^2 - 0.952 cos(1/2)\right) = 8.8 \times 10^{-7}\ mhos$$

Beam loading resistance is given by

$$R_b = \frac{1}{G_b} = \frac{1}{8.8 \times 10^{-7}} = 1.14 \times 10^6\ ohms$$

Example 9-4-1: Reflex Klystron

A reflex klystron operates under the following conditions:

$$V_0 = 600 \text{ V} \qquad L = 1 \text{ mm}$$

$$R_{sh} = 15\text{k}\Omega \qquad \frac{e}{m} = 1.759 \times 10^{11} \text{ (MKS system)}$$

$$f_r = 9 \text{ GHz}$$

The tube is oscillating at f_r at the peak of the $n = 2$ mode or $1\frac{3}{4}$ mode. Assume that tl transit time through the gap and beam loading can be neglected.

a. Find the value of the repeller voltage V_r.

b. Find the direct current necessary to give a microwave gap voltage of 200 V.

c. What is the electronic efficiency under this condition?

Solution

a. From Eq. (9-4-22) we obtain

$$\frac{V_0}{(V_r + V_0)^2} = \left(\frac{e}{m}\right)\frac{(2\pi n - \pi/2)^2}{8\omega^2 L^2}$$

$$= (1.759 \times 10^{11})\frac{(2\pi 2 - \pi/2)^2}{8(2\pi \times 9 \times 10^9)^2(10^{-3})^2} = 0.832 \times 10^{-3}$$

$$(V_r + V_0)^2 = \frac{600}{0.832 \times 10^{-3}} = 0.721 \times 10^6$$

$$V_r = 250 \text{ V}$$

b. Assume that $\beta_0 = 1$. Since

$$V_2 = I_2 R_{sh} = 2I_0 J_1(X')R_{sh}$$

the direct current I_0 is

$$I_0 = \frac{V_2}{2J_1(X')R_{sh}} = \frac{200}{2(0.582)(15 \times 10^3)} = 11.45 \text{ mA}$$

c. From Eqs. (9-4-11), (9-4-12), and (9-4-20) the electronic efficiency is

$$\text{Efficiency} = \frac{2X'J_1(X')}{2\pi n - \pi/2} = \frac{2(1.841)(0.582)}{2\pi(2) - \pi/2} = 19.49\%$$

UNIT-5 (MICROWAVE SEMICONDUCTOR DEVICES, ANTENNAS AND MICROWAVE MEASUREMENTS)

Syllabus: Microwave Semiconductor Devices: Gunn Oscillator – Principle of operation, Characteristics, Two valley model, IMPATT, TRAPATT diodes.

Antennas and Microwave Measurements: Sources of errors, Patterns to be Measured, Pattern Measurement Arrangement, Directivity Measurement, Gain Measurements (by comparison, Absolute and 3-Antenna Methods). Description of Microwave bench-different blocks and their features, errors and precautions, Microwave power measurements, Measurement of attenuation, frequency, VSWR (low, medium, high), Measurement of 'Q' of a cavity, Impedance measurements.

MICROWAVE SEMICONDUCTOR DEVICES:

Classification:

Solid state microwave devices are classified as

(a) Based on their electrical behavior

(i) Non-linear resistance type: example varistors (variable resistors)
(ii) Non-linear reactance type: example varactors (variable reactors)
(iii) Negative resistance type: example Tunnel diode, Impatt diode, Gunn diode
(iv) Controllable impedance type: example PIN diode

(b) Based on construction

(i) Point contact diodes
(ii) Schottky barrier diodes
(iii) Metal oxide semiconductor devices
(iv) Metal insulation devices.

Applications:

The applications of microwave solid state devices are given by

(a) The applications of varactor diode:

(i) Harmonic generation
(ii) Microwave frequency multiplication
(iii) Low noise amplification
(iv) Pulse generation and pulse shaping
(v) Tuning stage of a radio receiver
(vi) Active filters
(vii) Switching circuits and modulation of a microwave signal.

(b) Applications of PIN diode:

(i) Used as a switch
(ii) Used as a phase shifter
(iii) Used as a limiter

(c) Applications of schottky barrier diode:

(i) Low noise mixer
(ii) Balanced mixer in CW radar
(iii) Microwave detectors

(d) Applications of Gunn diode:

(i) Radar transmitters
(ii) Pulsed Gunn diode oscillators used in transponders for air traffic control(ATC) and in industry telemetry systems.
(iii) Broadband linear amplifier
(iv) Fast combinational and sequential logic circuits.
(v) Low and medium power oscillator in microwave receivers
(vi) As a pump sources in parametric amplifiers.

GUNN OSCILLATOR

Principle of operation:

Gunn effect diodes are named after J.B.Gunn who in 1963 discovered a periodic fluctuations of current passing through the n-type gallium arsenide (Ga As) specimen when the applied voltage exceeded a certain critical value. The Gunn effect is explained as follows: According to Gunn's observations, when the voltage is applied to a n-type gallium arsenide, the current in every specimen became a fluctuating function of time. This fluctuations occurs when the applied electric (applied voltage) field is exceeding certain threshold value, such as in the range of 2000-4000 volts/cm. In the Ga As this fluctuation took the form of a periodic oscillation. The frequency of this oscillation was determined mainly by the specimen and not by the external circuit. The period of oscillation was usually inversely proportional to the specimen length and closely equal to the transit time of electrons between the electrodes. The Ga As specimen is shown in figure below.

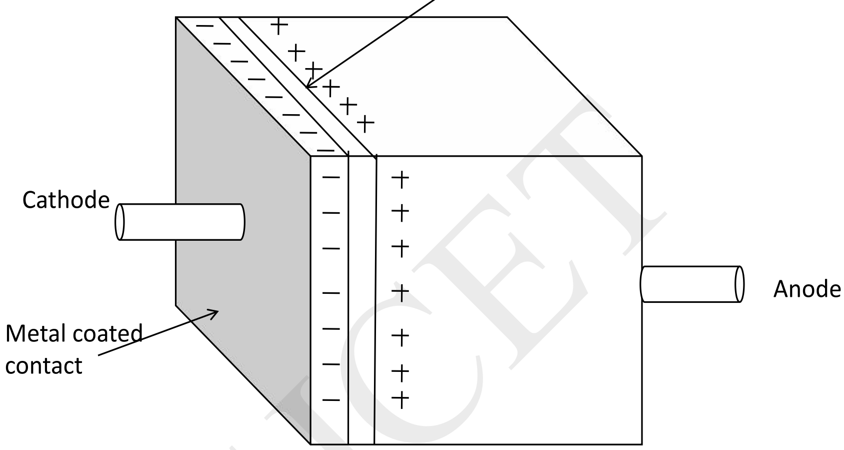


Fig: Schematic diagram for n-type GaAs diode.

The transit time of electrons between the cathode and anode is calculated by knowing the velocity of the electrons which is approximately equal to 10^7 cm/se for gallium arsenide. When the applied voltage or electric field is slowly increases, the carrier drift velocity is linearly increases as shown in figure below.

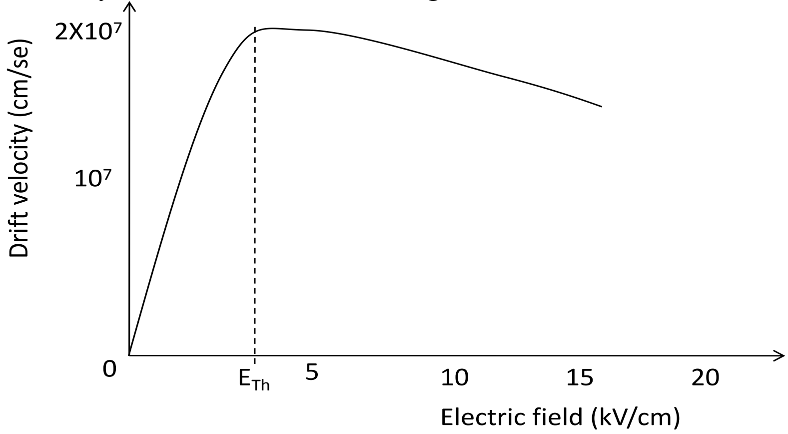


Fig: Drift velocity of electrons in n-type GaAs versus electric field

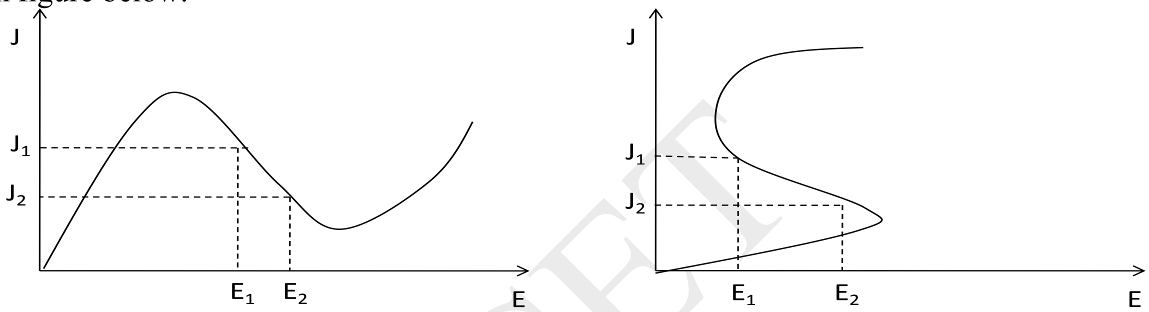
When the applied electric field is greater than the threshold value (E_{Th}), the drift velocity of the electrons or charge carriers decreases as shown in figure. Then the device exhibits the negative resistance. Gunn also discovered that the threshold electric field varied with the length and type of material. The formula for threshold electric field is given as

$$E_{Th} = \frac{V}{L}$$

Where 'V' is the applied voltage and 'L' is the length of the specimen. After threshold value of electric field the velocity decreases means the current also decreases.

Characteristics and Two valley model:

Differential negative resistance: RWH theory is known as Ridley-Watkins-Hilsum Theory. The fundamental concept of the RWH theory is the differential negative resistance in bulk solid state materials when either a voltage (electric field) or a current is applied to the terminals of the sample. There are two modes of negative-resistance devices such as voltage-controlled and current-controlled modes as shown in figure below.



(a) Voltage-controlled mode

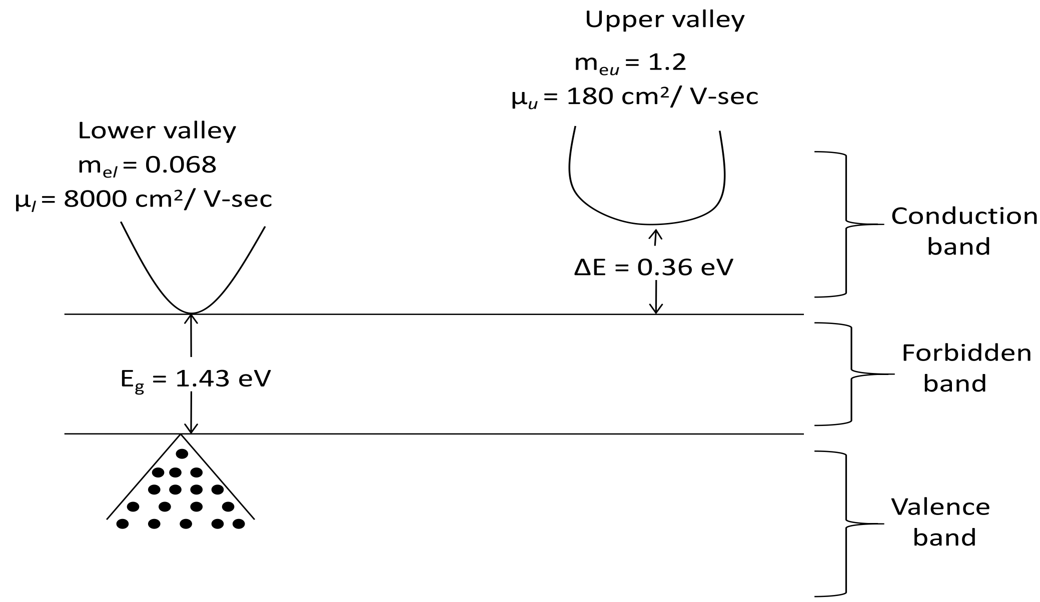
(a) Current-controlled mode

Fig: Diagram of negative resistance

In the voltage-controlled mode the current density can be multivalued, where as in the current-controlled mode the voltage can be multivalued. In a specimen a high field domain will be formed if it is under voltage-controlled mode and high current filament will be formed if it is under current-controlled mode. The negative resistance is given by

$$Negative\,resistance = \frac{E_2 - E_1}{J_2 - J_1}$$

Two-valley model theory: According to energy band theory of the n-type gallium arsenide (GaAs), the conduction band of the GaAs contains two sub bands which are called lower valley and upper valley as shown in figure below.

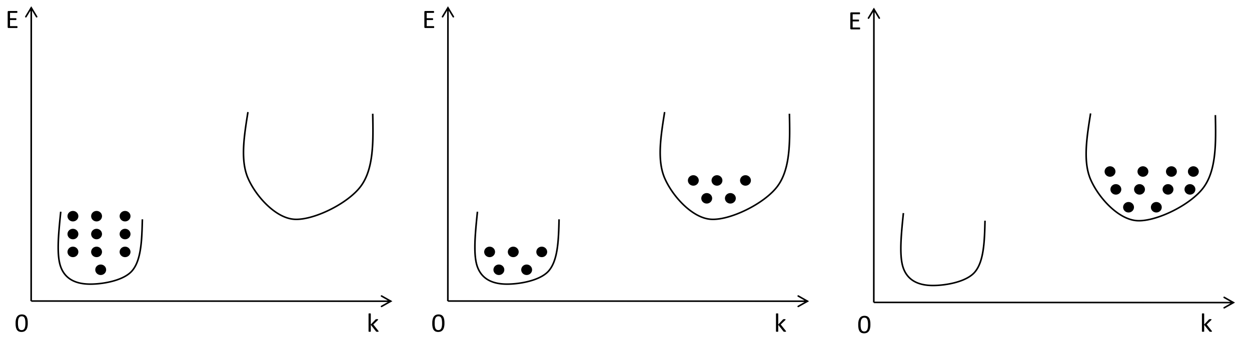


Fig(1): Two-valley model of electron energy versus wave number for n-type GaAs

The data for two valleys is given below.

Valley	Effective mass(m_e)	Mobility(μ)	Separation(ΔE)
Lower	M_{el} = 0.068	μ_l = 8000 cm^2/ V-sec	0.36 eV
upper	M_{el} = 1.2	μ_l = 180 cm^2/ V-sec	0.36 eV

The lower valley having low effective mass and high mobility. When the electrons are entering in to the lower valley of conduction band from the valence band, their effective mass will be decreases and mobility will be increases compared to upper valley. Under the open circuited condition, all the charge carriers present only within the valence band as shown in figure 1 above. When the biasing is applied, the charge carriers enter in to the lower valley of the conduction band as shown in figure 2(a), where 'k' is the wave number. When the applied electric field is less than the energy of lower valley (E_l) then all the electrons will be present within the lower valley.



(a) $E < E_l$ (b) $E_l < E < E_u$ (c) $E_u < E$

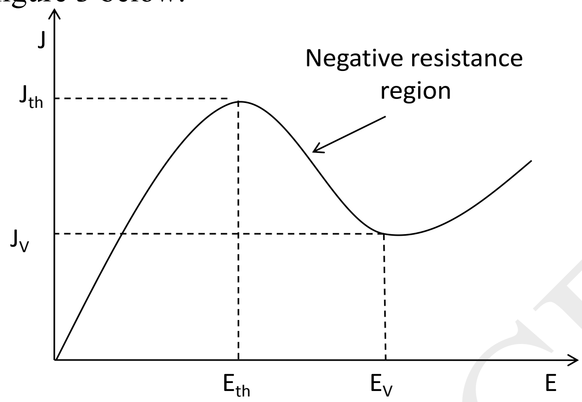
Fig(2): Transfer of electron densities

Since each energy state in the conduction band has its own energy level. Lower valley having less energy compared to upper valley, but lower valley having higher energy compared to valence band. When the electrons acquires sufficient energy from the applied field they can enter different energy states in the conduction band depends

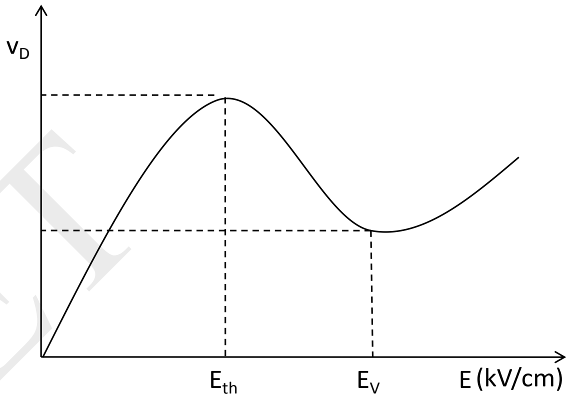
upon their energy compared to the energy of the states. When the electrons energy is lesser than the lower valley energy, they can jump up to the lower valley, but they cannot enter in to the upper valley. This situation is shown in figure 2(a). When the applied electric field is in between E_l and E_u, then the charge carriers will try to jump in to the upper valley. Where E_l is energy of lower valley and E_u is energy of upper valley. As shown in figure 2(b), when $E_l < E < E_u$ some of the electrons will enter from lower valley to upper valley. When the applied electric field is greater than the energy of upper valley, then all the electrons will enter from lower valley to upper valley as shown in figure 2(c).
If the electron densities in the lower and upper valley are 'n_l' and 'n_u' then the conductivity of the n-type GaAs is given by

$$\sigma = e(\mu_l n_l + \mu_u n_u)$$

Where 'e' is the charge of electron, 'μ' is the mobility of electrons. The V-I characteristics and electric field versus drift velocity characteristics are shown in figure 3 below.



Fig(3a): V-I characteristics



Fig(3b): Drift velocity versus electric field

As shown in figure 3a, when the applied electric field is slowly increasing from zero, the electrons will start to enter in to the lower valley of the conduction band and hence the current is linearly increases. The current density (J) also linearly increases. The drift velocity of electrons will increase linearly with the applied electric field. When the applied electric field is greater than the threshold value (E_{th}), the electrons will enter in to the upper valley and their effective mass will be increases and hence the mobility of electrons decreases. Once mobility decreases, the electron drift velocity also decreases. Therefore current density also decreases as shown in figure 3a. Then the device exhibits negative resistance. Therefore, the final conclusion is the device exhibits negative resistance when the charge carriers enter from lower valley to upper valley. After completion of electron transfer from lower valley to upper valley, again the current increases linearly because, the effective mass will be constant as long as the charge carriers present in the upper valley. When the charge carriers transfers from lower valley to upper valley, its mass is changing from one value to another value and hence the current decreases.
On the basis of RWH theory, the band structure of a semiconductor must satisfy the following three criteria in order to exhibit negative resistance.

(i) The separation energy between the bottom of the lower valley and the bottom of the upper valley must be several times larger than the thermal energy (about 0.026 eV at room temperature). That means $\Delta E > kT$ or $\Delta E > 0.026$ eV.

(ii) The separation energy between the valleys must be smaller than the energy gap between the conduction band and valance band. That is $\Delta E < E_g$. Otherwise the semiconductor will breakdown and become highly conductive before electron begin to transfer to the upper valley because hole-electron pair formation is created.

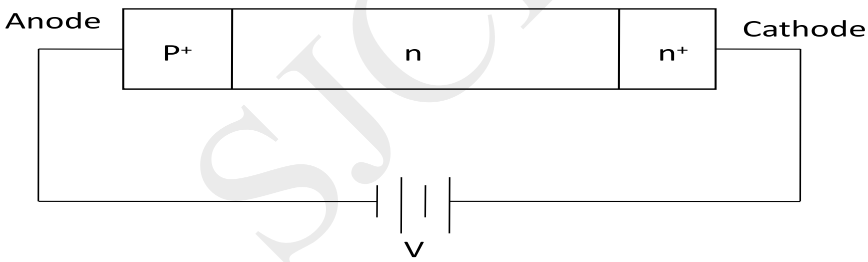
Electrons in the lower valley must have high mobility, small effective mass and low density of states where as the electrons in the upper valley must have low mobility, large effective mass and high density of states. In other words electron velocities must be larger in the lower valley than in the upper valley.

IMPATT diode

It is possible to make a microwave diode exhibit negative resistance by having delay between voltage and current in an avalanche together with transit time through the material. Such devices are called Avalanche transit time devices. They use carrier impact ionization to produce negative resistance at microwave frequencies. There are three distinct modes of avalanche oscillators.

(i) IMPATT: Impact Ionization Avalanche Transit Time device.
(ii) TRAPATT: Trapped Plasma Avalanche Triggered Transit devices.
(iii)BARITT: Barrier Injected Transit Time device.

IMPATT Diode: The abbreviation of IMPATT is Impact Ionization Avalanche Transit Time. The negative resistance can also be defined as that property of a device which causes the current through it to be 180^0 out of phase with the voltage across it. This is the kind of negative resistance exhibited by IMPATT diode i.e., if we show the voltage and current have a 180^0 phase difference, then negative resistance in IMPATT diode is proved. The schematic diagram of IMPATT diode is shown in figure 1 below.



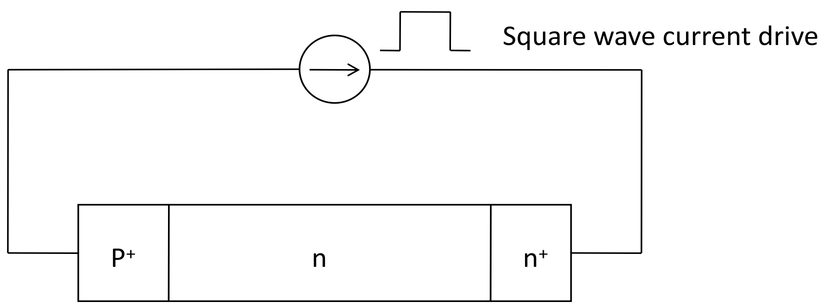
Fig(1): Schematic diagram of IMPATT diode

An extremely a high voltage is applied to the IMPATT diode eventually resulting in a very high current. A normal diode would very quickly breakdown under these conditions but IMPATT diode constructed such that it will withstand these conditions repeatedly. The reverse bias applied to the diode will cause to flow the minority carriers in the diode. Due to application of high voltage, these minority carriers will generate some additional electrons and holes by knocking them out of the crystal structures by so called Impact ionization. These additional carriers continue the process at the junction and it now snowballs into an avalanche. Due to this the avalanche current multiplication will be takes place.

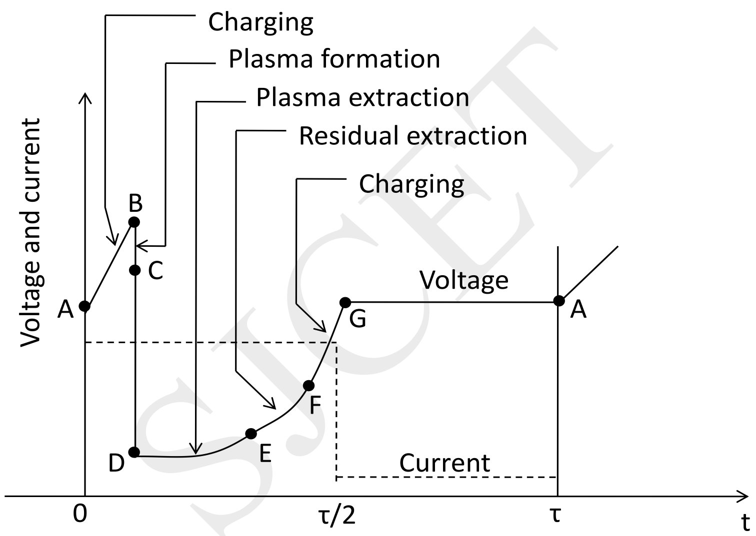
TRAPATT diode

The abbreviation TRAPATT stands for Trapped Plasma Avalanche Triggered Transit mode. The schematic structure and voltage & current waveforms of a TRAPATT diode is shown in figure 2 below. TRAPATT diode is a high efficiency microwave generator capable of operating from several hundred MHz to several GHz. The basic operation of the oscillator is semiconductor pn junction diode reverse biased to current densities well in excess of those encountered in normal avalanche operation. The operation of TRAPATT diode is explained as follows:

At Point ‘A’ the electric field is uniform throughout the sample and its magnitude is large but less than the required for avalanche breakdown. At ‘A’, the diode current is turned on.



Fig(2a): Schematic diagram of TRAPATT diode



Fig(2b): Voltage and current waveforms

Since the only charge carriers present are those caused by the thermal generation, the diode initially charge up like a linear capacitor, driving the magnitude of the electric field above the breakdown voltage. When the sufficient number of carriers is generated, the particle current exceeds the external current and the electric field is depressed throughout the depletion region, causing the voltage to decrease. This portion of the cycle is shown by the curve from point ‘B’ to point ‘C’. During this time interval the electric field is sufficiently large for the avalanche to continue, and dense plasma of electrons and holes is created. As some of the electrons and holes drift out of the ends of the depletion layer, the field is further depressed and “traps” the remaining plasma. The voltage decreases to point ‘D’.

A long time is required to remove the plasma because the total plasma charge is large compared to the charge per unit time in the external circuit. At point ‘E’ the plasma is removed, but a residual charge of electrons remains in one end of the depletion layer and a residual charge of holes in the other end. At point ‘F’ all the charge that was generated internally has been removed. This charge must be greater than or equal to that supplied by the external current; otherwise the voltage will exceed that at point 'A'. from point 'F' to point 'G' the diode charge up again like a fixed capacitor. At point 'G' the diode current goes to zero for half a period and the voltage remains constant at V_A until the current comes back on and the cycle repeats.

SOURCES OF ERRORS

Any measured quantity has a margin of error. For example, the complete value for the gain of an antenna might be 15 dbi±0.5dB indicating a half decibel uncertainty. Various errors and their sources are described here under

(a) Phase error and amplitude taper due to finite measurement distance:

Due to the finite measurement distance between the two antennas, phase error and amplitude
error will occur. Consider a radiation pattern of source antenna shown the figure below.

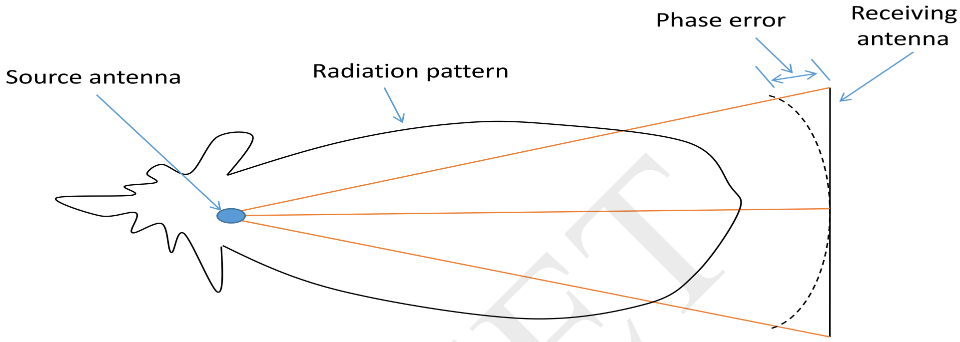


Fig: Phase error and amplitude taper

Due to the particular shape of the radiation pattern, it will modulate the amplitude of the signal reached the receiving antenna. Also all the incident waves will not have the same phase.

(b) Reflections:

Due to presence of obstacles (examples like trees, buildings, towers, etc) between the two antennas, some waves will undergo reflections, scattering, diffraction, shadowing, etc. The reflections are especially harmful in the measurement of low side lobes. A small reflection coupled to the AUT through the main lobe may completely mask the direct wave coupled through the side lobes.

(c) Other sources of error:

(i) Coupling to the reactive near field may be significant at low frequencies.
(ii) There is a scope for the alignment errors due to careless alignment of the source antenna.
(iii) Man-made interfering signals may couple to the sensitive receiver especially in outdoor ranges.
(iv) Due to large measurement distance between the two antennas, atmospheric effects such as scintillation and multipath propagation will be present.
(v) Incorrect use of cable such as cables with insufficient shielding, and unbalanced transmission lines may cause errors.
(vi) Impedance mismatch between the instruments and antennas may cause errors in the gain measurements.
(vii) Imperfections of the transmitter, receiver and positioner cause measurement errors.

PATTERNS TO BE MEASURED

In case of antenna measurements, two patterns to be measured are E-plane pattern and H-plane pattern. These two patters are shown in the figure below.

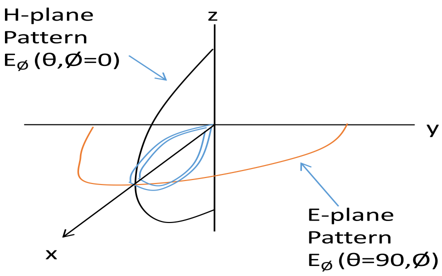


Fig: Patterns for horizontal antenna

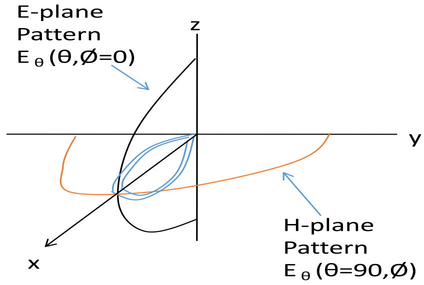


Fig: Patterns for vertical antenna

For Horizontal antennas,

(i) The φ-component of electric field (horizontal) is measured as a function of φ in the xy-plane ($\theta=90^0$). It is represented as $E_\varphi(\theta=90^0, \varphi)$ and is called as E-plane pattern.

(ii) The φ-component of electric field is measured as a function of θ in the xz-plane ($\varphi=0^0$). It is represented as $E_\varphi(\theta, \varphi=0^0)$ and is called as the H-plane pattern.

For Vertical antennas,

(i) The θ-component of electric field is measured as a function of φ in the xy-plane ($\theta=90^0$). It is represented as $E_\theta(\theta=90^0, \varphi)$ and is called as H-plane pattern.

(ii) The θ-component of electric field is measured as a function of φ in the xz-plane($\varphi=0^0$). It is represented as $E_\theta(\theta, \varphi=0^0)$ and is called as the E-plane pattern.

PATTERN MEASUREMENT ARRANGEMENT

The arrangement for measurement of radiation pattern is shown in the figure below. It contains transmitting antenna primary antenna, Antenna Under Test (AUT) called secondary antenna, mount for rotating the primary antenna, detector or receiver and indicator. The primary antenna will radiate the signal towards the secondary antenna. The secondary antenna will be rotated with the help of antenna drive unit. The indicator will be used to indicate or to measure the relative magnitude of the received field. There are two requirements for conducting the experiments with the above arrangement such as distance requirement and uniform illumination requirement.

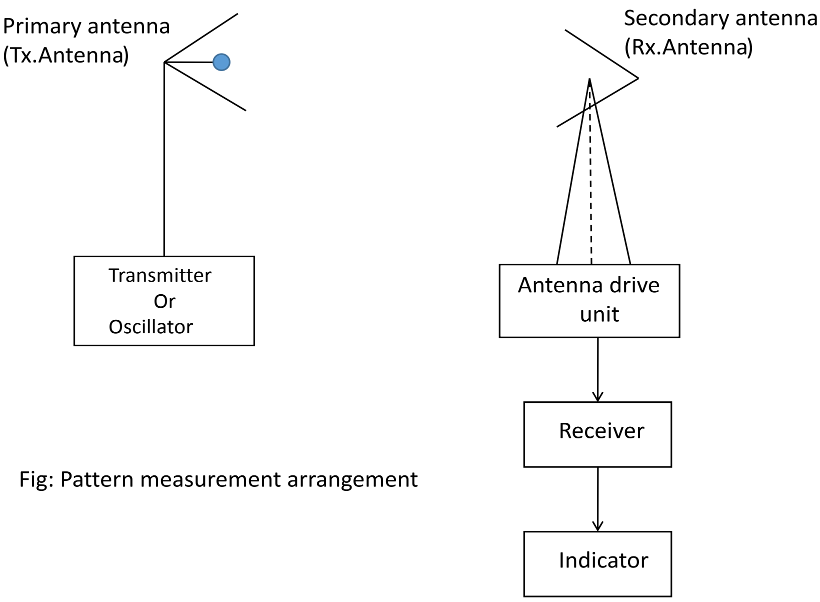


Fig: Pattern measurement arrangement

The distance between the two antennas must be related to the following equation

$$r \geq \frac{2d^2}{\lambda}$$

Or

$$r = \frac{d^2}{8\delta}$$

Where d is the maximum linear dimension of the either antenna, λ is the wavelength and δ is phase difference error.
The other requirement for an accurate field pattern measurement is, the primary antennas should produce a plane of wave of uniform amplitude and phase over the distance r.

DIRECTIVITY MEASUREMENT

The directivity of antenna is defined as

$$Directivity(D) = \frac{Maximum\ radiation\ intensity}{Average\ radiaiton\ intensity}$$

$$D = \frac{4\pi \times Maximum\ radiaiton\ intensity}{\int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} Radiaiton\ intensity \times \sin\theta d\varphi}$$

Form the above equation the directivity can be determined knowing the radiation intensity with the help of radiation pattern. In the above equation the integral part in the denominator can be determined using any one of the following two methods:

Orange slice method:

In this method, set of patterns is obtained by measuring the radiation intensity versus θ for a discrete value of φ. Then each pattern is multiplied continuously by sinθ weight factor and then summed together.

Conical cut method:

In this method, set of patterns is obtained by measuring the radiation intensity versus φ for a discrete value of θ. Then each pattern is multiplied continuously by sinθ weight factor and then summed together.

GAIN MEASUREMENT

Gain measurement by using comparison method:

The set up require for the measurement of gain of the antenna by using the comparison method is
shown in the figure below

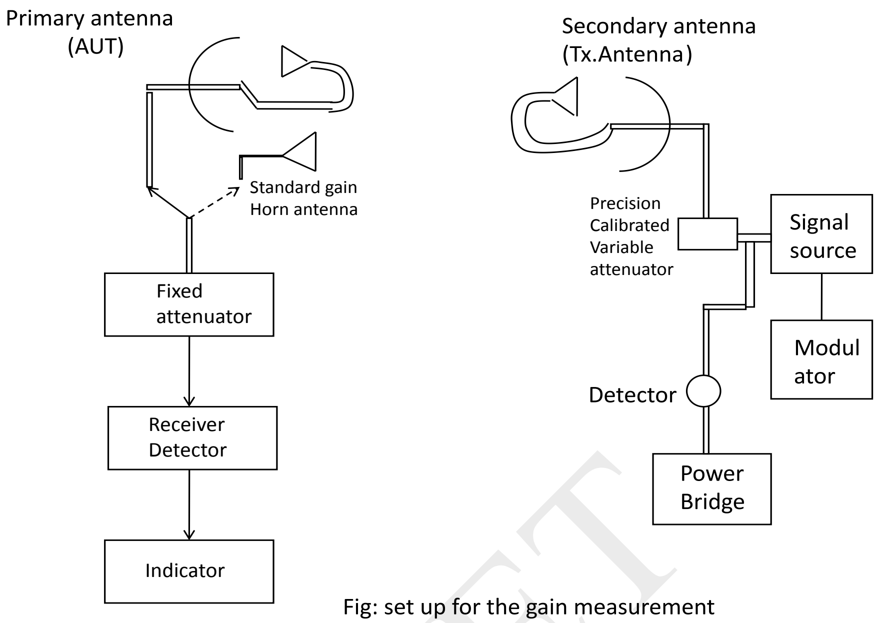


Fig: set up for the gain measurement

The transmitting section contains, signal source, modulator, precision calibrated variable attenuator, detector, Power Bridge and arbitrary transmitting antenna. The modulator modulates the signal generated by the signal source. This modulated signal is transmitted towards the receiving antenna. The precision calibrated variable attenuator is used to adjust the power to the required level. The power bridge is used to sense the variations in the frequency of operation if any during the experiment. The receiver section contains two antennas such as antenna under test (AUT) and standard gain horn antenna (reference antenna), fixed attenuator, receiver detector and indicator. The purpose of fixed attenuator is, to avoid the impedance mismatch between the two antennas and the receiver. The experimental procedure is explained as follows:

(i) Connect the standard horn antenna to the receiver with the help of switch‘s’ and orient the antenna towards the transmitting antenna.

(ii) Adjust the input to the transmitting antenna to a convenient level with the help of precision calibrated variable attenuator.

(iii) Note down the attenuator dial setting and let it be W_1.

(iv) Note down the reading of Power Bridge and let it be P_1.

(v) Now replace the standard horn antenna with AUT (antenna under test).

(vi) Again adjust the precision calibrated variable attenuator such that, the receiver indicates the same previous reading as was with horn antenna.

(vii) Again note down the readings of variable attenuator and power Bridge and let it be W_2 and P_2 respectively.

(viii) When $P_1 = P_2$, then calculate the gain by using the formula

$$Gain(G) = \frac{W_2}{W_1}$$

Or gain in dB is $G(dB) = W_2(dB) - W_1(dB)$

(ix) When $P_1 \neq P_2$, then calculate the gain by using the formula

$$G = \frac{W_2}{W_1} \times \frac{P_1}{P_2} = G_p \times P$$

Or gain in dB is $G(dB) = G_p(dB) + P(dB)$

Gain Measurement by using Absolute method:

The experimental set up for the measurement of gain by using the absolute method is shown the figure below.

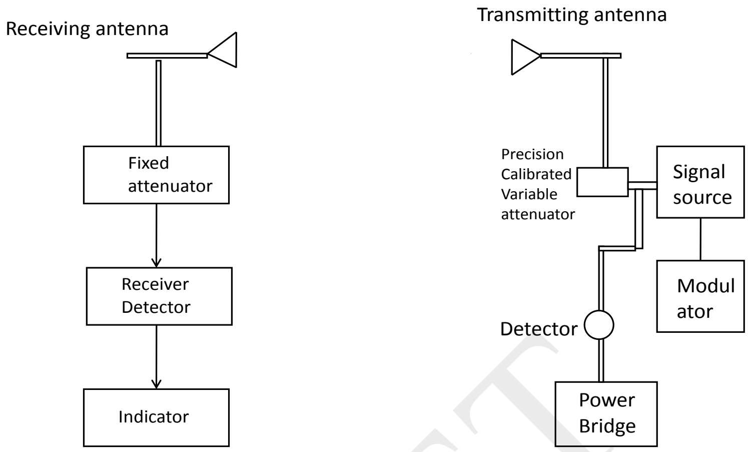


Fig: set up for the gain measurement

The transmitting section contains, signal source, modulator, precision calibrated variable attenuator, detector, Power Bridge and transmitting antenna. The modulator modulates the signal generated by the signal source. This modulated signal is transmitted towards the receiving antenna. The precision calibrated variable attenuator is used to adjust the power to the required level. The power bridge is used to sense the variations in the frequency of operation if any during the experiment. The receiver section contains receiving antenna, attenuator pad or fixed attenuator, receiver detector and indicator. In this method, the two antennas (transmitting and receiving) must be identical. The experimental procedure is explained as follows:

(i) Transmit the power with transmitting antenna towards the receiving antenna with the help of signal source and let this transmitted power is P_T.

(ii) Note down the power received with the receiving antenna with the help of receiver and indicator and let it be P_R.

(iii) Calculate the value of gain by using the Friiss transmission equation given by

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi R}\right)^2$$

Where P_R = Power received

P_T = Power transmitted

G_T = Gain of the transmitting antenna

G_R = Gain of the receiving antenna

R = Distance between the two antennas

λ = Wavelength

Since the two antennas (transmitting and receiving) are identical, $G_T = G_R = G$

Therefore the gain (G) is given by

$$G = \frac{4\pi R}{\lambda}\sqrt{\frac{P_R}{P_T}}$$

Gain Measurement by using 3-Antenna Method:

When two identical antennas are not available, then three antenna method will be used to
measure the gain of the antenna. In three antenna method, the gain will be measured by using three different antennas. The experimental set up for the measurement of gain by using three antenna method is shown in the figure below. In this method, three equations will be obtained with the help of three antennas. Let the three antennas are A_1, A_2 and A_3. Three equations will be obtained as follows:

Case-I (Antenna A_1 is transmitting and antenna A_2 is receiving):

By using the friiss transmission equation the, the relation between the received power and transmitted power is given by

$$P_{R2} = P_{T1}G_{T1}G_{R2}\left(\frac{\lambda}{4\pi R}\right)^2 \qquad -- 1$$

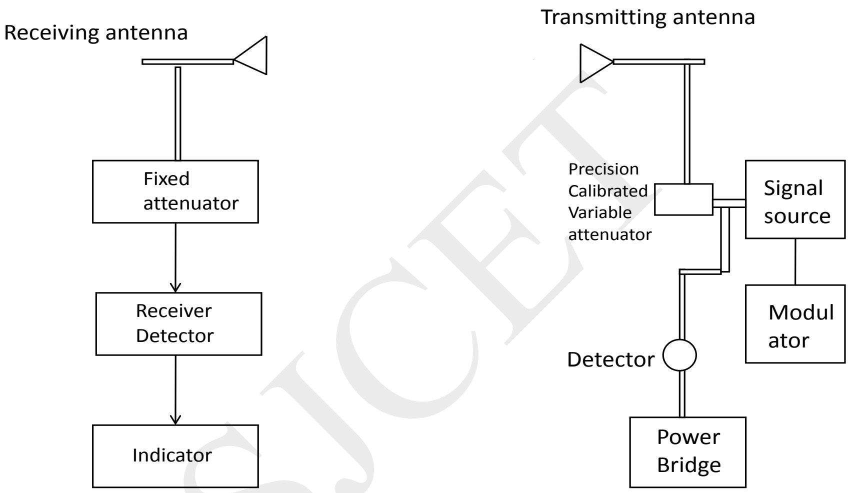


Fig: set up for the gain measurement

Where P_{T1} is the power transmitted by antenna A_1, P_{R2} is the power received by the antenna A_2, G_{T1} is the gain of the antenna A_1, G_{R2} is the gain of the receiving antenna A_2 and R is the distance between the transmitting and receiving antenna.

Case-II (Antenna A_2 is transmitting and antenna A_3 is receiving):

By using the friiss transmission equation the, the relation between the received power and transmitted power is given by

$$P_{R3} = P_{T2}G_{T2}G_{R3}\left(\frac{\lambda}{4\pi R}\right)^2 \qquad -- 2$$

Where P_{T2} is the power transmitted by antenna A_2, P_{R3} is the power received by the antenna A_3, G_{T2} is the gain of the antenna A_2, G_{R3} is the gain of the receiving antenna A_3 and R is the distance between the transmitting and receiving antenna.

Case-III (Antenna A_3 is transmitting and antenna A_1 is receiving):

By using the friiss transmission equation the, the relation between the received power and transmitted power is given by

$$P_{R1} = P_{T3}G_{T3}G_{R1}\left(\frac{\lambda}{4\pi R}\right)^2 \qquad -- 3$$

Where P_{T3} is the power transmitted by antenna A_3, P_{R1} is the power received by the antenna A_1, G_{T3} is the gain of the antenna A_3, G_{R1} is the gain of the receiving antenna A_1 and R is the distance between the transmitting and receiving antenna.
But

$$G_{T1} = G_{R1} = G_1$$
$$G_{T2} = G_{R2} = G_2$$
$$G_{T3} = G_{R3} = G_3$$

Then the above three equations (1, 2, 3) becomes

$$P_{R2} = P_{T1}G_1G_2\left(\frac{\lambda}{4\pi R}\right)^2 \qquad -- 4$$
$$P_{R3} = P_{T2}G_2G_3\left(\frac{\lambda}{4\pi R}\right)^2 \qquad -- 5$$
$$P_{R1} = P_{T3}G_3G_1\left(\frac{\lambda}{4\pi R}\right)^2 \qquad -- 6$$

Equations 4, 5 and 6 consist of three unknowns such as G_1, G_2, and G_3. By solving these three equations we can obtained the gain of any antenna (either the gain of A_1 or A_2 or A_3).

<u>Description of Microwave bench-different blocks and their features</u>

The basic microwave bench setup for measuring the various microwave parameters is shown in the following figure.

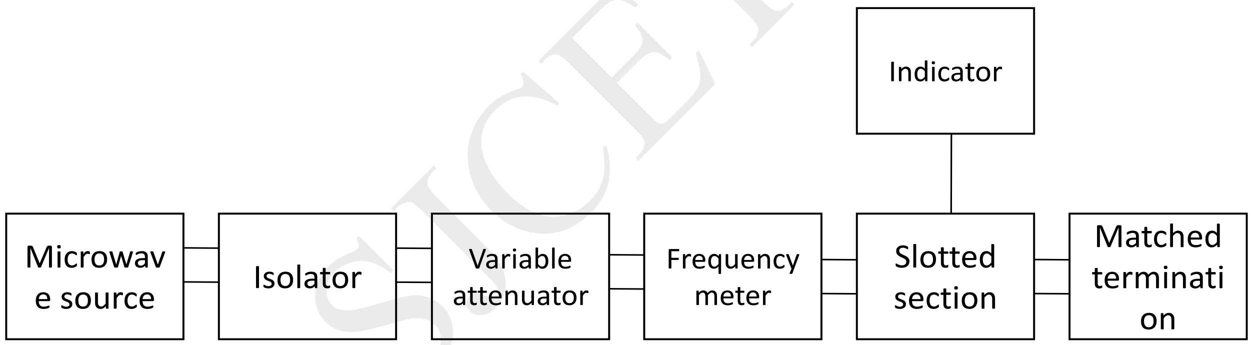


Fig: Microwave bench setup

The function of each and every block in the bench setup is explained as follows:
Microwave source is a oscillator which will generate the microwave signal of required frequency. Examples of microwave oscillators are Reflex klystron oscillator, Magnetron oscillator, Gunn oscillator, etc. The function of isolator is to avoid the reflections not to reaching the source due to mismatch of the load. Variable attenuator will be used to set the required value of the signal strength for a particular experiment. Frequency meter will be used to measure the frequency of the signal flowing through the bench setup. Slotted section will be used to measure the very important parameters such as V_{max} , V_{min} , VSWR, reflection coefficient, guide wavelength, distance of V_{max}, and V_{min}, etc. Matched termination is matched load connected to the second port of the slotted section. Indicator is a either CRO or VSWR meter or some other device which will be used to read the values to measured.

Errors and precautions

While conducting the microwave measurements, we must follow certain precautions otherwise measurements errors will occur. The major precautions to be followed while measuring the microwave parameters are:

1) When the microwave source is a reflex klystron oscillator, then the reflex klystron power supply must be properly checked such that the beam voltage should be in minimum position and repeller voltage must be in maximum position. After switch on the power supply we need to wait until getting the stable value of beam current by setting the particular value of beam voltage. Cooling fan must be used. The bench setup must be horizontal otherwise alignment errors will occur.

2) When the microwave source is a Gunn diode, then it must be operated in the negative resistance region. While measuring the values in the negative resistance region, the values must be note down as early as possible.

Microwave power measurement-Bolometers

The Bolometer is temperature sensitive device. There are two types of bolometers such as barretters and thermistors. Barretters are positive temperature coefficient device and thermistor is a negative temperature coefficient device. Positive temperature coefficient means the resistance of a device will increase with increase in temperature where as negative temperature coefficient means the resistance of a device will decrease with temperature. The following figure represents the measurement of microwave power.

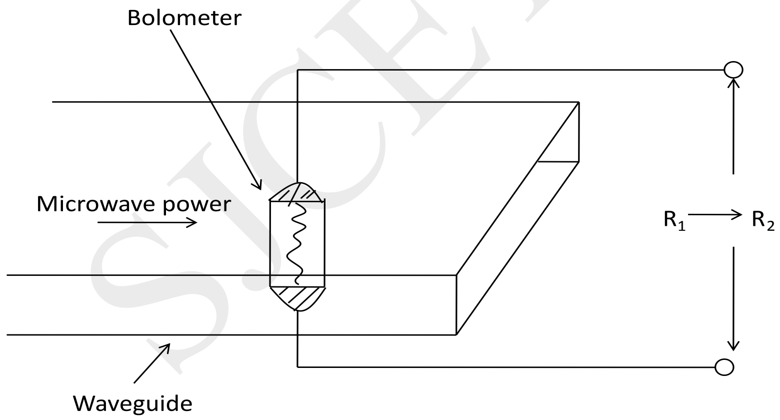


Fig: Basic principle of microwave power measurement using bolometer

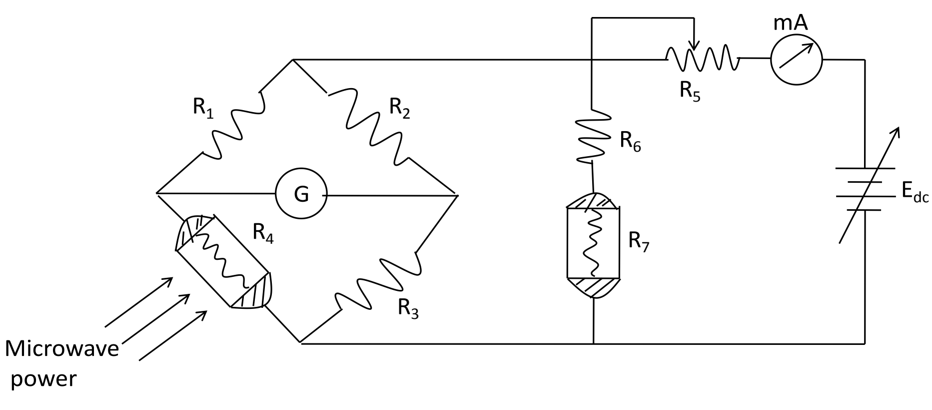


Fig: Measurement of microwave power

The microwave power to be measured must be applied to the bolometer. Then the power will be absorbed by the bolometer resistance and it dissipates this power in the form of heat. Due to this heat, the surrounding temperature will be changed. Therefore, the resistance of bolometer changes. The difference between the resistances of bolometer before application of power (R_1) and after application of power (R_2) will be proportional to the power to be measured. The second figure shows the bridge method of power measurement in which the bolometer itself act as the one of the arm. The procedure for measuring the power by using the second figure is as follows:

(i) Adjust the resistance R_5 to get the balance condition in the bridge and note down the d.c. voltage and let it be E_1

(ii) Apply the microwave power to be measured to the bolometer and again adjust the resistance R_5 to get the balance condition in the bridge and note down the d.c.voltage. let it be E_2

(iii) The difference between the two d.c. voltages will gives the microwave power to be measured.

Measurement of attenuation

Attenuation measurement using Power ratio method:

The bench setup for the measurement of attenuation using the power ratio method is shown in the following figure. The procedure for attenuation measurement using the power ratio method will be explained as follows:

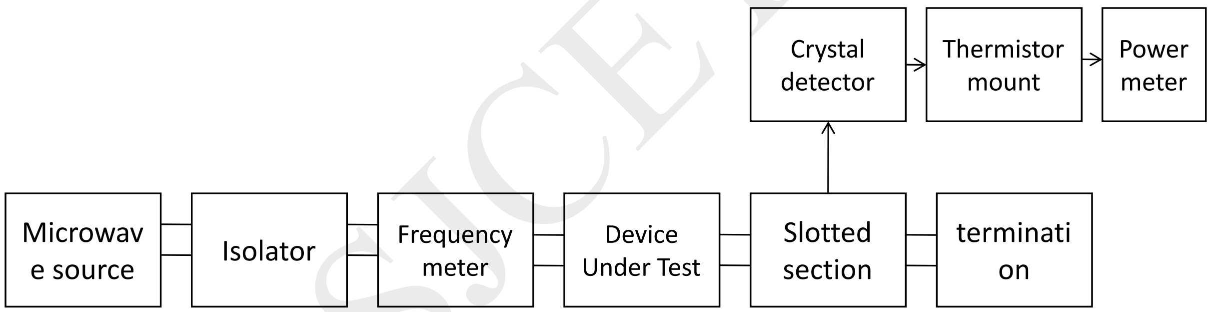


Fig: Microwave bench setup for attenuation measurement

(i) Measure the power with the help of bench setup without connecting the DUT (Device Under Test) and let it be $P_{in.}$

(ii) Without disturbing the bench setup, Connect the DUT and measure the power with the help of power meter and let it be P_{out}.

(iii) Calculate the attenuation by using the formula
Attenuation = $10 \log(P_{in}/P_{out})$

Attenuation measurement using RF substitution method:

The bench setup for the measurement of attenuation using the RF substitution method is shown in the following figure.

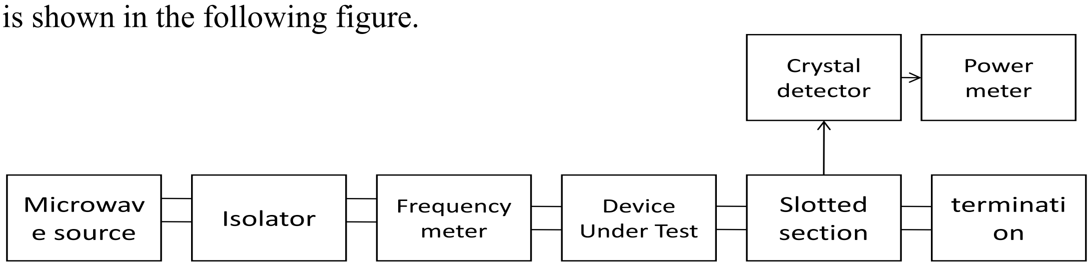


Fig: Microwave bench setup for attenuation measurement

The procedure for attenuation measurement using the power ratio method will be explained as follows:

(i) Measure the power by connecting the DUT and let it be P watts
(ii) Replace the DUT with precession calibrated variable attenuator.
(iii) Adjust the precession calibrated variable attenuator to read the same power as it was obtained with DUT.
(iv) The value on the variable attenuator gives the attenuation of the DUT.

Frequency measurement

The bench setup for the frequency measurement using the frequency meter or wave meter is shown in the following figure. The procedure for the measurement of frequency by using the frequency meter is as follows:

(i) Set up the signal in the bench setup by properly adjusting the power supply and variable attenuator.
(ii) Rotate the frequency meter until getting the dip (suddenly the signal in the indicator will reduces to zero) in the signal of indicating meter.
(iii) Directly note down the values of frequency from the frequency meter.

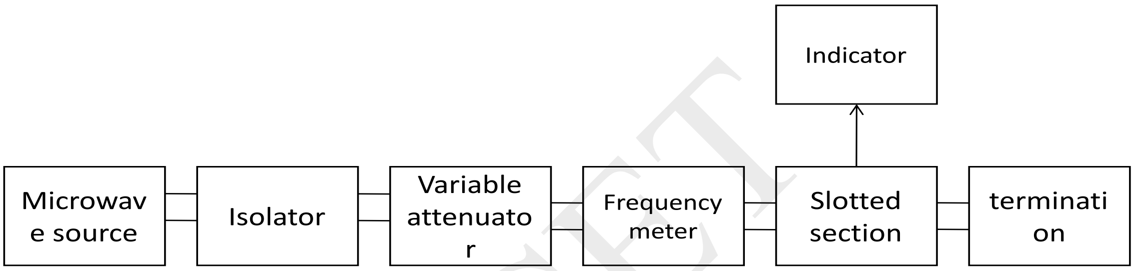


Fig: Microwave bench setup for frequency measurement

Standing wave measurements –measurement of low and high VSWR

To measure the low VSWR values, the bench setup is shown in the following figure.

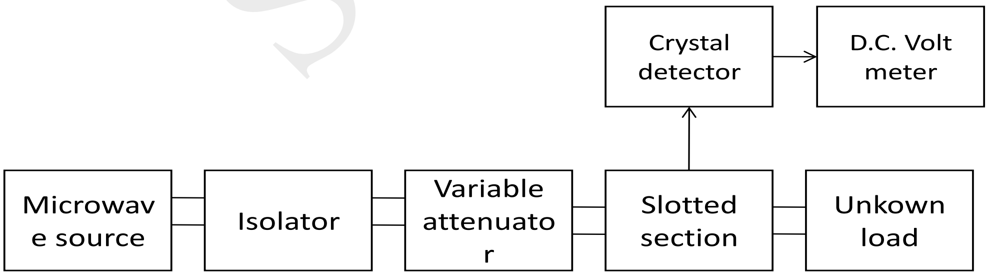


Fig: Microwave bench setup for measurement of low VSWR

The procedure for measurement of low VSWR is as follows:

(i) Set up the signal in the bench setup by properly adjusting the power supply and variable attenuator.
(ii) Measure the maximum voltage(V_{max}) and minimum voltage (Vmin) by moving the probe carriage of slotted section.
(iii) Calculate the VSWR by using the formula

$$VSWR = V_{max}/V_{min}$$

Measurement of high VSWR:

To measure the high VSWR values, the bench setup is shown in the following figure. The method used to measure the high VSWR is called as the double minima method.

The procedure for measuring the high VSWR is as follows:

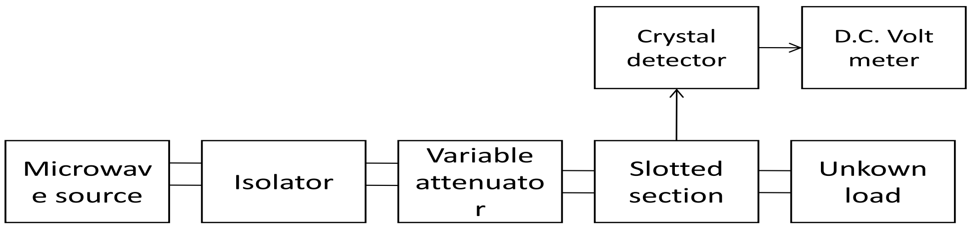
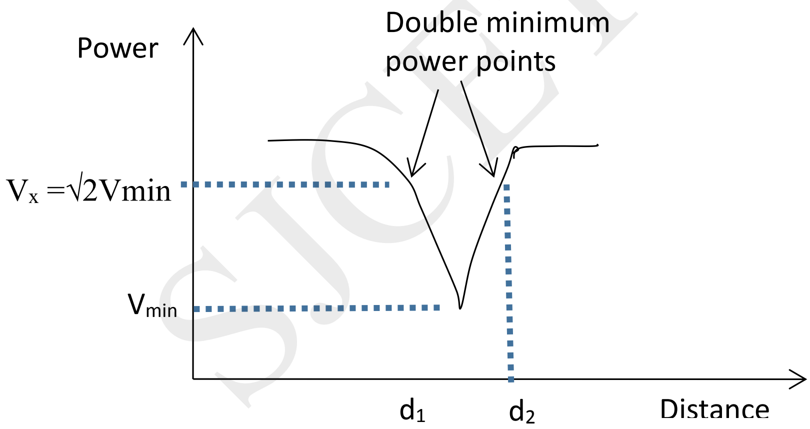


Fig: Microwave bench setup for measurement of low VSWR

(i) Set up the signal in the bench setup by properly adjusting the power supply and variable attenuator.
(ii) Locate the minima by adjusting the probe carriage of slotted section.
(iii) Find out the distance at which the power is twice the minimum value on both sides of the minima point as shown in the following figure.



(iv) Calculate the guide wavelength(λ_g)
(v) Calculate the VSWR by using the following formula

$$VSWR = \lambda_g/\pi(d_2-d_1)$$

<u>Measurement of Cavity-Q</u>

The Bench setup for the measurement of Q-factor or Quality factor of a cavity resonator is shown in the following figure. The method used for the measurement of Q-factor is called transmission method.

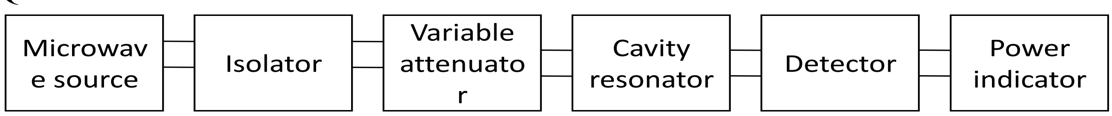


Fig: Microwave bench setup for measurement of Q-factor

The procedure for the measurement of Q-factor is as follows:

(i) Set up the signal in the bench setup by properly adjusting the power supply and variable attenuator.

(ii) Vary the frequency of the microwave signal and note down the power output for various values of frequency.
(iii) Draw the graph between the power output versus frequency
(iv) From the graph calculate the 3 dB bandwidth.
(v) Calculate the Q-factor by using the following formula

$$Q = 1/\text{Bandwidth}$$

Impedance measurements

Impedance measurement by using the slotted line method:

The bench setup for the impedance measurement by using the slotted line method is shown in the following figure.

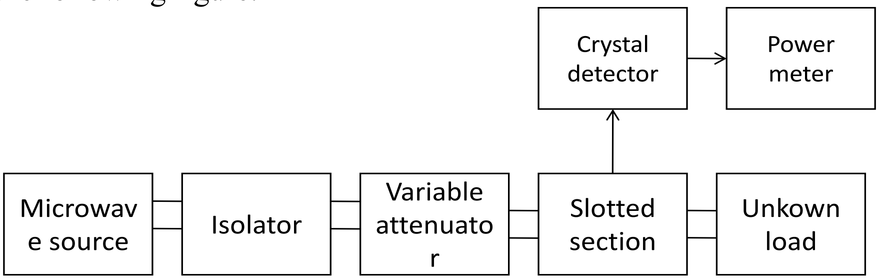


Fig: Microwave bench setup for measurement of impedance using slotted line

The procedure for the measurement of impedance is as follows:

(i) Set up the signal in the bench setup by properly adjusting the power supply and variable attenuator.
(ii) Connect the unknown load whose impedance is to be measured and locate the minima with the help of slotted line
(iii) Replace the load with movable short and again locate the minima
(iv) Observe the shift in the minima. If the minima is shifted to left side, then the load is inductive and if the shift in minima is right side then the load is capacitive.
(v) Findout the VSWR of the load using the VSWR measurement procedure.
(vi) Calculate the impedance by using the smith chart.

Impedance measurement by using the Reflectometer method

The bench setup for the impedance measurement by using the reflectometer method is shown in the following figure.

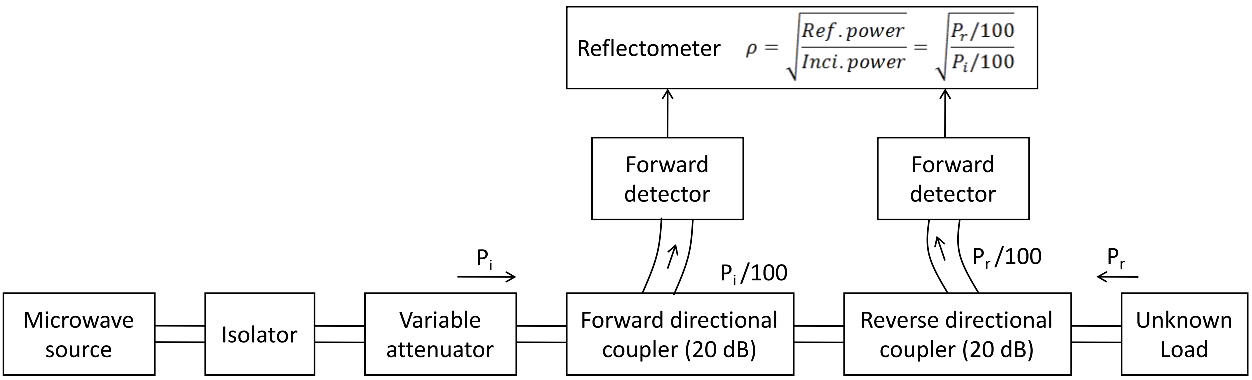


Fig: Measurement of impedance using reflectometer.

The procedure for the measurement of impedance is as follows:

(i) Make the arrangements as per the diagram shown in the bench setup.

(ii) Set up the signal in the bench setup by properly adjusting the power supply and variable attenuator.

(iii) The forward directional coupler will divert some the incident signal towards the reflectometer and reverse directional coupler will divert the some of the reflected signal towards the reflectometer. The reflectometer is meter which will directly gives the reflection coefficient values from the incident and reflected powers.

(iv) Note down the value of reflection coefficient(ρ).

(v) Calculate the value of the impedance(Z_L) of the unknown load by using the following formula

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0}$$